Key Stage 3 mathematics: mastery professional development materials – overall structure and mapping to the national curriculum

Teaching for mastery is teaching that aims for deep and sustainable learning; learning that is rooted in an appreciation of the connectedness of mathematical ideas and based on an understanding of the underlying structures. It emphasises the need to go beyond being able to memorise facts and practise procedures and routines.

Such teaching requires us to 'look through' the national curriculum statements of content and descriptions of what students need to be able to do. We must discern what students need to be aware of and understand in order to do these things fluently. These materials therefore offer a more 'fine grained' description of the key themes and big ideas of the curriculum by detailing:
- six broad mathematical themes
- a number of core concepts within each theme
- a set of 'knowledge, skill and understanding' statements within each core concept
- a collection of focused key ideas within each statement of knowledge, skill and understanding.

The diagram on page 2 and Table 1 on pages 3–13 detail the complete Key Stage 3 curriculum structure that forms the foundation for the NCETM secondary mastery professional development materials.

Please note: Numbering of themes, core concepts, 'knowledge, skills and understanding' statements, and key ideas is for ease of reference only. Whilst the numbering represents one possible teaching order, it is not intended to be prescriptive. Each guidance document details the prior learning required so that the sequencing can be adapted to fit your own scheme.

Table 2 on pages 14–20 indicates where the national curriculum Key Stage 3 mathematics programme of study statements are covered, followed by related endnotes.
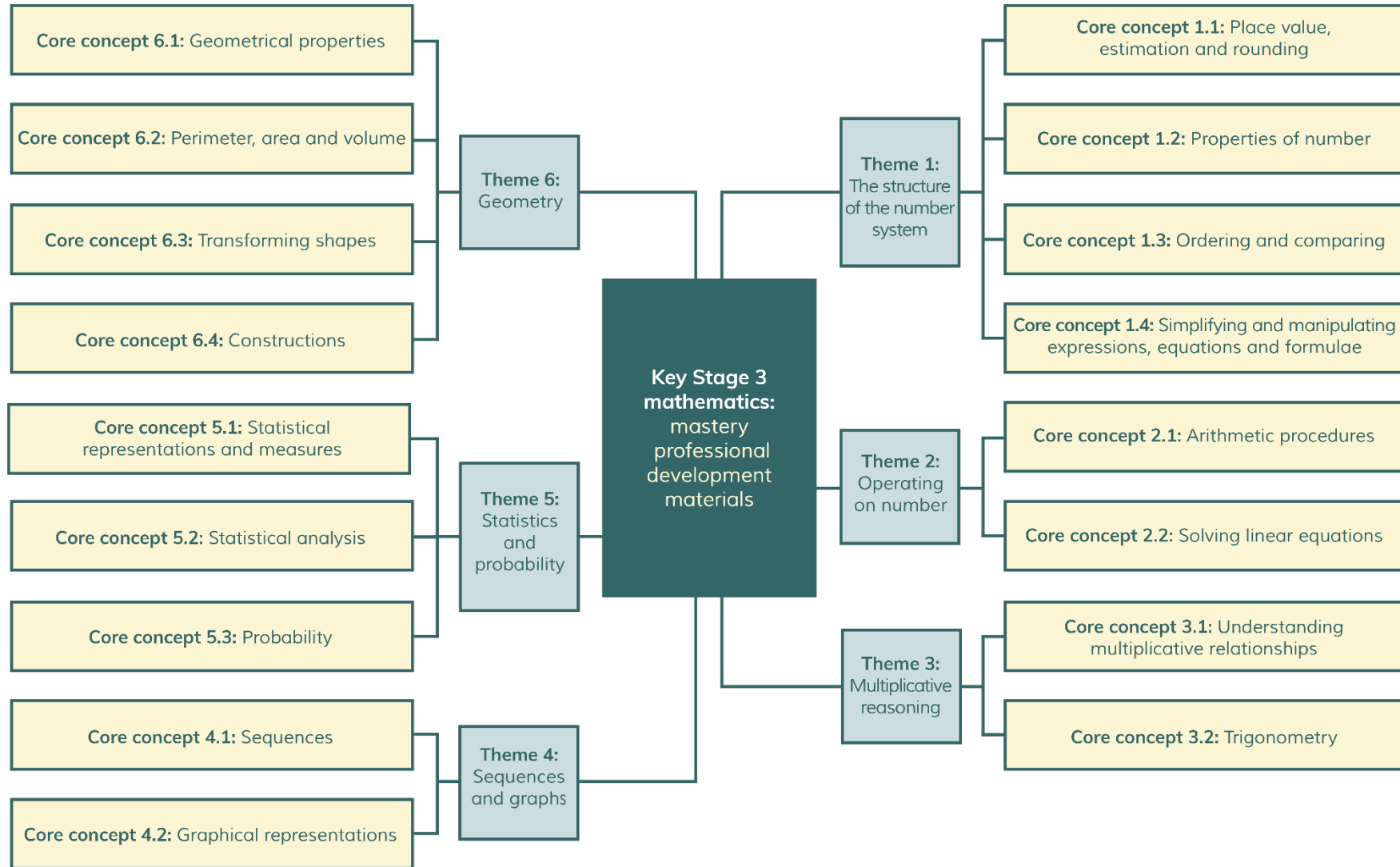
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| Core concept 6.1: Geometrical properties |
| Core concept 6.2: Perimeter, area and volume |
| Core concept 6.3: Transforming shapes |
| Core concept 6.4: Constructions |

