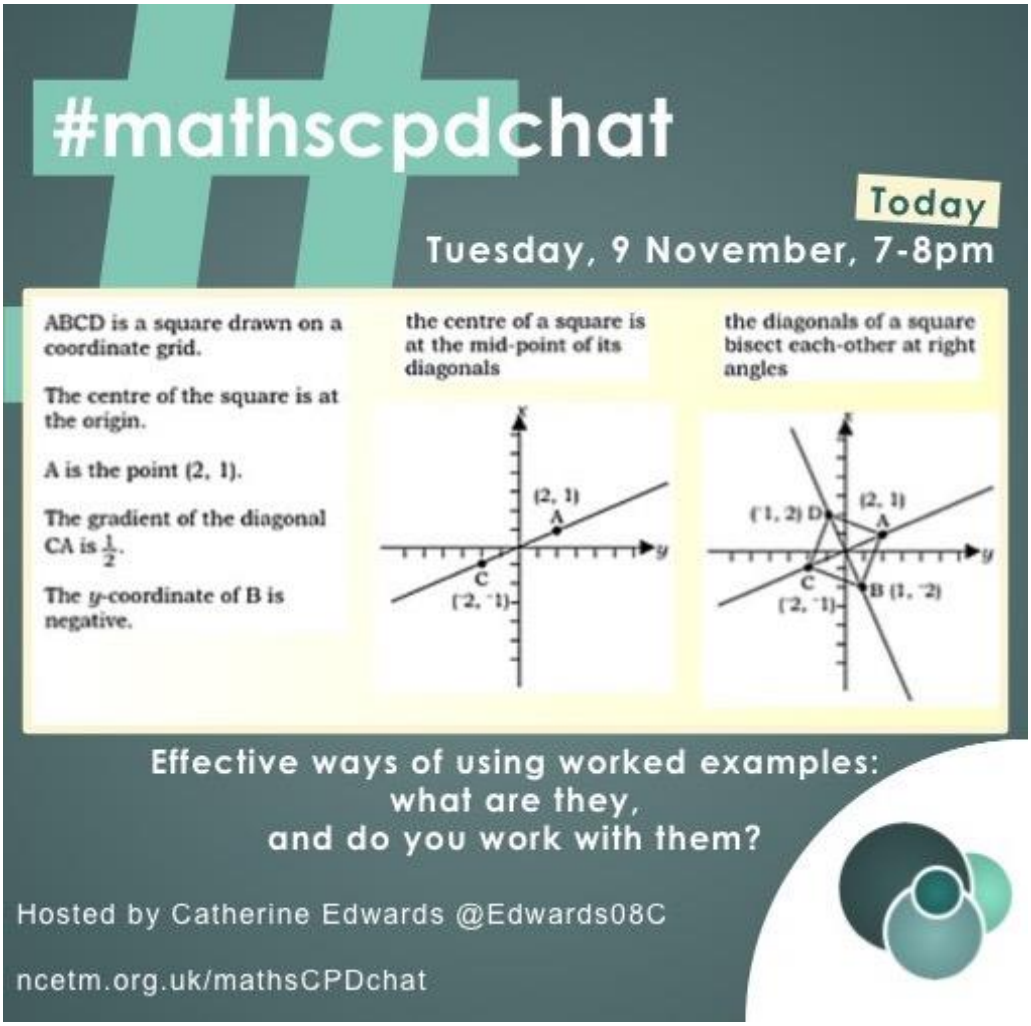
#mathscpdchat 9 November 2021

Effective ways of using worked examples: what are they, and do you work with them?
Hosted by Catherine Edwards

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



The links shared during this discussion were:

Math Mistakes which is a website by Michael Pershan who is a math(s) teacher in New York. The purpose of the site is to collect, organise and make sense of mistakes that students make while doing mathematics. Michael also hopes to use the mistakes to support the devising of worked examples that students can learn from. It was shared by Catherine Edwards

GLT Book Club: Teaching Math with Examples which is a Greenshaw Learning Trust Book Club video (June 2021) in which Michael Pershan talks to maths teachers about his work and his book Teaching Math with Examples. It was shared by Catherine Edwards

