Secondary Magazine Issue 68

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 68 of the Secondary Magazine.

Contents

Contributors to this issue include: Alison Clark-Wilson, Paul Ernest, Mary Pardoe, Richard Perring, Peter Ransom and Geoff Wake.

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From the editor

Welcome to the new school year, and to this issue of the NCETM Secondary Magazine. Although you may feel that you already have plenty to do as the term starts, the 5 things to do section points out important opportunities that you might be sorry to miss – such as the December NCETM conference about the benefits for you and your students of professional networking and collaboration.

As a result of participating in Mathematics Knowledge Networks many teachers and their students are already enjoying achievements beyond their previous expectations. For example, in response to the new KS3 Programme Of Study, the mathematics teachers in an all-ability secondary school had introduced into their scheme of work references to rich mathematical tasks such as those from NRICH and Bowland Maths. But they were not confident that they knew how to use such tasks effectively with their students. So the 15 teachers formed themselves into a network funded by the National Centre. By working together they developed understanding of the wide range of problem-solving approaches and strategies that they themselves already used, which opened their minds to what students might also do. This gave them confidence to encourage students to think for themselves. You can read more about the impact of this professional development collaboration in their report.

The teachers in this Mathematics Knowledge Network were fortunate to be guided, encouraged and inspired by Dr Jennifer Piggott, and by her article Rich Tasks and Contexts. In this article Dr Piggott shows how and why rich tasks and contexts provide powerful learning opportunities for students working in 'communities of enquiry and collaboration' that promote communication and imagination.

In this issue the subject of Focus on…, *Quadrigon*, is a rich starting point for classroom enquiry, as is the idea for using ICT in the classroom. You will also find rich ideas by following links in the other articles.

STEM Knowledge Networks (STEM KNs)
The National STEM Centre, in partnership with the National Centre, is keen to encourage teachers of mathematics in schools and colleges to form STEM Knowledge Networks (STEM KNs).
Funding of up to £2 000 may be available to support the development of a network, either within a single school, college or other training provider institution, or drawn from several establishments. STEM Knowledge Networks will be supported through regular contact with the National STEM Centre and the Regional Coordinator for the NCETM.

Closing date for applications is **30 September**. For more information, including details of how to apply, visit the National STEM Centre website.

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It's in the News!
The World's Fastest Texter

The fortnightly *It's in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but a framework which you can personalise to fit your classroom and your learners.

Is there a mobile phone nearby? How quickly can you type the sentence: "The razor-toothed piranhas of the genera Serrasalmus and Pygocentrus are the most ferocious freshwater fish in the world. In reality they seldom attack a human." into the phone? Have a go! If you beat 25.94 seconds then you could be in for a world record!

This *It's in the News!* provides a context for students to design and carry out an experiment or survey to test the hypothesis that it's faster to text using a touch screen rather than a push button phone. This follows from the news that a Salford woman out shopping may have a claim to being the world's fastest texter using a new touch screen mobile.

This resource is not year group specific and so will need to be read through and possibly adapted before use. The way in which you choose to use the resource will enable your learners to access some of the Key Processes from the Key Stage 3 Programme of Study.

Download this *It's in the News!* resource - in PowerPoint format

The Interview

Name: Geoff Wake

About you: I started working as a flight simulation engineer after university and must have been among the first people to have flown a computer-controlled flight simulator - it's strange to think that there are now better versions for use on your phone! However, I didn't find that work very interesting and decided that teaching could allow me to work both with people (much more fun than machines) and maths!

After five years teaching in a South London comprehensive I had a stressful summer holiday, during which I got married, moved to Derbyshire to teach in a tertiary college, and bought a house in which we have lived ever since. I taught a lot of A-level maths, started using computers (with David Tall's calculus software) and Mechanics in Action kits. When I had an opportunity to work with the Mechanics in Action Project team at the University of Manchester, I jumped at the chance – and I've been working at Manchester University ever since on a range of research and curriculum development projects, such as the PRIMAS and COMPASS projects. For a time, until recently, I also looked after our PGCE in Secondary Mathematics.

The most recent use of mathematics in your job was…
I write the comprehension papers for AS Use of Mathematics and the pilot A Level Use of Mathematics – that is where I do most of my mathematics these days. This summer, therefore, I've been working on exam papers that look at radioactive decay and using calculus to calculate where to fill wine and cocktail glasses so that they are half full. Day-to-day, I approximate how much I'll be charged in shops and I'm amazed at how often mistakes are made: not long ago I was charged for grapes rather than potatoes, and I think the check-out assistant was surprised that when she told me the total I immediately suggested there must be some mistake.