Theme 6: Geometry

| Core concept 5.1: Statistical representations and measures |
| Core concept 5.2: Statistical analysis |
| Core concept 5.3: Probability |

Theme 5: Statistics and probability

Key Stage 3 mathematics: mastery professional development materials

Theme 1: The structure of the number system

| Core concept 1.1: Place value, estimation and rounding |
| Core concept 1.2: Properties of number |
| Core concept 1.3: Ordering and comparing |
| Core concept 1.4: Simplifying and manipulating expressions, equations and formulae |

Theme 2: Operating on number

| Core concept 2.1: Arithmetic procedures |
| Core concept 2.2: Solving linear equations |

Theme 3: Multiplicative reasoning

| Core concept 3.1: Understanding multiplicative relationships |
| Core concept 3.2: Trigonometry |

| Core concept 4.1: Sequences |
| Core concept 4.2: Graphical representations |

Theme 4: Sequences and graphs

Table 1: NCETM secondary mastery professional development materials: Key Stage 3 curriculum structure

Core concepts		'Knowledge, skills and understanding' statements		Key ideas (* = key ideas exemplified in guidance documents)	
1.1	Place value, estimation and rounding	**1.1.1**	Understand the value of digits in decimals, measure and integers	**1.1.1.1**	Understand place value in integers
				1.1.1.2*	Understand place value in decimals, including recognising exponent and fractional representations of the column headings
				1.1.1.3	Understand place value in the context of measure
				1.1.1.4	Order and compare numbers and measures using <, >, =
		1.1.2	Round numbers to a required number of decimal places	**1.1.2.1**	Round numbers to up to three decimal places
				1.1.2.2	Round numbers to any number of decimal places
		1.1.3	Round numbers to a required number of significant figures	**1.1.3.1**	Understand the concept of significant figures
				1.1.3.2*	Round integers to a required number of significant figures
				1.1.3.3	Round decimals to a required number of significant figures
		1.1.4	Estimate calculations by rounding	**1.1.4.1**	Understand what is meant by a sensible degree of accuracy
				1.1.4.2*	Estimate numerical calculations
				1.1.4.3	Estimate and check if solutions to problems are of the correct magnitude
				1.1.4.4	Determine whether calculations using rounding will give an underestimate or overestimate
				1.1.4.5	Understand the impact of rounding errors when using a calculator, and the way that these can be compounded to result in large inaccuracies
				1.1.4.6	Calculate possible errors expressed using inequality notation $a < x \leq b$
1.2	Properties of number	**1.2.1**	Understand multiples	**1.2.1.1**	Understand what a multiple is and be able to list multiples of n
				1.2.1.2*	Identify and explain whether a number is or is not a multiple of a given integer
		1.2.2	Understand integer exponents and roots	**1.2.2.1**	Understand the concept of square and cube
				1.2.2.2	Understand the concept of square root and cube root
				1.2.2.3	Understand and use correct notation for positive integer exponents
				1.2.2.4	Understand how to use the keys for squares and other powers and square root on a calculator

Theme 1

		1.2.3	Understand and use the unique prime factorisation of a number	1.2.3.1	Understand what a factor is and be able to identify factors of positive integers
				1.2.3.2	Understand what a prime number is and be able to identify prime numbers
				1.2.3.3	Understand that a positive integer can be written uniquely as a product of its prime factors
				1.2.3.4*	Use the prime factorisation of two or more positive integers to efficiently identify the highest common factor
				1.2.3.5	Use the prime factorisation of two or more positive integers to efficiently find their lowest common multiple
1.3	Ordering and comparing	**1.3.1**	Work interchangeably with terminating decimals and their corresponding fractions	**1.3.1.1**	Understand that 1 can be written in the form $\frac{n}{n}$ (where n is any integer) and vice versa
				1.3.1.2	Understand that fractions of the form $\frac{a}{b}$ where a > b are greater than 1 and use this awareness to convert between improper fractions and mixed numbers
				1.3.1.3*	Understand that a fraction represents a division and that performing that division results in an equivalent decimal
				1.3.1.4	Appreciate that any terminating decimal can be written as a fraction with a denominator of the form 10^n (e.g. 0.56 = $\frac{56}{100}$, $\frac{560}{1000}$, etc.)
				1.3.1.5*	Understand the process of simplifying fractions through dividing both numerator and denominator by common factors
				1.3.1.6	Know how to convert from fractions to decimals and back again using the converter key on a calculator
				1.3.1.7	Know how to enter fractions as divisions on a calculator and understand the limitations of the decimal representation that results
		1.3.2	Compare and order positive and negative integers, decimals and fractions	**1.3.2.1**	Compare negative integers using < and >
				1.3.2.2	Compare decimals using < and >
				1.3.2.3	Compare and order fractions by converting to decimals
				1.3.2.4	Compare and order fractions by converting to fractions with a common denominator
				1.3.2.5	Order a variety of positive and negative fractions and decimals using appropriate methods of conversion and recognising when conversion to a common format is not required
				1.3.2.6	Appreciate that, for any two numbers there is always another number in between them