Michael Pershan on Worked Examples which is an ERRR podast interview with Michael Pershan in which he talks about principles that underlie learning from 'examples, self-explanation and feedback'. It was shared by Catherine Edwards

Mr Barton Maths Podcast - Michael Pershan:Teaching with worked examples- part 2 which is a podcast interview where Craig Barton talks to Michael Pershan about his ideas and teaching approaches that are related to mistakes students make, and the use of worked examples in the teaching and learning of mathematics. It was shared by Catherine Edwards

Algebra by Example which are 'Example-based Problem Sets for Algebra' that are intended to give students 'practice solving problems and practice modelling, analysing, critiquing and articulating mathematical arguments'. It was shared by Catherine Edwards

BerwickMaths which is a BerwickMaths ('The Centre of Worked Examples') in which the author aims to clarify the meaning of, and help maths teachers 'deliver', 'instruction that involves prompt shifts progressively from highly-prompted examples to unprompted examples'. It was shared by Catherine Edwards

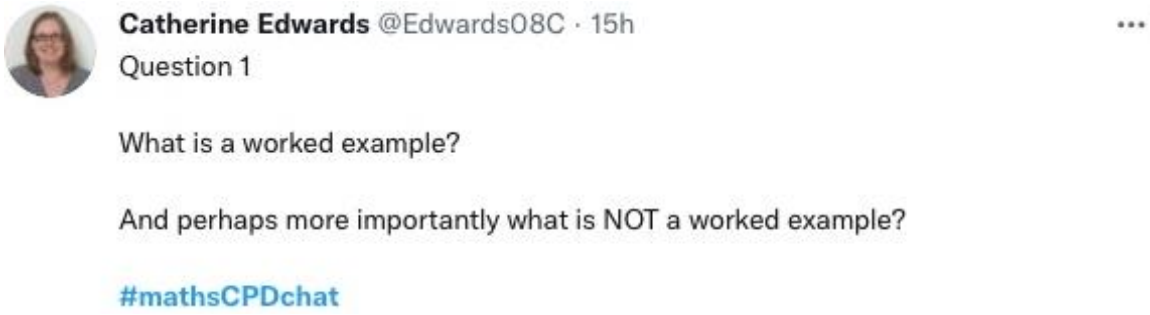
Teaching Maths with Worked Examples which is an online page created by Karen Hancock. It contains some of her thoughts about, and experiences of, working in mathematics with worked examples, together with some of the worked examples that she has herself designed/devised. It was shared by Catherine Edwards

Tom Manners Online CPD which is are online video sessions for maths teachers 'on different areas of representation, the importance of language that we use', and some about 'the NCETM's approach to Teaching for Mastery'. It was shared by Tom Manners

