Primary & Early Years Magazine 77

National Centre for Excellence in the Teaching of Mathematics

Welcome to another issue of our Primary Magazine. This magazine has been serving primary teachers for 76 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are accessible through the Primary Magazine Archive.

Contents

In each issue we have a selection of interesting and useful articles. New National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study. This edition focuses on exploring how to design learning for short division.

Where's the Maths in That? shares ideas for ensuring that mathematics is taught and experienced across the curriculum. In the coming months, this series of articles that will explore opportunities for mathematics and mathematical thinking within the new science programme of study. This month the theme is *Evolution and inheritance for Y6*.

Finally, Maths in the Staff Room provides simple plans for CPD meetings in your school to be led by a member of staff. These are short meetings that can be used exactly as indicated or adapted to meet the CPD needs of the school. We continue our series focusing on the features of great teaching in the context of maths which was explored in full in Issue 73. In this issue we explore organising mathematical learning.

But first, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

News

Sample KS1 and KS2 test papers for 2016 are now available

On 29 June the DfE published the final framework for KS1 and KS2 assessment for use by test writers. There will be one performance descriptor for each key stage. No manipulatives will be permitted in any of the papers for KS1 or KS2. Some measuring equipment and a mirror will be permitted for KS2. Sample test papers for KS1 and KS2 are also available with accompanying mark schemes. If you are interested in carrying out these sample tests with your current Y2 and Y6 before the holidays to see how they fair when compared to the old levelled tests, then please do share your findings in the Primary Forum.

PISA assessments

We regularly hear politicians referring to the international comparison assessments such as PISA and TIMSS. This document from the National Foundation for Educational Research (NFER) shares how data from the 2012 PISA assessments shape up for the UK, with a number of examples of our position in the international rankings for maths.

Rubbers

Last month this news item shot around social media after Guy Claxton was reported to have claimed that "erasers are the instrument of the devil". Whether you agree or disagree, this article can serve to help you to stop and reflect on the use of rubbers in your maths lessons, and the value you place on pupil mistakes as part of the learning process.

London Mathematical Society CPD Grants

Did you know that the London Mathematics Society (LMS) provides opportunities for schools/ teachers to bid for grants of up to £400 to support teachers with maths-specific CPD? Although there are certain conditions that need to be met, these grants are available to all teachers. Application deadlines for grants are 31 August, 30 November, 31 January and 30 April each year, so you might like to start considering an application over the summer.

NCETM National Curriculum support

Have you explored our National Curriculum Planning Tool yet? This interactive tool will support you in the following ways: your subject knowledge; making connections within and across the primary curriculum; suggest helpful papers, pupil activities, exemplification of expectations, and links to the suite of NCETM videos. There are also sections on the Bar Model, Teaching Fractions, Progression in Reasoning, and Developing a Scheme of Work - all accessible via buttons on the main National Curriculum information page.

Mathematics CPD and networking events

Don't forget to use our Professional Development Calendar if you are looking for courses or training run by high quality providers of maths CPD, or indeed any events in the next few months where you can meet and share experiences with other maths teachers. Look for CPD Standard Holders (gold rosette) and/or

Accredited Professional Development Leads (purple rosette). A list of providers, classified by the same system, can be found in the CPD Providers Directory.

Mastery

A reminder of the new section of the NCETM website that pulls together all the work the Centre, and the Maths Hubs are doing in the field of mastery. The NCETM has also started three new threads of discussion in the Maths Café community. Please contribute your views and experiences: constructive criticism is always welcome.

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New National Curriculum in Focus

New National Curriculum in Focus *is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series here.*

Designing learning for short division

The new curriculum requires children to learn to use standard written methods sooner than has been taught in recent years. In their 2011 report Good practice in primary mathematics: evidence from 20 successful schools, Ofsted identified that in the most successful schools pupils were moved to standard written methods swiftly and once pupils were secure with interim methods were moved quickly on to more efficient methods.

What is important is that, if pupils are to be expected to move to a standard written method more quickly than previously expected, we need to ensure that that they do this not only with procedural fluency but with conceptual understanding.

So can this be achieved for short division?