Some mathematics that amazed you is…
I'm constantly amazed by mathematics in art and art in mathematics. I'll give just two examples.

> *Art in mathematics*: I have been very impressed by Japanese Sangaku. These wooden tablets, which were hung at the entrance to Buddhist and Shinto temples, presented geometrical problems for the congregation. I love the nested circle problems – which are so visually pleasing – and have tried constructing some using dynamic geometry software.
>
> *Mathematics in art*: I was intrigued by the account in Judith Field's book The Invention of Infinity: Mathematics and Art in the Renaissance of how artists in the renaissance painted with a true sense of perspective using geometry – again I tried building a dynamic geometry file to explore this and had more success this time!

I suppose these examples hint at my general appreciation of visual representation in mathematics.

Why mathematics?
At school I just found that somehow I was in tune with the logical structure of the subject. When I was 12 years old I had a teacher who taught us differentiation from first principles using many diagrams with coloured chalk. I'm sure most of the class were lost, possibly forever, but I was intrigued! When I got to mechanics A-level I found connecting mathematics to reality a real challenge, and had to draw on the

persistence I first encountered when making sense of algebra. I suppose, overall, I enjoy a sense of achievement in constructing mathematical arguments and solving problems.

A significant mathematics-related incident in your life was…
A number of years ago I was developing some mathematics for science resources and one day in a college was interviewing a chemistry student about her work in chemistry. This was tricky for me as I really know very little chemistry, but when we looked through the graphs in her report of an experiment she had done I found I could use my mathematics to interpret what had happened in the experiment. It struck me then how empowering maths can be and how everyone should have access to that. That moment was very influential in my vision for Freestanding Mathematics Qualifications that I went on to develop.

A mathematics joke that makes you laugh is...
A mathematician and an engineer are on a desert island. They find two palm trees each with one coconut on it. The engineer climbs up the first tree, gets the coconut and eats it. The mathematician climbs up the second tree, gets the coconut, climbs up the first tree and puts it there. "Now we've reduced it to a problem that we know how to solve."

The best book you have ever read is…
Almost always the book I have just finished. I guess I'm the sort of person who becomes completely immersed in the author's world, which I suppose is what they try to achieve. I rarely go back to a book, but recently I have re-read Nineteen Eighty-Four, and although in reality 1984 was nothing like the world Orwell envisaged, I somehow think we are very slowly creeping towards it!
A mathematics book that I like using if I have the opportunity to teach something interesting is The Heart of Mathematics: An invitation to effective thinking, by Edward Burger and Michael Starbird. It's full of interesting mathematics with many illustrations and can provide inspiration for many lessons.

Who inspired you?
The people that I first worked with in Crown Woods School in Eltham gave me the confidence to try things out in the classroom. In particular Trevor Weight, the Head of Department, who supported both staff and pupils (sadly Trevor passed away a few years ago but, as he would have liked, it was while he was refereeing a school hockey match). And more lately, my colleagues in Manchester, and of these of course Julian Williams, who inevitably has been a great influence and has supported all aspects of my work.

If you weren't doing this job you would…
be a journalist or maybe an architect. One of our sons is working towards becoming an architect and the interdisciplinary nature of it is really appealing!

What do you see in your crystal ball for mathematics education?
Well, as all good mystics say: "It's very cloudy but I see emerging...a time when mathematics in schools is re-energised, and better connects with the world in which young people operate. New technologies are harnessed so that we have classrooms where pupils enquire, explore, model, investigate…, there is excitement and intrigue and, as well as there being a new mathematical world, we also make connections to other subjects and disciplines." Of course, this is all possible now, but we are too restricted in schools through having to comply with testing and inspection that lacks vision and imagination. There's a lot for us all to do!