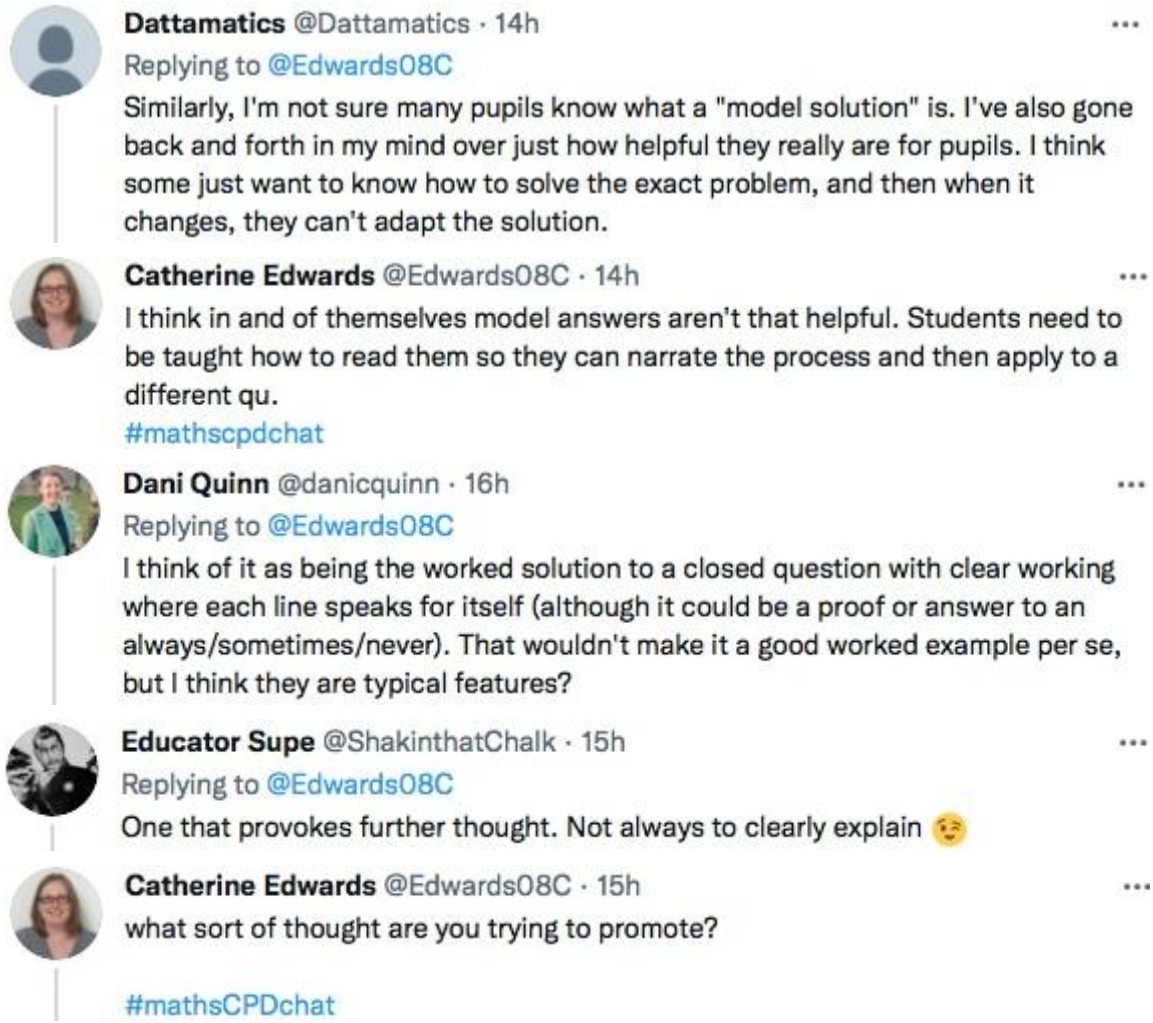
During this busy discussion many more tweets and 'conversations' were in response to the host's first question, rather than to any of her other three questions. The screenshots below show most of those replies and conversations, which focussed on various issues, such as: distinguishing between 'static' and 'live' ways of using worked examples, endeavouring to establish a common and correct use of mathematical language by all teachers and students in a school, whether learning from worked examples is enhanced when students use individual

whiteboards, and the physical or IT materials that help some teachers use worked examples as effectively as possible. **Click on any of these screenshots-of-a-tweet to go to that actual tweet on Twitter.**

The following conversations were generated by this tweet from Catherine Edwards:



and included these 'first responses' from Dattamatics, Catherine Edwards, Dani Quinn, Educator Supe, and Mary Pardoe:



Educator Supe @ShakinthatChalk · 15h

Depends on the task at hand. Maybe boundary conditions, maybe as simple as where is the error here. The error in thinking (and one that gets levelled at text books) is to view the worked example(s) as a complete programme of understanding.