Let's consider the Y6 statement from the programme of study:

> *divide numbers up to four digits by a one-digit number using the formal written method*

Appendix 1 of the National Curriculum suggests this method for short division:

Short division

98 ÷ 7 becomes

$$\begin{array}{r} 1\ 4 \\ 7\overline{)\,9^{2}\ 8} \end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r} 8\ \ 6\ \text{r}\,2 \\ 5\overline{)\,4\ 3^{3}\ 2} \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r} 4\ \ 5\ \text{r}\,1 \\ 1\ 1\overline{)\,4\ 9^{5}\ 6} \end{array}$$

Answer: $45\frac{1}{11}$

Before pupils can begin to learn to do this there are a number of skills and concepts that need to have been developed in order to carry out short division:

- recall fluently multiplication facts to 12 x 12, recognise multiples
- visualise and understand how a four-digit number can be partitioned and recombined into multiples of 1000, 100, 10 and 1 with both concrete and abstract representations. (i.e. base 10 (concrete), place value counters or arrow cards)
- visualise the relative quantity of the numbers
- know the value of a digit because of its position in a number
- understand the effect of multiplying by 10, 100 and 1000
- understand that multiplication and division are inverses and use this relationship to estimate and check answers

- decide when it is more efficient to calculate mentally
- understand the concept of a remainder after division
- understand that division is (left) distributive over addition, eg (a + b) ÷ c = (a ÷ c) + (b ÷ c)

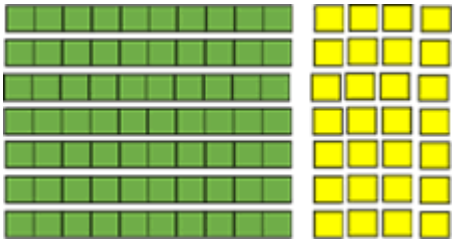
Scaffolding learning through procedural and conceptual variation

One particular feature of the teaching seen in Shanghai has been the use of teaching with conceptual and procedural variation. You can read in more detail what this means in National Curriculum in Focus from Issue 73.

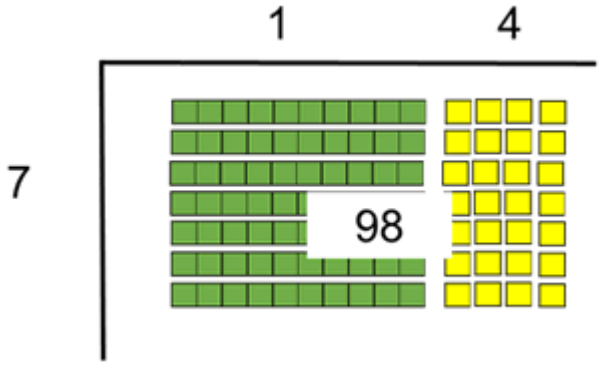
So what might conceptual and procedural variation look like in the context of teaching short division of four-digit numbers by a single-digit number?

An effective representation of multiplication is to consider this as an array.

e.g. Looking at the columns we can see 7 x 14 = (7 x 10) + (7 x 4), but also reveals from the rows that 14 x 7 = 7 x 14 = 98:



This can help to understand the abstract notation of often referred to as the 'bus stop' method:



There is one lot of seven 10s and four lots of seven ones.

So how is this translated to a written method for division?

Firstly, in choosing examples we should be mindful of the different skills that need to be developed. In previous articles we have separated out examples that require regrouping from those that do not. So when dividing a four-digit number by a single digit number using short division we should begin with examples that do not require regrouping to develop a secure method.

Let's consider the problem: ***A biscuit factory makes the same number of biscuits every hour. If it makes 3636 biscuits in three hours, how many biscuits are made every hour?***

By Y6 we should expect pupils to recognise that this is a division problem. By first acting out the problem with concrete resources, this will enable any uncertain pupils to see that the actions they are performing are related to the abstract problem 3636 ÷ 3 = ?

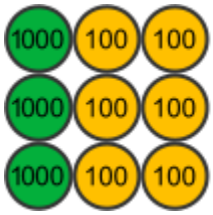
Building 3636 with place value counters will present this:

How many times can we share three thousand between three?

What's the same, what's different?

	Th	H	T	1s
	1			
3	3	6	3	6

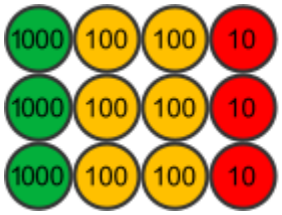
How many times can we share six hundred between three?



What's the same, what's different?

	Th	H	T	1s
	1	2		
3	3	6	3	6

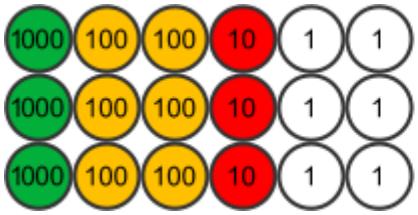
How many time can we share thirty (or three tens) between three?