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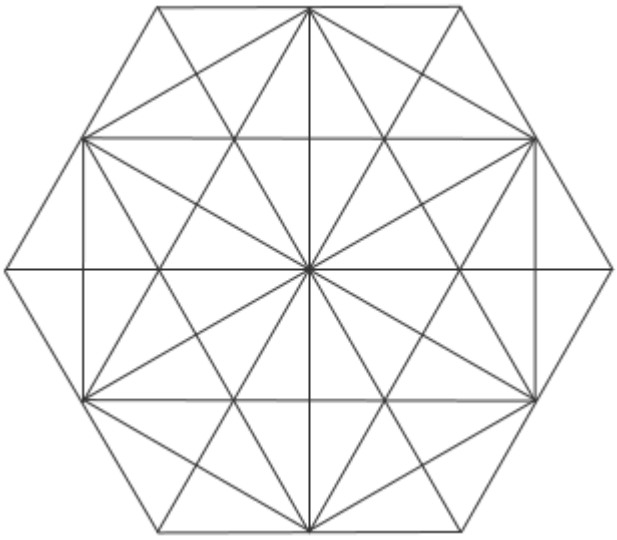
Focus on…Quadrigon

A mathematical game invented during the 1970s by a mathematics teacher, W. Ransome, and described in *Mathematics Teaching, Number 75*, is played on a hexagonal board showing part of a tessellation of 30˚, 60˚, 90˚ triangles.



In the game, called Quadrigon, 2 or 3 players alternately fit coloured triangular counters, which are copies of the smallest right-angled triangles, onto triangles on the board.

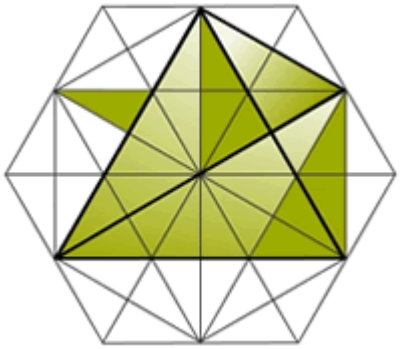
W. Ransome devised a particular set of rules for the game, in which the object is to complete quadrilaterals on the grid – hence the name 'Quadrigon'. But many different rules could be thought up and explored.

The great richness of this situation is in the opportunities that the board itself provides for students to explore their own examples of mathematical objects and relationships between them, ask their own questions, and consolidate what they know in creative and aesthetically satisfying ways.

I have observed students seeing in the Quadrigon board, and investigating, many different objects and relationships, such as those suggested by the following questions – which are just a few of the many questions that students might ask themselves.

In what ways are different right-angled triangles on the board related to each other?

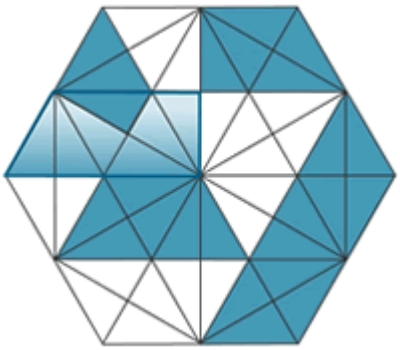
For example:



- It is easy to see, by noticing the number of the smallest right-angled triangles composing each larger triangle, that the areas of the similar right-angled triangles shown in this diagram are in the ratios 1:3:4:9:12. Can I show using Pythagoras' theorem that their lengths are in the ratios 1:√3:2:3:2√3?

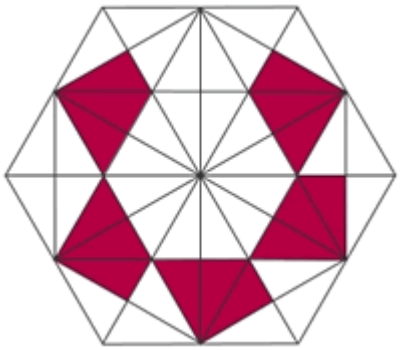
- Is it possible to obtain each triangle shown in the diagram from every other one that is shown by combining mathematical transformations of rotation, reflection and enlargement?

What trapezia are on the board? Many pairs of trapezia are congruent, but are any pairs similar?



Many shapes are reflections of other shapes.

For example, can I identify one of these five trapezia from which every one of the other four can be obtained by a reflection in a line shown on the board? If so, which line on the board is the mirror line for each reflection?



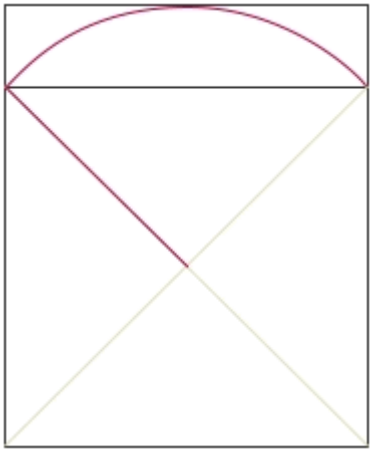
What kinds of special quadrilateral can be found on the board? What kinds can't be found? Why not?