Catherine Edwards @Edwards08C · 15h

as always context is king!
what do you use to supplement worked examples?
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Mary Pardoe @PardoeMary · 15h

Replying to @Edwards08C

A worked example might consist of a sequence of visual of prompts? Such as (in this sequence each one of the six single diagrams might be displayed separately)
...
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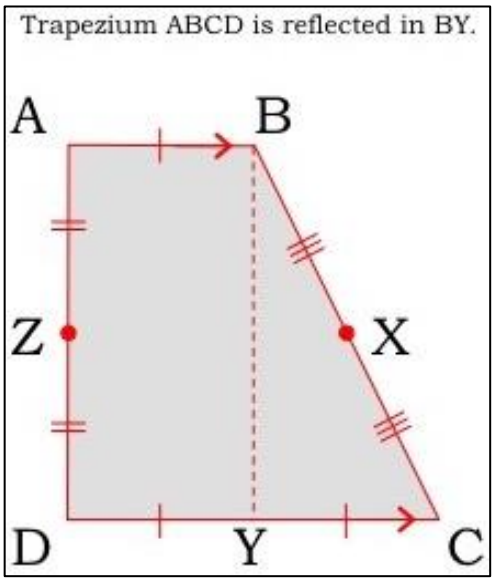
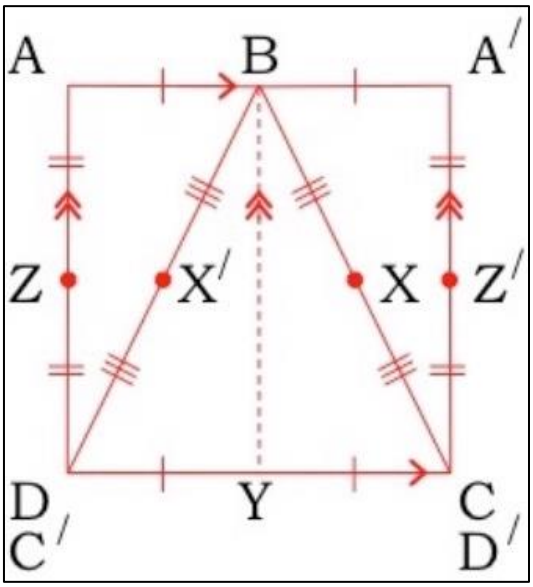
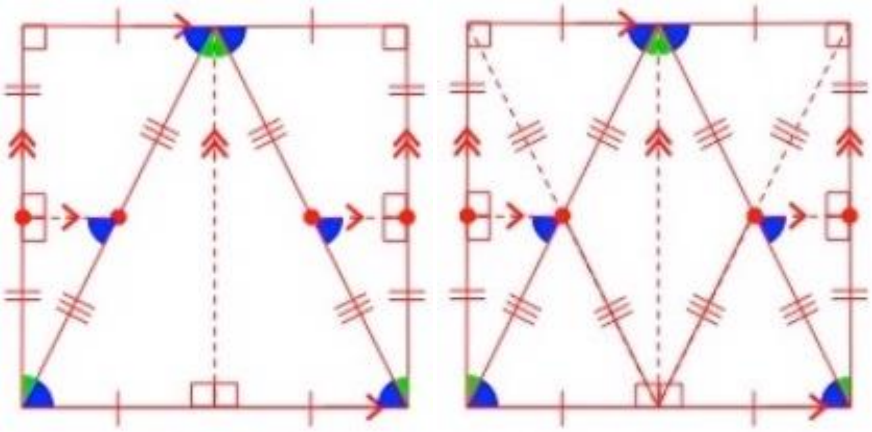
… first this …