What's the same, what's different?

	Th	H	T	1s
	1	2	1	
3	3	6	3	6

How many times can we share six (or six ones) between three?



What's the same, what's different?

	Th	H	T	1s
	1	2	1	2
3	3	6	3	6

The variations from left to right draw the pupils' attention to the meaning of the abstract representation. Opportunities should be given for pupils to articulate these similarities and differences by posing questions that help to reveal the pupils' deepening understanding. For example – why do you think these are different? Why do you think these are the same? Pupils should be encouraged to seek the similarities

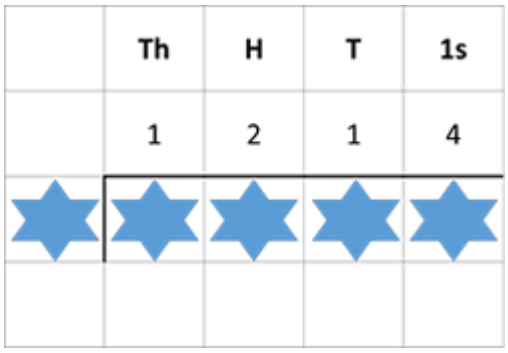
and differences and at each successive step compare what has changed and what has remained the same (see Issues 75, 74 and 73 for further examples in the context of other operations).

Vary the numbers slightly in the same word problem. For example this sequence will enable pupils to notice slight variations in each problem in terms of the recording:

3639 ÷ 3 =
3669 ÷ 3 =
3969 ÷ 3 =
6969 ÷ 3 =

All of the above problems require simple division and no regrouping when dividing by three. Pupils should then have the opportunity to practise for fluency in the context of problems with other divisors, dividends and contexts, moving away from the need to use the concrete resources when pupils are confident that they understand the method.

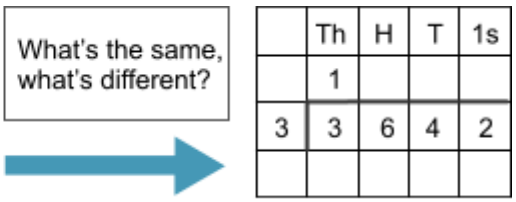
Pupils' understanding could be deepened by asking them to finding possible answers for the numbers hidden by the stars:



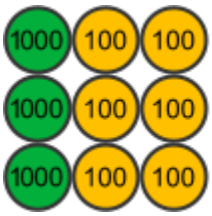
However another problem such as 3642 ÷ 3 will then enable a shift in the children's emerging thinking and generalising of the procedure to situations where regrouping is required.

As before, build the number with place value counters:

How many times can we share three thousand between three?



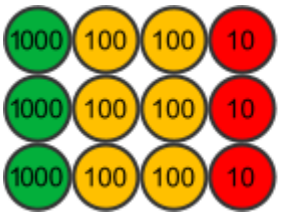
How many times can we share six hundred between three?



What's the same, what's different?

	Th	H	T	1s
	1	2		
3	3	6	4	2

How many times can we share four tens between three?



What's the same, what's different?

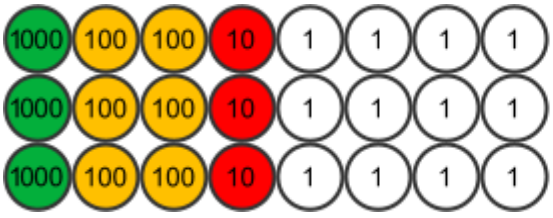
	Th	H	T	1s
	1	2	1	
3	3	6	4	$^{1}2$

At this point the children will discover that they can share four tens between three only once, and that there is a **remaining** ten (and two ones) to deal with (referring to this as a 'remaining ten' will enable connections from earlier or later with the term 'remainder'. This can be described as a 'pivot point' for their learning because the procedure they have become familiar with without regrouping is now being challenged, enabling an opportunity to deepen their understanding.

Invite pupils to discuss what can be done next. One suggestion might be to place the **remaining** ten in the next column and finish the column off with the remaining ones. It will therefore be important to have noticed and mentioned in previous examples the fact that in each row of every column the value of each counter is the same.

Experience from previous examples in other operations, and previous experience in short division, should enable some pupils to suggest regrouping the one ten into ten ones:

After regrouping the remaining ten we can ask: *How many times can we share twelve (ones) between three?*



What's the same, what's different?