Theme 1

		1.3.3	Interpret and compare numbers in standard form A × 10^n, 1 ≤ A < 10	1.3.3.1*	Be able to write any integer in a range of forms, e.g. 53 = 5.3 × 10, 530 × $\frac{1}{10}$, 5300 × 0.01, etc.
				1.3.3.2	Understand that very large numbers can be written in the form a × 10^n, (where 1 < a ≤ 10) and appreciate the real-life contexts where this format is usefully used
				1.3.3.3	Understand that very small numbers can be written in the form a × 10^{-n}, (where 1 < a ≤ 10) and appreciate the real-life contexts where this format is usefully used
Theme 1	1.4 Simplifying and manipulating expressions, equations and formulae	1.4.1	Understand and use the conventions and vocabulary of algebra including forming and interpreting algebraic expressions and equations	1.4.1.1	Understand that a letter can be used to represent a generalised number
				1.4.1.2	Understand that algebraic notation follows particular conventions and that following these aids clear communication
				1.4.1.3	Know the meaning of and identify: term, coefficient, factor, product, expression, formula and equation
				1.4.1.4*	Understand and recognise that a letter can be used to represent a specific unknown value or a variable
				1.4.1.5*	Understand that relationships can be generalised using algebraic statements
				1.4.1.6	Understand that substituting particular values into a generalised algebraic statement gives a sense of how the value of the expression changes
		1.4.2	Simplify algebraic expressions by collecting like terms to maintain equivalence	1.4.2.1	Identify like terms in an expression, generalising an understanding of unitising
				1.4.2.2	Simplify expressions by collecting like terms
		1.4.3	Manipulate algebraic expressions using the distributive law to maintain equivalence	1.4.3.1*	Understand how to use the distributive law to multiply an expression by a term such as 3(a + 4b) and 3p^2(2p + 3b)
				1.4.3.2	Understand how to use the distributive law to factorise expressions where there is a common factor, such as 3a + 12b and 6p^3 + 9p^{2}b
				1.4.3.3	Apply understanding of the distributive law to a range of problem-solving situations and contexts (including collecting like terms, multiplying an expression by a single term and factorising), e.g. 10 − 2(3a + 5), 3(a ± 2b) ± 4(2ab ± 6b), etc.
		1.4.4	Find products of binomials	1.4.4.1*	Use the distributive law to find the product of two binomials
				1.4.4.2	Understand and use the special case when the product of two binomials is the difference of two squares
				1.4.4.3	Find more complex binomial products

		1.4.5	Rearrange formulae to change the subject	1.4.5.1* Understand that an additive relationship between variables can be written in a number of different ways
				1.4.5.2 Understand that a multiplicative relationship between variables can be written in a number of different ways
				1.4.5.3 Apply an understanding of inverse operations to a formula in order to make a specific variable the subject (in a wide variety of increasingly complex mix of operations)
Theme 2	2.1 Arithmetic procedures	2.1.1	Understand and use the structures that underpin addition and subtraction strategies	2.1.1.1* Understand the mathematical structures that underpin addition and subtraction of positive and negative integers
				2.1.1.2* Generalise and fluently use written addition and subtraction strategies, including columnar formats, with decimals
		2.1.2	Understand and use the structures that underpin multiplication and division strategies	2.1.2.1* Understand the mathematical structures that underpin multiplication and division of positive and negative integers
				2.1.2.2 Factorise multiples of 10^n in order to simplify multiplication and division of both integers and decimals, e.g. 300 × 7000, 0.3 × 0.007, 0.9 ÷ 0.03, etc.
				2.1.2.3* Generalise and fluently use written multiplication strategies to calculate accurately with decimals
				2.1.2.4 Generalise and fluently use written division strategies to calculate accurately with decimals
		2.1.3	Know, understand and use fluently a range of calculation strategies for addition and subtraction of fractions	2.1.3.1 Understand the mathematical structures that underpin the addition and subtraction of fractions
				2.1.3.2 Generalise and fluently use addition and subtraction strategies to calculate with fractions and mixed numbers
		2.1.4	Know, understand and use fluently a range of calculation strategies for multiplication and division of fractions	2.1.4.1* Understand the mathematical structures that underpin the multiplication of fractions
				2.1.4.2* Understand how to multiply unit, non-unit and improper fractions
				2.1.4.3 Generalise and fluently use strategies to multiply with mixed numbers (e.g. $2\frac{3}{4} \times 1\frac{2}{3}$)
				2.1.4.4 Understand the mathematical structures that underpin the division of fractions
				2.1.4.5 Divide a fraction by a whole number
				2.1.4.6 Divide a whole number by a fraction
				2.1.4.7 Divide a fraction by a fraction