What symmetrical shapes are possible? What kinds of symmetry are possible/not possible? Why?

You and your students could use this sheet of Quadrigon boards to explore possibilities, which will prompt you to ask new questions of your own.

'Quadrigon' should not be confused with 'Quadriagon', which is the name that Wolfgang Von Wersin, a German glassware designer, gave in 1956 in *The Book of Rectangles, Spatial Law and Gestures of The Orthogons Described*, to one of the 12 special rectangles known as orthogons – of which the Golden Rectangle is the most familiar.

To produce a Quadriagon first draw a square. Then rotate a half-diagonal of the square to create an arc. Extend two sides of the square to form a rectangle – the Quadriagon – that includes the arc, and one side of which is tangential to the arc.



The longest side of the Quadriagon created from a unit square is $(1 + \sqrt{2})/2$, whereas the longest side of the Golden Rectangle created from a unit square is $(1 + \sqrt{5})/2$, which is the Golden Ratio.

I can find no information about W. Ransome other than that in 1976 this person was teaching mathematics at Highgate School, and might possibly be Wilson Ransome who is the author of several books about mathematics.

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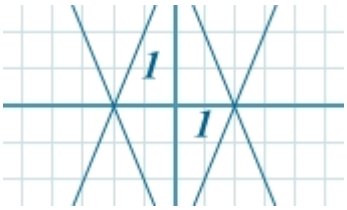

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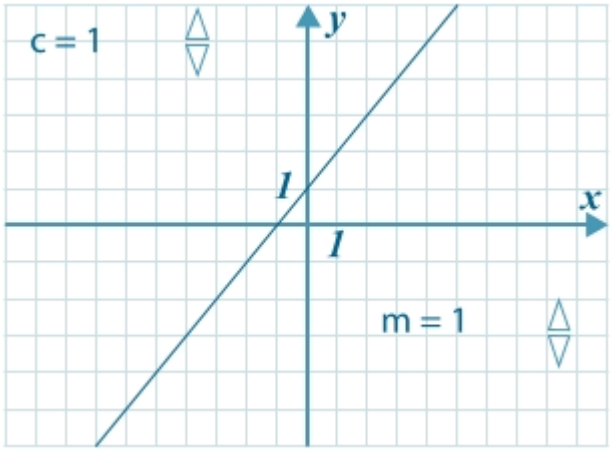
An idea for ICT in the classroom - exploring linear graphs dynamically

Many schools have access to an ICT resource that students could use to carry out hands-on tasks to explore functions and graphs dynamically. The new 'traditional' approaches tend to focus on students changing certain parameters, and teachers 'hoping' that they work systematically to come to wished-for conclusions!

For example, varying 'm' and 'c' within $y = mx + c$ to learn about gradient and intercept properties of linear functions.

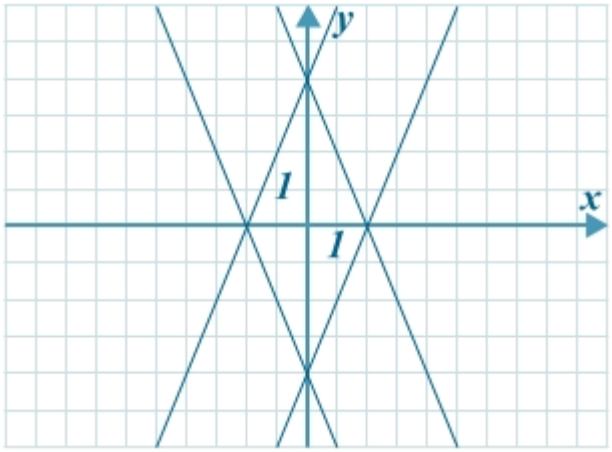


However, this could be seen as an approach that lacks connectivity with other aspects of the mathematics curriculum! What follows is an alternative suggestion, which makes links with the properties of two-dimensional shapes, and can be extended as an introduction to simultaneous linear equations.