	Th	H	T	1s
	1	2	1	4
3	3	6	4	$^{1}2$

Comparing the concrete representation with the abstract notation will be an interesting discussion point by asking where is the remaining ten that we regrouped into ones?

Provide further sequences of examples where one ten requires regrouping into ten ones and withdrawing from the concrete resources as pupils develop confidence in recording the procedure, e.g.

3642 ÷ 3

3645 ÷ 3
3648 ÷ 3

4852 ÷ 4
4856 ÷ 4

7784 ÷ 7

8896 ÷ 8

At this point you may wish to ask pupils to observe any emerging patterns - in the above the thousands and hundreds digits are multiples of the divisor and the tens digit is 'one more' than the divisor. To deepen understanding, ask pupils to suggest another where the tens will need to be regrouped into ten ones.

Pupils could then be asked to conjecture what will happen if instead of the problem being 3642 ÷ 3 the problem is 3651 ÷ 3. At this point there would be two tens that need regrouping into 20 ones. This might be cumbersome with the resources on the table so demonstrating the process with the interactive whiteboard may be more helpful. This will then allow a discussion to evaluate how efficient it is using the resources compared with using just the written notation.

Compare the abstract notation to make sense of the procedure:

	Th	H	T	1s
	1	2	1	4
3	3	6	4	$^{1}2$

What's stayed the same? What's changed?

	Th	H	T	1s
	1	2	1	7
3	3	6	5	$^{2}1$

Provide a sequence of problems for pupils to work through that involves regrouping two tens and then other multiples of tens for different divisors. Again ask pupils to identify any patterns to help them predict whether any regrouping is required.

Other 'pivot' points to include in teaching of short division:

- dividing a four-digit number by a single-digit number where the hundreds require regrouping
- dividing a four-digit number by a single-digit number where the thousands require regrouping
- dividing a four-digit number by a single-digit number where the tens and hundreds, tens and thousands or hundreds and thousands require regrouping
- dividing a four-digit number by a single-digit number where all digits require regrouping
- dividing a four-digit number by a single-digit number where one digit is zero
- dividing a four-digit number by a single-digit number where two or three digits are zero
- dividing a four-digit number by a single-digit number where the thousands, hundreds or tens is less than the divisor (non-zero)
- dividing a four-digit number by a single-digit number where there is a remainder.

To assess pupils' understanding, provide a selection of division problems and ask them to notice which ones will require regrouping without them having to perform the entire calculation.

Pupils can deepen their understanding by solving problems involving reasoning about the written method such as Division Rules from NRICH.

Further resources

- Primary Magazine Issue 61: Maths to Share - Division.

You can read more about conceptual and procedural variation here.

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Where's the Maths in That? – Maths across the curriculum

Where's the Maths in That? *explores how mathematics can be embedded into other subjects in the context of the new curriculum. The subject in this new series is* ***science*** *and over the next few months we will explore the different themes for the KS1 and KS2 science programmes of study and how maths can be embedded in and enhance understanding of scientific ideas. You can find previous features in this series here.*

In this edition we look at the theme of **Evolution and inheritance for Y6** and how a scheme of work for this might incorporate mathematical skills.

The statutory requirements are that children are taught to:

- recognise that living things have changed over time and that fossils provide information about living things that inhabited the Earth millions of years ago
- recognise that living things produce offspring of the same kind, but normally offspring vary and are not identical to their parents
- identify how animals and plants are adapted to suit their environment in different ways and that adaptation may lead to evolution.

Below are some suggestions for how mathematical skills and thinking can be incorporated into this unit of work.

Invite pupils to compare themselves to their biological parents (if this is appropriate to do so). Identify similarities and differences that might have been inherited from each parent. Which physical characteristics can you/can't you inherit from your parents (e.g. hair length is a choice). This activity could also be done by comparing biological siblings to compare characteristics that have been inherited from both parents.

Ask pupils to sort themselves into two groups, then into another pair of sub-group. How many groups now? Now sub-group again. Invite pupils to consider in each group how they can now subdivide. This activity then enables them to see that everyone is unique because they may vary by one or more characteristics. This activity will develop further their ideas of variation and diversity.

Investigate and comparing the measurements of different parts of the pupils' bodies and the ratios between two body parts. The ratios between the same two body parts in each person will be roughly the same. Pupils can then explore whether a play doll (e.g. Barbie) is built in proportion to a human body.