		2.1.5	Use the laws and conventions of arithmetic to calculate efficiently	2.1.5.1	Know the commutative law and use it to calculate efficiently
				2.1.5.2	Know the associative law and use it to calculate efficiently
				2.1.5.3	Know the distributive law and use it to calculate efficiently
				2.1.5.4	Calculate using priority of operations, including brackets, powers, exponents and reciprocals
				2.1.5.5*	Use the associative, distributive and commutative laws to flexibly and efficiently solve problems
				2.1.5.6	Know how to fluently use certain calculator functions and use a calculator appropriately
2.2	Solving linear equations	2.2.1	Understand what is meant by finding a solution to a linear equation with one unknown	2.2.1.1	Recognise that there are many different types of equations of which linear is one type
				2.2.1.2	Understand that in an equation the two sides of the 'equals' sign balance
				2.2.1.3*	Understand that a solution is a value that makes the two sides of an equation balance
				2.2.1.4	Understand that a family of linear equations can all have the same solution
		2.2.2	Solve a linear equation with a single unknown on one side where obtaining the solution requires one step	2.2.2.1	Solve a linear equation requiring a single additive step
				2.2.2.2	Solve a linear equation requiring a single multiplicative step
		2.2.3	Solve a linear equation with a single unknown where obtaining the solution requires two or more steps (no brackets)	2.2.3.1	Understand that an equation needs to be in a format to be 'ready' to be solved, through collecting like terms on each side of the equation
				2.2.3.2	Know that when an additive step and a multiplicative step are required, the order of operations will not affect the solution
				2.2.3.3*	Recognise that equations with unknowns on both sides of the equation can be manipulated so that the unknowns are on one side
				2.2.3.4	Solve complex linear equations, including those involving reciprocals
		2.2.4	Solve efficiently a linear equation with a single unknown involving brackets	2.2.4.1	Appreciate the significance of the bracket in an equation
				2.2.4.2	Recognise that there is more than one way to remove a bracket when solving an equation
				2.2.4.3	Solve equations involving brackets where simplification is necessary first

Theme 2

	3.1	Understanding multiplicative relationships	3.1.1	Understand the concept of multiplicative relationships	3.1.1.1*	Appreciate that any two numbers can be connected via a multiplicative relationship
Theme 3					3.1.1.2	Understand that a multiplicative relationship can be expressed as a ratio and as a fraction
					3.1.1.3	Be able to calculate the multiplier for any given two numbers
					3.1.1.4	Appreciate that there are an infinite number of pairs of numbers for any given multiplicative relationship (equivalence)
			3.1.2	Understand that multiplicative relationships can be represented in a number of ways and connect and move between those different representations	3.1.2.1*	Use a double number line to represent a multiplicative relationship and connect to other known representations
					3.1.2.2*	Understand the language and notation of ratio and use a ratio table to represent a multiplicative relationship and connect to other known representations
					3.1.2.3	Use a graph to represent a multiplicative relationship and connect to other known representations
					3.1.2.4	Use a scaling diagram to represent a multiplicative relationship and connect to other known representations
			3.1.3	Understand that fractions are an example of a multiplicative relationship and apply this understanding to a range of contexts	3.1.3.1	Find a fraction of a given amount
					3.1.3.2	Given a fraction and the result, find the original amount
					3.1.3.3	Express one number as a fraction of another
			3.1.4	Understand that ratios are an example of a multiplicative relationship and apply this understanding to a range of contexts	3.1.4.1	Be able to divide a quantity into a given ratio
					3.1.4.2	Be able to determine the whole, given one part and the ratio
					3.1.4.3*	Be able to determine one part, given the other part and the ratio
					3.1.4.4	Use ratio to describe rates (e.g. exchange rates, conversions, cogs, etc.)
			3.1.5	Understand that percentages are an example of a multiplicative relationship and apply this understanding to a range of contexts	3.1.5.1	Describe one number as a percentage of another
					3.1.5.2	Find a percentage of a quantity using a multiplier
					3.1.5.3*	Calculate percentage changes (increases and decreases)
					3.1.5.4	Calculate the original value, given the final value after a stated percentage increase or decrease
					3.1.5.5	Find the percentage increase or decrease, given start and finish quantities