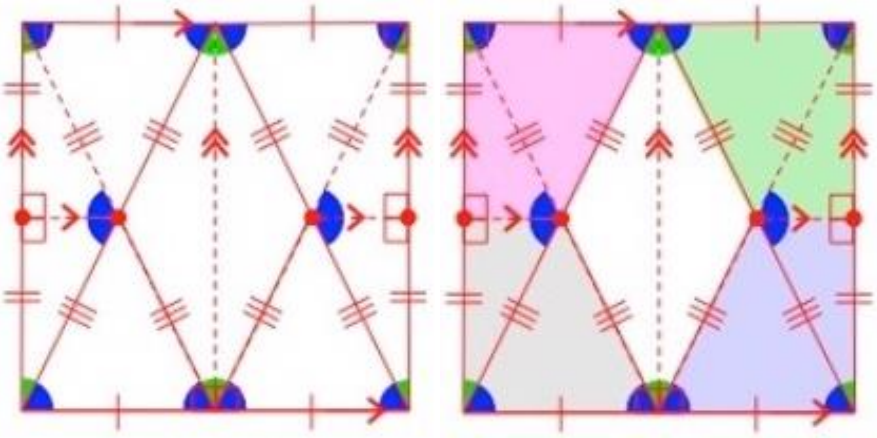
… then this …



… then these (left to right) …



… then these (left to right) …



Catherine Edwards @Edwards08C · 15h

looking at @danicquinn point- is that a worked example or is it a prompt which leads to a worked example?

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Mary Pardoe @PardoeMary · 15h

Could be a worked example … the diagrams are pre-drawn … see my later tweet …

#mathsCPDchat

Mary Pardoe @PardoeMary · 16h

Replying to @Edwards08C

By prompting/generating thinking as you go through it?
e.g. … show these one at a time … each time a diagram is shown ask what knowledge they have that they can use to 'amplify' the diagram …
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this conversation involving Sharon Malley, Catherine Edwards and Richard Dare:

Sharon Malley @mathsmumof2 · 15h

Replying to @Edwards08C

It shows each stage of a mathematical calculation to produce an answer to a problem. I'm interested in the difference between live worked examples and static fully worked examples #mathscpdchat

Catherine Edwards @Edwards08C · 15h

me too- I think there is a place for both, but which one when?
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Sharon Malley @mathsmumof2 · 15h

Live first, then static to discuss and examine maybe 🤔 #mathsCPDchat

Catherine Edwards @Edwards08C · 16h

Does it change for different topics/groups?

I like static then live for angle chasing

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Richard Dare @dare_richard · 15h

I like @mrbartonmaths 'silent teacher' to reveal the example step-by-step and defer narrating it and discussing it until a little later.

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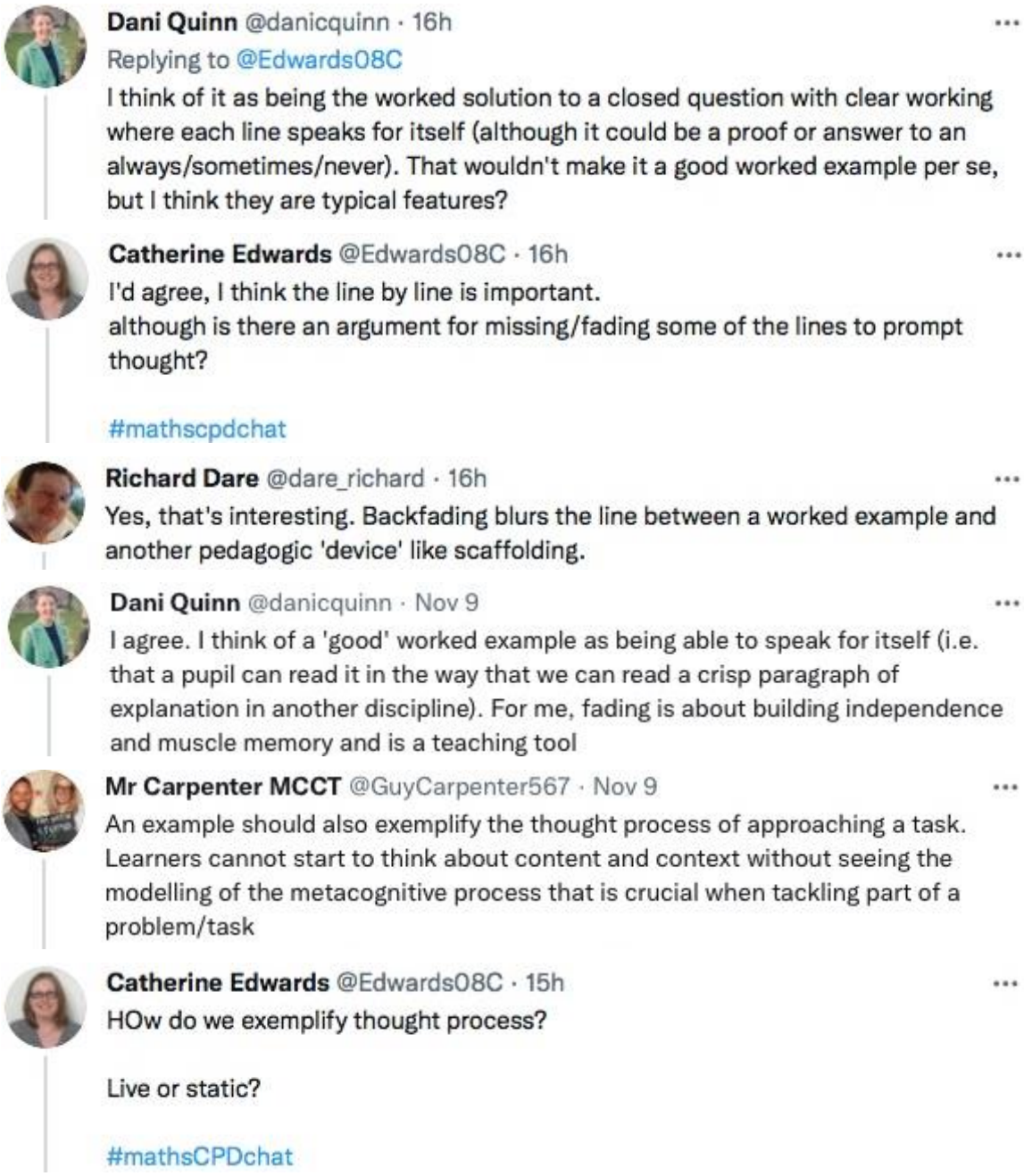
Sharon Malley @mathsmumof2 · 15h