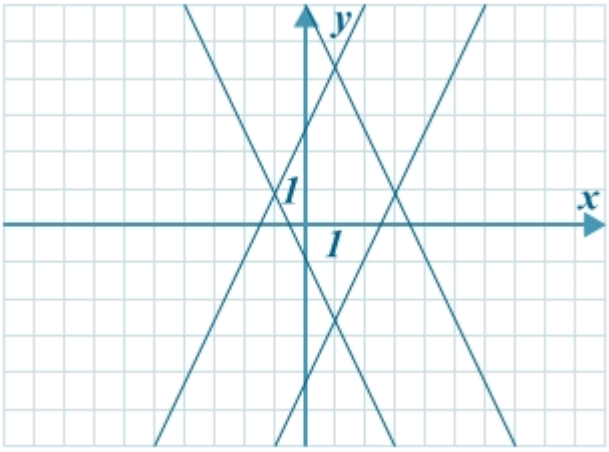
Challenge the students to produce on-screen diagrams that show the outlines of different quadrilaterals – you can support this by providing some templates for them to try to match.

The level of difficulty can be varied by positioning the quadrilaterals differently on the page.

So, drawing a rhombus with a line of symmetry on the y-axis is less of a challenge than…



…drawing the outline of the same rhombus 'shifted' a little to the right.



Both challenges lead to the need for students to connect ideas concerning gradient and intercept properties of straight-line graphs.

Whilst working on a variety of challenges within this task, depending on the way that you have presented the task, the students could have opportunities to:

- plot the graphs of linear functions and recognise that equations of the form $y = mx + c$ correspond to straight-line graphs
- appreciate that if lines are parallel, they have the same gradient
- recognise that if lines cross on the y-axis, they have the same intercept value.

Asking questions such as: 'What is similar about the set of functions that produced the square and the rectangle?' and 'What do you notice about the gradients of the four equations that produced each shape?' will encourage students to extend their mathematical thinking.

Teaching and learning mathematics – a free web journal!

The *Philosophy of Mathematics Education journal* publishes research and ideas about the teaching and learning of mathematics from a philosophical viewpoint. Since it became freely available many teachers of mathematics at all levels have found its papers informative and eye-opening. It is accessed via the homepage of the editor, Paul Ernest. The journal is published on the web annually, and the current issue and all past issues can be accessed via this link. The next issue (Number 25) will be published during the autumn (2010).

The aim of the journal is to question and discuss deep ideas that underpin the mathematics curriculum and the teaching and learning of mathematics. It encourages teachers and researchers to share their thoughts and findings about a very broad range of theoretical and philosophical issues, from algebraic errors to zero as a sign.

What has philosophy got to do with the teaching and learning of mathematics?
A widely accepted belief is that:

> *All mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics.* René Thom

Researchers have found that:

> *Teachers' views, beliefs and preferences about mathematics do influence their instructional practice.* Alba Thompson

So a strong justification for the importance of philosophy for the teaching and learning of mathematics is that philosophies of mathematics and of education, often expressed in informal ideas and beliefs, both play a central role in the way the mathematics curriculum is structured and interpreted. Such philosophies also have a powerful impact through the ways that maths teachers present their subject and use resources such as ICT in the classroom.

What issues does the journal discuss?
Some of the many questions and topics that have been discussed are:

- is there a best way to teach mathematics? Do investigations, problem solving and discovery learning work? Can ICT make the teaching of mathematics better or more effective?
- what are teachers' personal beliefs and philosophies of mathematics? Are these reflected in our teaching, and if so, how and to what extent? Are there gaps between our beliefs and practices? Do schools unintentionally encourage such gaps?
- what are the aims of teaching and learning maths? Are these aims valid? Whose aims are they? For whom? Based on what values? Who gains and who loses? How does mathematics contribute to the overall goals of society and education?
- what is mathematics? How can its unique characteristics be accommodated in a philosophy? Is mathematics discovered or invented? Is it timeless and 'god-given' or socially constructed? There is a raging controversy over these questions!
- how is mathematics learned? What learning theories have been developed? This leads to another controversy concerning constructivist and social theories of learning mathematics. How much do children create their own knowledge? How can and should maths learning be assessed? What is mathematical ability and how can it be fostered? Is mathematics accessible to all?

- is there a best method for researching the teaching and learning of mathematics? Has the philosophical conflict between the scientific (quantitative) and the interpretative (qualitative) research paradigms been resolved?

There have also been themed issues treating such topics as:

- mathematics in art
- social justice in mathematics teaching
- mathematics and the use of signs.

The next issue (Number 25) has *Critical Mathematics Education* as its theme. This raises more controversy by asking political questions such as:

- can politics and human values have any relevance for mathematics?
- how can mathematics education help citizens relate to society and enhance democracy?
- how can mathematics critically assess itself and its uses in education and society?