Theme 3			**3.1.6**	Understand proportionality	**3.1.6.1** Understand the connection between multiplicative relationships and direct proportion
					3.1.6.2 Recognise direct proportion and use in a range of contexts including compound measures
					3.1.6.3 Recognise and use inverse proportionality in a range of contexts
	3.2	Trigonometry	**3.2.1**	Understand the trigonometric functions	**3.2.1.1*** Understand that the trigonometric functions are derived from measurements within a unit circle
					3.2.1.2 Recognise the right-angled triangle within a unit circle and use proportion to scale to similar triangles
					3.2.1.3* Know how the sine, cosine and tangent ratios are derived from the sides of a right-angled triangle
			3.2.2	Use trigonometry to solve problems in a range of contexts	**3.2.2.1** Choose appropriate trigonometric relationships to use to solve problems in right-angled triangles
					3.2.2.2 Use trigonometric ratios to find a missing side in a right-angled triangle
					3.2.2.3 Use trigonometric ratios to find a missing angle in a right-angled triangle
Theme 4	**4.1**	Sequences	**4.1.1**	Understand the features of a sequence	**4.1.1.1*** Appreciate that a sequence is a succession of terms formed according to a rule
					4.1.1.2 Understand that a sequence can be generated and described using term-to-term approaches
					4.1.1.3 Understand that a sequence can be generated and described by a position-to-term rule
			4.1.2	Recognise and describe arithmetic sequences	**4.1.2.1** Understand the features of an arithmetic sequence and be able to recognise one
					4.1.2.2* Understand that any term in an arithmetic sequence can be expressed in terms of its position in the sequence (*n*th term)
					4.1.2.3 Understand that the *n*th term allows for the calculation of any term
					4.1.2.4 Determine whether a number is a term of a given arithmetic sequence
			4.1.3	Recognise and describe other types of sequences (non-arithmetic)	**4.1.3.1** Understand the features of a geometric sequence and be able to recognise one
					4.1.3.2 Understand the features of special number sequences, such as square, triangle and cube, and be able to recognise one
					4.1.3.3 Appreciate that there are other number sequences
	4.2	Graphical representations	**4.2.1**	Connect coordinates, equations and graphs	**4.2.1.1** Describe and plot coordinates, including non-integer values, in all four quadrants
					4.2.1.2 Solve a range of problems involving coordinates
					4.2.1.3* Know that a set of coordinates, constructed according to a mathematical rule, can be represented algebraically and graphically

Theme 4			**4.2.1.4**	Understand that a graphical representation shows all of the points (within a range) that satisfy a relationship
		4.2.2 Explore linear relationships	**4.2.2.1**	Recognise that linear relationships have particular algebraic and graphical features as a result of the constant rate of change
			4.2.2.2	Understand that there are two key elements to any linear relationship: rate of change and intercept point
			4.2.2.3*	That writing linear equations in the form $y = mx + c$ helps to reveal the structure
			4.2.2.4	Solve a range of problems involving graphical and algebraic aspects of linear relationships
		4.2.3 Model and interpret a range of situations graphically	**4.2.3.1**	Understand that different types of equation give rise to different graph shapes, identifying quadratics in particular
			4.2.3.2	Read and interpret points from a graph to solve problems
			4.2.3.3*	Model real-life situations graphically
			4.2.3.4*	Recognise that the point of intersection of two linear graphs satisfies both relationships and hence represents the solution to both those equations
Theme 5	**5.1** Statistical representations and measures	**5.1.1** Understand and calculate accurately measures of central tendency and spread	**5.1.1.1***	Understand what the mean is measuring, how it is measuring it and calculate the mean from data presented in a range of different ways
			5.1.1.2	Understand what the median is measuring, how it is measuring it and find the median from data presented in a range of different ways
			5.1.1.3*	Understand what the mode is measuring, how it is measuring it and identify the mode from data presented in a range of different ways
			5.1.1.4	Understand what the range is measuring, how it is measuring it and calculate the range from data presented in a range of different ways
		5.1.2 Construct accurately statistical representations	**5.1.2.1**	Construct bar charts from data presented in a number of different ways
			5.1.2.2*	Construct pie charts from data presented in a number of different ways
			5.1.2.3	Construct pictograms from data presented in a number of different ways
			5.1.2.4	Construct scatter graphs from data presented in a number of different ways

5.2	Statistical analysis	**5.2.1**	Interpret reasonably statistical measures and representations	**5.2.1.1**	Understand that the different measures of central tendency offer a summary of a set of data
				5.2.1.2	Understand how certain statistical measures may change as a result in changes of data
				5.2.1.3	Understand range as a measure of spread, including a consideration of outliers
				5.2.1.4	Understand that the different statistical representations offer different insights into a set of data
				5.2.1.5*	Use the different measures of central tendency and spread to compare two sets of data
				5.2.1.6	Use the different statistical representations to compare two sets of data
				5.2.1.7	Recognise relationships between bivariate data represented on a scatter graph
		5.2.2	Choose appropriately statistical measures and representations	**5.2.2.1**	Given a statistical problem, choose what data needs to be analysed to explore that problem
				5.2.2.2*	Given a statistical problem, choose appropriate statistical measures to explore that problem
				5.2.2.3	Given a statistical problem, choose appropriate representations to explore that problem
				5.2.2.4	Given a statistical problem, choose appropriate measures and representations to effectively summarise and communicate conclusions
5.3	Probability	**5.3.1**	Explore, describe and analyse the frequency of outcomes in a range of situations	**5.3.1.1**	Understand that some outcomes are equally likely, and some are not
				5.3.1.2	Understand that the likelihood of events happening can be ordered on a scale from impossible to certain
				5.3.1.3*	Understand that the likelihood of outcomes can be determined by designing and carrying out a probability experiment
		5.3.2	Systematically record outcomes to find theoretical probabilities	**5.3.2.1**	Systematically find all the possible outcomes for two events using a range of appropriate diagrams
				5.3.2.2	Systematically identify all possible outcomes for more than two events using appropriate diagrams, e.g. lists
				5.3.2.3	Find theoretical probabilities from sets of outcomes organised in a systematic way from a range of appropriate representations
		5.3.3	Calculate and use probabilities of single and combined events	**5.3.3.1***	Understand that probability is a measure of the likelihood of an event happening and that it can be assigned a numerical value
				5.3.3.2	Calculate and use theoretical probabilities for single events
				5.3.3.3	Understand that the probabilities of all possible outcomes sum to one
				5.3.3.4	Calculate and use theoretical probabilities for combined events using a variety of appropriate representations, including Venn diagrams