It is such an effective strategy to ensure student focus is on the correct thing and of course it is a live worked example which you then as a static example you then narrate and explain #mathsCPDchat

these from Dani Quinn, Catherine Edwards, Richard Dare, Guy Carpenter and Peter Williams:



Peter Williams @MathsImpact · 15h

I find that live works much better.

I'm also a big fan of the "my turn" "your turn" approach.

I want students to be fully focused on what I'm doing when I model, not on recording it in their books.

Pens down, watching and listening.

#mathscpdchat

Catherine Edwards @Edwards08C · 15h

oh absolutely, it's really upset 9 set 1 this year, that they can't just copy down a pretty example and I expect them to listen to what I'm saying!

#mathsCPDchat

this long conversation started from one of Guy Carpenter's comments, which tweet had also been part of a conversation shown above, and consisted of tweets from Dani Quinn, Sharon Malley, Catherine Edwards, Richard Dare and Tom Manners:

Mr Carpenter MCCT @GuyCarpenter567 · Nov 9

An example should also exemplify the thought process of approaching a task. Learners cannot start to think about content and context without seeing the modelling of the metacognitive process that is crucial when tackling part of a problem/task

Dani Quinn @danicquinn · 16h

Replying to @GuyCarpenter567 @dare_richard and @Edwards08C

For sure. I love live modelling, whether silent or narrated (time and place for each). I feel sad when the whole working out is revealed with a click on PowerPoint. It's not what the experience of doing maths is like!

Mr Carpenter MCCT @GuyCarpenter567 · Nov 9

Replying to @danicquinn @dare_richard and @Edwards08C

Agreed. Non engaging and boring.

Mr Carpenter MCCT @GuyCarpenter567 · Nov 9

I think it was @danicquinn that did a session on language and modelling responses from being specific to general. A real game changer for modelling the path to expertise and transferring key core knowledge and skills to similar problems.

Sharon Malley @mathsmumof2 · Nov 9

For my @leicesterscitt trainees I show 2 videos I recorded of myself where I show the exact same working but with different language in my explanation #mathsCPDchat

Dani Quinn @danicquinn · Nov 9

How does the language differ? This is so interesting. What did they notice?