The journal includes and welcomes contributions from people in all roles in mathematics education; from students and teachers, researchers and world-class scholars. If you have a preliminary idea, a course assignment or a finished paper, feel free to send it to me, the editor – you are guaranteed a friendly response! Your own ideas are probably more interesting and useful to other teachers and researchers than you realise. Currently many issues have had over 5 000 hits and the overall webpage more than 100 000 visitors.

Paul Ernest, editor, *Philosophy of Mathematics Education* journal

5 things to do this fortnight

- This is a good time to book your place on the NCETM National CPD Conference *Professional Learning Networks: learning better through learning together.*
  Date: 1 December 2010
  Time: 10am - 4pm
  Location: Sheffield. Venue: TBC

  Join teachers from across the country to explore the benefits of networking and collaboration to enhance your mathematics teaching and learning. Hear from practitioners who have created networks of all sizes across their department, school or college, local cluster group, HEIs and industry to discover how collaboration by individuals and organisations can help tackle issues, deliver practical resources and increase pupil attainment.
  The day will include:

  - an exhibition showcasing NCETM-funded projects that focus on creating networks to share ideas between individuals, departments and institutions, tackle issues in mathematics learning and support professional development
  - interactive group sessions to explore the power of working and learning together
  - workshops to present examples of collaboration and networks from schools, colleges, subject associations and organisations
  - keynote presentations from inspiring individuals who utilise networks.

  Details of the programme will be published on the portal next month. The conference is free to attend and is open to teachers, lecturers and advisers from all sectors. To book your place please email events@ncetm.org.uk.

- Have you explored the maths careers website? It is packed with articles about how mathematics is used in life all over the world. Students can read the career profiles of people who are applying their mathematical knowledge and qualifications in a wide range of occupations such as environmental protection, business and sport. The site, which was re-launched last December, now contains much new material, including pages for teachers and careers advisers from which you can download an imaginative variety of useful free resources and activities.

- The 2011 Mathematical Association Annual Conference, *Mathematics: The Big Picture*, will take place at Loughborough University from 14 to 16 April 2011. More details are on the MA website.

- The next London Mathematical Society Popular Lecture is *Clutching at Random Straws*, and will be delivered by Matt Parker at the University of Birmingham on 29 September, commencing at 6.30 pm. To attend you must apply to the LMS by 24 September.

- Have you considered joining the Royal Institution? If you are a member you can take two children to the free Royal Institution Maths masterclass: chases and escapes at 10.30 am on 27 October.

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Diary of a subject leader

Issues in the life of an anonymous Subject Leader

We know that mathematics is everywhere – and the summer vacation is no exception! During our break in France we spent a few days with a French colleague at his home in the Burgundy area. I met him back in 1990 at a conference on history in mathematical education, organised by the British Society for the History of Mathematics, and we have corresponded and collaborated ever since.

We spent some time bringing each other up to date with what we were doing, and plotted a couple of new ventures. He teaches a lot of his mathematics through English so was keen to learn some more of the terminology we use. In the past I have accommodated mathematics teachers from France who spend a fortnight with us and go into school to observe lessons and participate in the daily life of a teacher. They always find it interesting to see the faculty meetings in school and the half-termly mathematics teacher meetings, and also the Royal Institution mathematics masterclasses. It is worth finding out what is happening near to you and getting involved – the richness of the activities is just the thing to enthuse young mathematicians.

My French colleague took us to an absolutely fantastic library of old books in the museum at Semur-en-Auxois where we spent some time perusing books from the 16th century onwards (yes – you can browse through them).

There is a very good selection of mathematical books and I was interested to find some references to Galileo's compass of proportion since I'm working on a little project using it with students.

Another morning saw us interchanging electronic copies of other old texts. There are many opportunities for using bits of these in the classroom, and a search on Google books can often give electronic versions free of charge, including The Diarian Repository, which consists of all the mathematical questions published in The Ladies' Diary from 1704 to 1770. These vary in difficulty (but all the answers are there!).

While we were in France we paid tribute to Vauban (1633-1707), the French engineer who fortified many French towns in the reign of The Sun King (Louis XIV) by visiting Neuf-Brisach. The symmetry of this World

Heritage site is stupendous and the mathematics that can be planned around such sites can give rise to a lot of functional activity.
Plenty of material to keep me busy!