Theme 5

Theme 6	**6.1**	Geometrical properties	**6.1.1**	Understand and use angle properties	**6.1.1.1***	Understand that a pair of parallel lines traversed by a straight line produces sets of equal and supplementary angles
					6.1.1.2*	Know and understand proofs that in a triangle, the sum of interior angles is 180 degrees
					6.1.1.3	Know and understand proofs for finding the interior and exterior angle of any regular polygon
					6.1.1.4	Solve problems that require use of a combination of angle facts to identify values of missing angles, providing explanations of reasoning and logic used
			6.1.2	Understand and use similarity and congruence	**6.1.2.1***	Recognise that similar shapes have sides in proportion to each other but angle sizes are preserved
					6.1.2.2	Recognise that for congruent shapes both side lengths and angle sizes are preserved
					6.1.2.3	Understand and use the criteria by which triangles are congruent
					6.1.2.4	Recognise rotational symmetry in shapes
			6.1.3	Understand and use Pythagoras' theorem	**6.1.3.1**	Be aware that there is a relationship between the lengths of the sides of a right-angled triangle
					6.1.3.2*	Use and apply Pythagoras' theorem to solve problems in a range of contexts
	6.2	Perimeter, area and volume	**6.2.1**	Understand the concept of perimeter and use it in a range of problem-solving situations	**6.2.1.1**	Use the properties of a range of polygons to deduce their perimeters
					6.2.1.2	Recognise that there is constant multiplicative relationship (π) between the diameter and circumference of a circle
					6.2.1.3	Use the relationship $C = \pi d$ to calculate unknown lengths in contexts involving the circumference of circles
			6.2.2	Understand the concept of area and use it in a range of problem-solving situations	**6.2.2.1***	Derive and use the formula for the area of a trapezium
					6.2.2.2	Understand that the areas of composite shapes can be found in different ways
					6.2.2.3*	Understand the derivation of, and use the formula for, the area of a circle
					6.2.2.4	Solve area problems of composite shapes involving whole and/or part circles, including finding the radius or diameter given the area
					6.2.2.5*	Understand the concept of surface area and find the surface area of 3D shapes in an efficient way
			6.2.3	Understand the concept of volume and use it in a range of problem-solving situations	**6.2.3.1**	Be aware that all prisms have two congruent polygonal parallel faces (bases) with parallelogram faces joining the corresponding vertices of the bases
					6.2.3.2	Use the constant cross-sectional area property of prisms and cylinders to determine their volume

	6.3	Transforming shapes	6.3.1	Understand and use translations	6.3.1.1	Understand the nature of a translation and appreciate what changes and what is invariant
					6.3.1.2	Understand the minimum information required to describe a translation (vertical and horizontal displacement)
					6.3.1.3	Translate objects from information given in a variety of forms
			6.3.2	Understand and use rotations	6.3.2.1	Understand the nature of rotations and appreciate what changes and what is invariant
					6.3.2.2*	Understand the minimum information required to describe a rotation (centre of rotation, size and direction of rotation)
					6.3.2.3	Rotate objects using information about centre, size and direction of rotation
			6.3.3	Understand and use reflections	6.3.3.1	Understand the nature of reflections and appreciate what changes and what is invariant
					6.3.3.2*	Understand the minimum information required to describe a reflection (line of reflection)
					6.3.3.3	Reflect objects using a range of lines of reflection (including non-vertical and non-horizontal)
			6.3.4	Understand and use enlargements	6.3.4.1	Understand the nature of enlargements and appreciate what changes and what is invariant
					6.3.4.2	Understand the minimum information required to describe an enlargement (centre of enlargement and scale factor)
					6.3.4.3	Enlarge objects using information about the centre of enlargement and scale factor
	6.4	Constructions	6.4.1	Use the properties of a circle in constructions	6.4.1.1	Understand a circle as the locus of a point equidistant from a fixed point
					6.4.1.2	Use intersecting circles to construct triangles and rhombuses from given lengths
			6.4.2	Use the properties of a rhombus in constructions	6.4.2.1	Be aware that the diagonals of a rhombus bisect one another at right angles
					6.4.2.2	Be aware that the diagonals of a rhombus bisect the angles
					6.4.2.3*	Use the properties of a rhombus to construct a perpendicular bisector of a line segment
					6.4.2.4	Use the properties of a rhombus to construct a perpendicular to a given line through a given point
					6.4.2.5	Use the properties of a rhombus to construct an angle bisector