Sharon Malley @mathsmumof2 · Nov 9

I solved an equation with an unknown on both sides, one with correct modelling using language like 'unknowns' 'inverse operation' 'maintain equality' etc the other where I 'change the side, change the sign' numbers 'disappear' etc #mathsCPDchat

Sharon Malley @mathsmumof2 · Nov 9

The correct language one is much longer, the 'incorrect' one is shorter so appears more efficient but it is much more difficult to understand why each stage is happening if you don't already know what to do #mathsCPDchat

Catherine Edwards @Edwards08C · Nov 9

"talk like a mathematician"

I've set myself a mini goal to be really strict about the correct use of language this year

#mathsCPDchat

Sharon Malley @mathsmumof2 · Nov 9

It pays massive dividends if you can do it consistently across all staff. I asked a y9 student if she understood what I meant by 'maintain equality' in the context of solving equations she replied with 'yes like we do with fractions and multiplying by 1' #mathsCPDchat

Catherine Edwards @Edwards08C · Nov 9

do you discuss and choose vocabulary in faculty time?

how do non specialists feel about using the language?

curious as we ahve started talking about some similar ideas

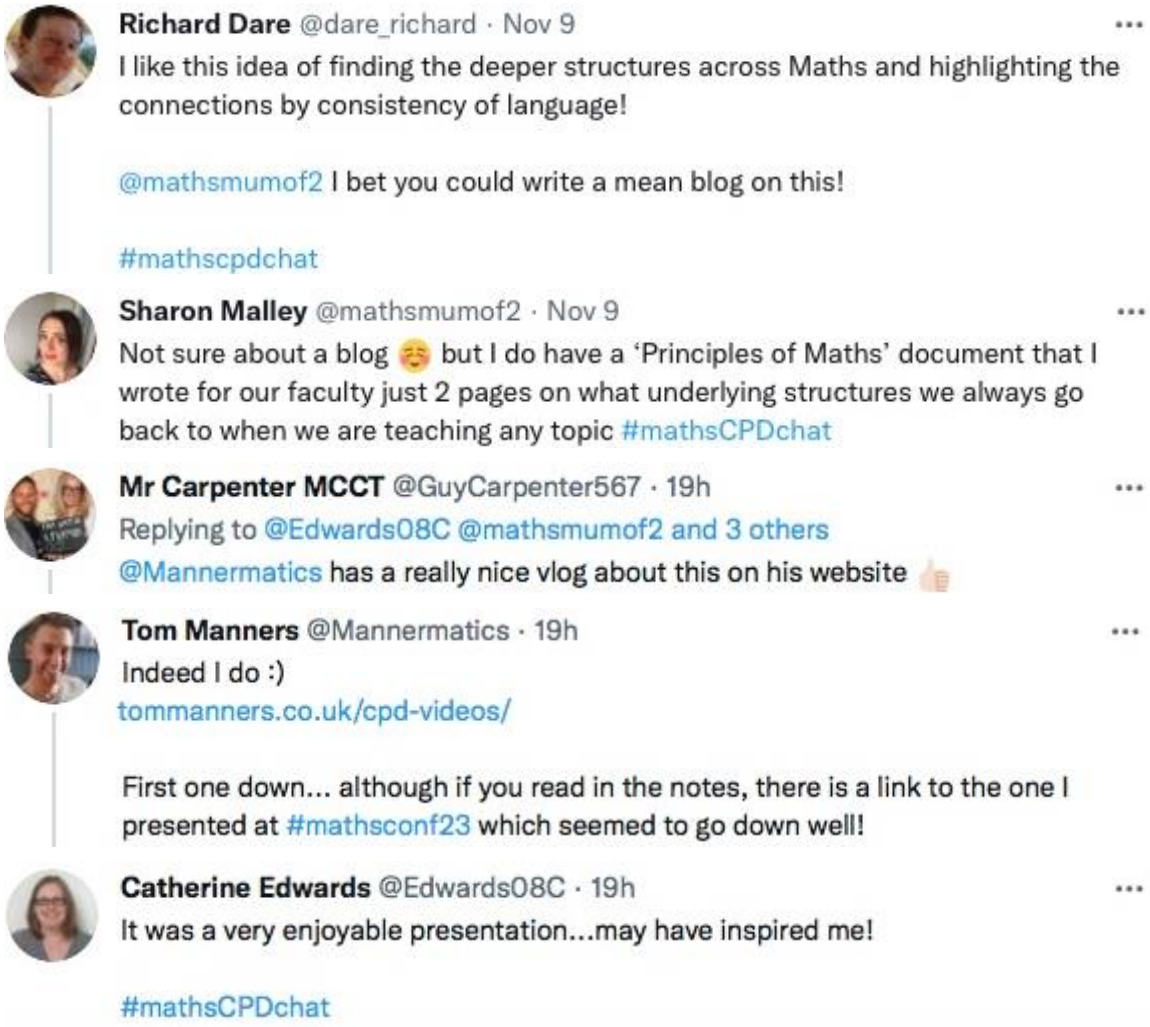
#mathsCPDchat

Sharon Malley @mathsmumof2 · Nov 9

We plan examples centrally and in faculty time discuss how to model with a massive emphasis on correct language and definitions of terms. This reflects the whole school culture on vocabulary though. A small maths team so no non-specialists as yet. #mathsCPDchat

Sharon Malley @mathsmumof2 · Nov 9

We look at different examples depending on our current need but actually the language of it is defined at the start is then used across all topics e.g. commutativity or maintain equality it is the language of maths not a topic to topic vocabulary. #mathscpdchat



this brief conversation 'branched off' from one of Richard Dare's comments in a conversation shown above, and was with Catherine Edwards:

Richard Dare @dare_richard · 15h

Oh, mistake analysis! Great stuff, but - yes - I'd say another strategy. One I'd use after using worked examples.