Knowing ratios of body parts can also enable you to calculate approximations of unknown measurement. Can you estimate how tall someone might be based on the length of their feet? Forearm? Femur? Link this then to the work of palaeontologists and how they have been able to estimate the heights of different types of dinosaurs.

Play this simulation to experience the evolution of interdependent creatures. Use the bar chart to interpret what is happening to the populations of the two groups of bugs.

Use the World Wildlife Fund endangered species page to transfer the data about population of different endangered animals to a bar chart. Pupils will need to choose an appropriate scale on which to plot the data.

An interesting problem solved by Italian mathematician Fibonacci was about the reproduction rate of rabbits in a year, if a pair of adult rabbits (two-months old or more) reproduced another pair of rabbits every month. You can view this problem in this NRICH article.

Suggested link

- National STEM Centre eLibrary - Evolution and inheritance.

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Maths in the Staff Room – Short Professional Development Meetings

Maths in the Staff Room *provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school. You can find previous features in this series here*

Maths Homework

In Issue 73 we provided a meeting to review the effectiveness of teaching mathematics in relation to the two recent reports on effective pedagogy and teaching:

- Exploring Effective Pedagogy in Primary Schools: Evidence from Research (EPPSE)
- What Makes Great Teaching?

This meeting follows on from that first reflection and provides a more in depth understanding of how a group of teachers might address any areas of weakness from the audit carried out. In this issue we explore **organising mathematical learning**.

Meeting Aims

- Review current characteristics of classroom organisation in maths lessons
- Identify changes in practice that might develop improved pupil outcomes.

Timing

- 1 x 90 minute meeting

Prior to the meeting it would be helpful for the subject leader to visit some maths lessons informally to focus on observing the structural organisation in classrooms for mathematical learning.

Observation should focus on:

- preparation for the lesson
- organisation of pupils - including transitions within/between classes
- choice of equipment
- choice of tasks
- use of time.

1.0 Share the aim of the session

2.0 Start by sharing outcomes of classroom observations – without naming teachers or classes specifically – by presenting evidence for the above five focuses

3.0 Share this quote from the EPPSE report to begin considering research evidence for high quality classroom organisation

> *"Teachers in excellent schools gave a great deal of **thought to the resources they used**. These were **prepared ahead of time, well managed during lessons**, were particularly **fit for purpose** and **tailored to the individual needs of their pupils**. These teachers also made*

productive use of instructional time *by maintaining* ***good pace*** *and by ensuring that every second of their lessons counted. Pupils in these classes had the* ***highest ratings of self-reliance****."*

p18, Exploring Effective Pedagogy in Primary Schools: Evidence from Research

4.0 Explore this quotation through the following questions:

4.1 How are resources chosen for your lessons?

Some answers might be…

- search on the internet first
- refer to the maths scheme
- look at last year's planning
- identify the learning objective and consider the resources that are required to support the steps in learning.
- …

Consider the risks and benefits of each of the responses. For example a risk of downloading from the internet may not provide sufficient quality assurance, consistency in presentation from previous teaching/learning, or match with the school's calculation policy.

The use of resources may also identify that some teachers are unable to access resources that they need for teaching - e.g. concrete materials - or when there are insufficient resources for all the pupils to use them in a meaningful way.

Resources chosen by the teacher may also be limited by their confidence to use the resources. E.g. Cuisenaire or Dienes apparatus may be in cupboards, but rarely chosen because the teacher requires support to use them effectively.

Identify any action points for the selection of resources.

4.2 What routines do you have to enable you to maximise learning time?

Routines can help minimise time taken up for organisational aspects of a lesson. If children are familiar with a routine then they are likely to settle into their learning more quickly and understand what is expected of them in each lesson to be 'ready for learning'.

Some answers might be…

- pupils sit in the same seats each time
- maths books are put out on tables before the lesson for children to look at for feedback from marking
- a maths puzzle or problem related to the lesson is shown while pupils are settling
- teacher uses a suite of familiar games/activities that can be adapted for different objectives so rules do not need to be taught each time. E.g. matching pairs, bingo, 4 in a row/grid games

- pupils know to stick or write learning intention/success criteria into books at the beginning of lesson
- equipment is prepared for each table so that learning can begin immediately
- pupils are confident to organise themselves
- …

Identify any action points for maximising use of time.

5.0 Review the action points identified and then ask teachers to set themselves an individual target to change in their classroom and share what they hope to do and will hope to see an improvement in their maths lessons. This could then be used as a subsequent focus for further observations.