Theme 6

Table 2: Coverage of the national curriculum Key Stage 3 mathematics programme of study

		KS3 programme of study Pupils should be taught to:	1. The structure of the number system	2. Operating on number	3. Multiplicative reasoning	4. Sequences and graphs	5. Statistics and probability	6. Geometry
Number	**N1**	understand and use place value for decimals, measures and integers of any size	1.1.1					
	N2	order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥	1.1.1 1.3.2					
	N3	use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property	1.2.1 1.2.3					
	N4	use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative		2.1.1–2.1.4				
	N5	use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals		2.1.5				
	N6	recognise and use relationships between operations including inverse operations		2.1.1 2.1.2				
	N7	use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations	1.2.2					
	N8	interpret and compare numbers in standard form $A \times 10^n$ $1 \le A < 10$, where n is a positive or negative integer or zero	1.3.3					

Number	**N9**	work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.372 and $\frac{3}{8}$)	1.3.1					
	N10	define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%			3.1.5			
	N11	interpret fractions and percentages as operators			3.1.3 3.1.5			
	N12	[use standard units of mass, length, time, money and other measures, including with decimal quantities][i]						
	N13	round numbers and measures to an appropriate degree of accuracy (for example, to a number of decimal places or significant figures)	1.1.2 1.1.3					
	N14	use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a < x \le b$	1.1.4					
	N15	use a calculator [and other technologies][ii] to calculate results accurately and then interpret them appropriately		2.1.5				
	N16	appreciate the infinite nature of the sets of integers, real and rational numbers.	1.3.2					
Algebra	**A1**	use and interpret algebraic notation, including: • ab in place of a × b • 3y in place of y + y + y and 3 × y • a^2 in place of a × a, a^3 in place of a × a × a; a^2b in place of a × a × b • $\frac{a}{b}$ in place of a ÷ b • coefficients written as fractions rather than as decimals • brackets	1.4.1					
	A2	substitute numerical values into formulae and expressions, including scientific formulae	1.4.1					

	A3	understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors	1.4.1					
	A4	simplify and manipulate algebraic expressions to maintain equivalence by: • collecting like terms • multiplying a single term over a bracket • taking out common factors • expanding products of two or more binomials	1.4.2–1.4.4					
	A5	[understand and use standard mathematical formulae][iii]; rearrange formulae to change the subject	1.4.5					
	A6	model situations or procedures by translating them into algebraic expressions or formulae and by using graphs				1.4.1 4.2.3		
	A7	use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)		2.2.1–2.2.4				
Algebra	A8	work with coordinates in all four quadrants				4.2.1		
	A9	recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in x and y and the Cartesian plane				4.2.2 4.2.3		
	A10	interpret mathematical relationships both algebraically and graphically				4.2.3		
	A11	reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically				4.2.2		
	A12	use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations				4.2.3		
	A13	find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs				4.2.3		

Algebra	A14	generate terms of a sequence from either a term-to-term or a position-to-term rule				4.1.1		
	A15	recognise arithmetic sequences and find the nth term				4.1.2		
	A16	recognise geometric sequences and appreciate other sequences that arise.				4.1.3		
Ratio, proportion and rates of change	R1	[change freely between related standard units (for example time, length, area, volume/capacity, mass)]iv						
	R2	use scale factors, scale diagrams and maps			3.1.4			
	R3	express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1			3.1.3			
	R4	use ratio notation, including reduction to simplest form			3.1.4			
	R5	divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio			3.1.4			
	R6	understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction			3.1.1–3.1.4			
	R7	relate the language of ratios [and the associated calculations to the arithmetic of fractions and to linear functions]v			3.1.4			
	R8	solve problems involving percentage change, including: percentage increase, decrease and original value problems [and simple interest in financial mathematics]vi			3.1.5			
	R9	solve problems involving direct and inverse proportion, including graphical and algebraic representations			3.1.6			
	R10	use compound units such as speed, unit pricing and density to solve problems.			3.1.6			

Geometry and measures	**G1**	derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)						6.2.1–6.2.3
	G2	calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes						6.2.1
	G3	[draw and measure line segments and angles in geometric figures, including interpreting scale drawings]**vii**						
	G4	derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line						6.4.1 6.4.2
	G5	[describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively]**viii** and rotationally symmetric						6.1.2
	G6	[use the standard conventions for labelling the sides and angles of triangle ABC]**ix**, and know and use the criteria for congruence of triangles						6.1.2
	G7	[derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures (for example, equal lengths and angles) using appropriate language and technologies]**x**						
	G8	identify properties of, and describe the results of, translations, rotations and reflections applied to given figures						6.3.1–6.3.3
	G9	identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids						6.1.2 6.3.4
	G10	[apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles]**xi**						

Geometry and measures	**G11**	understand and use the relationship between parallel lines and alternate and corresponding angles						6.1.1
	G12	derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons						6.1.1
	G13	apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' theorem, and use known results to obtain simple proofs						6.1.1–6.1.3
	G14	use Pythagoras' theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles						3.2.1 3.2.2 6.1.3
	G15	[use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D][xii]						
	G16	interpret mathematical relationships both algebraically and geometrically.						6.2
Probability	**P1**	record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0–1 probability scale					5.3.1	
	P2	understand that the probabilities of all possible outcomes sum to 1					5.3.3	
	P3	enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams					5.3.2	
	P4	generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities.					5.3.3	

Statistics	**S1**	describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)					5.1.1, 5.2.1 and 5.2.2	**Statistics**
	S2	construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data					5.1.2	
	S3	describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs.					5.2.1	

	Notes	Relevant sections of the professional development materials
N12	[i] Students will have met standard units of mass, length, time, money and other measures, including decimal quantities, at Key Stage 2. As such, we have not focused on these as a stand-alone key idea at Key Stage 3; rather, measures may be used as a context throughout students' learning.	**1.1** Place value, estimation and rounding **6.2** Perimeter, area and volume
N15	[ii] The national curriculum states that teachers should use their judgement about when ICT tools should be used. We have therefore not explicitly covered ICT in the key ideas, as use of ICT is best determined by teachers based on the needs of their students and the available resources.	Use of dynamic software: **3.2** Trigonometry Use of statistical software: **5** Statistics and probability
A5	[iii] Students will have been introduced to the idea of a mathematical formula at Key Stage 2. As such, we have not focused on these as a separate key idea at Key Stage 3; rather, use of formulae may be used as a context for work on algebra including changing the subject of a formula throughout this Key Stage.	**1.4.1.3** Know the meaning of and identify: term, coefficient, factor, product, expression, formula and equation **5.1** Statistical representations and measures **5.2** Statistical analysis **6.2** Perimeter, area and volume
R1	[iv] Students will have converted between related standard units of mass, length, time, money and other measures, including decimal quantities, at Key Stage 2. For example, between litres and millilitres and hours and minutes. They will also have converted between related metric and imperial measures using approximate equivalents. As such, we have not focused on these as a stand-alone key idea at Key Stage 3; rather, conversion may be used as a context throughout students' learning.	**1.4.1.3** Know the meaning of and identify: term, coefficient, factor, product, expression, formula and equation **6.2** Perimeter, area and volume
R7	[v] Understanding that the same relationship between two entities can be expressed in different ways is an important aspect of mathematics. Once the basics of a concept are grasped, its relationship to other representations and forms helps to deepen understanding and can improve efficiency when calculating. In this case: • when working with ratios, students should explore the connections with fractions • when exploring linear functions and the features of straight line graphs, links to ratio should be made.	**3.1.4** Understand that ratios are an example of a multiplicative relationship and apply this understanding to a range of contexts **4.2.2.2** Understand that there are two key elements to any linear relationship: rate of change and intercept point **4.2.3.3** Model real-life situations graphically
R8	[vi] Rather than this being a specific key idea in these materials, we have endeavoured to use money and finance as a context for studying percentage increase.	**3.1.5** Understand that percentages are an example of a multiplicative relationship and apply this understanding to a range of contexts

G3	[vii] Students will have constructed and measured line segments and angles at Key Stages 1 and 2, including within the contexts of 2-D shape. As such, we have not focused on these as a stand-alone key idea at Key Stage 3; rather, students should be given opportunity to construct and measure in a wide range of contexts, including geometric and statistical.	**4.2** Graphical representations **5.1** Statistical representations and measures **6** Geometry (especially **6.4** Constructions)
G5	[viii] Students will have described, sketched and constructed using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively symmetrical at Key Stage 2. Note that rotational symmetry is a new idea at Key Stage 3. As such, we have not focused on these as a separate key idea at Key Stage 3; rather, students should continue to use and apply these ideas in a range of contexts as they continue to study geometry.	**3.2** Trigonometry **6** Geometry (especially **6.4** Constructions)
G6	[ix] Students will have used the standard conventions for labelling the sides and angles of triangle ABC at Key Stage 2, including from 2-D representations. As such, we have not focused on these as a separate key idea at Key Stage 3; rather, students should continue to use and apply these ideas in a range of contexts as they continue to study geometry.	**3.2** Trigonometry **6** Geometry
G7	[x] Students will have worked with the properties of 2-D shapes at Key Stages 1 and 2. As such, we have not focused on these as a stand-alone key idea at Key Stage 3; rather, students should be given opportunity to use and apply these existing knowledge and skills as they continue to study geometry.	**3.2** Trigonometry **6** Geometry
G10	[xi] Students will have applied the properties of angles at a point, angles at a point on a straight line and vertically opposite angles at Key Stage 2. As such, we have not focused on these as a stand-alone key idea at Key Stage 3; rather, students should be given opportunity to use and apply these knowledge and skills as they continue to study geometry.	**6.1** Geometrical properties
G15	[xii] Students will have used the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D at Key Stage 2, including from 2-D representations. As such, we have not focused on these as a separate key idea at Key Stage 3; rather, students should be given opportunity to use and apply these knowledge and skills as they continue to study geometry.	**3.2** Trigonometry **6** Geometry