Whether worked examples are necessarily direct-instruction, I think is moot. @mrbartonmaths's use is d.i., whilst I'm not sure @mpershan's model is

#mathscpdchat

a tweet from Guy Carpenter was a reply to one of Catherine Edwards' comments in a conversation shown above:

Catherine Edwards @Edwards08C · 15h

HOw do we exemplify thought process?

Live or static?

#mathsCPDchat

Mr Carpenter MCCT @GuyCarpenter567 · 16h

Replying to @Edwards08C @danicquinn and @dare_richard

Both. 'Here I am going to do …… because ……' Also when looking at misconceptions that have happened 'Learner A has done…… is this right? Why do you think that? Well we can see that if I do this……' And so on.

this conversation started from one of Peter Williams' comments, which had been part of a conversation shown above, and consisted of tweets from Dani Quinn, Nathalie Leighton, Richard Dare and Catherine Edwards:

Peter Williams @MathsImpact · 15h

I find that live works much better.

I'm also a big fan of the "my turn" "your turn" approach.

I want students to be fully focused on what I'm doing when I model, not on recording it in their books.

Pens down, watching and listening.

#mathscpdchat

Dani Quinn @danicquinn · 17h

Replying to @MathsImpact @Edwards08C and 2 others

100%
I would love a nationwide moratorium on:
clicky-clicky modelling (i.e. powerpoint instead of live)
copying down
asking 'do you understand?'
going straight to independent work after a model

Nathalie Leighton @LeightonM4ths · 19h
Replying to @danicquinn @MathsImpact and 3 others
Definitely needs a mini whiteboard activity between modelling and independant practice
#mathscpdchat

Richard Dare @dare_richard · 18h
Replying to @LeightonM4ths @danicquinn and 3 others
YES!

Catherine Edwards @Edwards08C · 18h
ugh, I hate mini whiteboards

why would you want to rub your work out?

Mistakes are how we learn.

plus the pens always run out!

#mathsCPDchat

Richard Dare @dare_richard · 18h
We'd burn through books if students had to write on them big enough for me to see all students' responses!

They're wonderful!

Yes, mistakes are great - celebrate them.

It's the boards' display value which is immense.

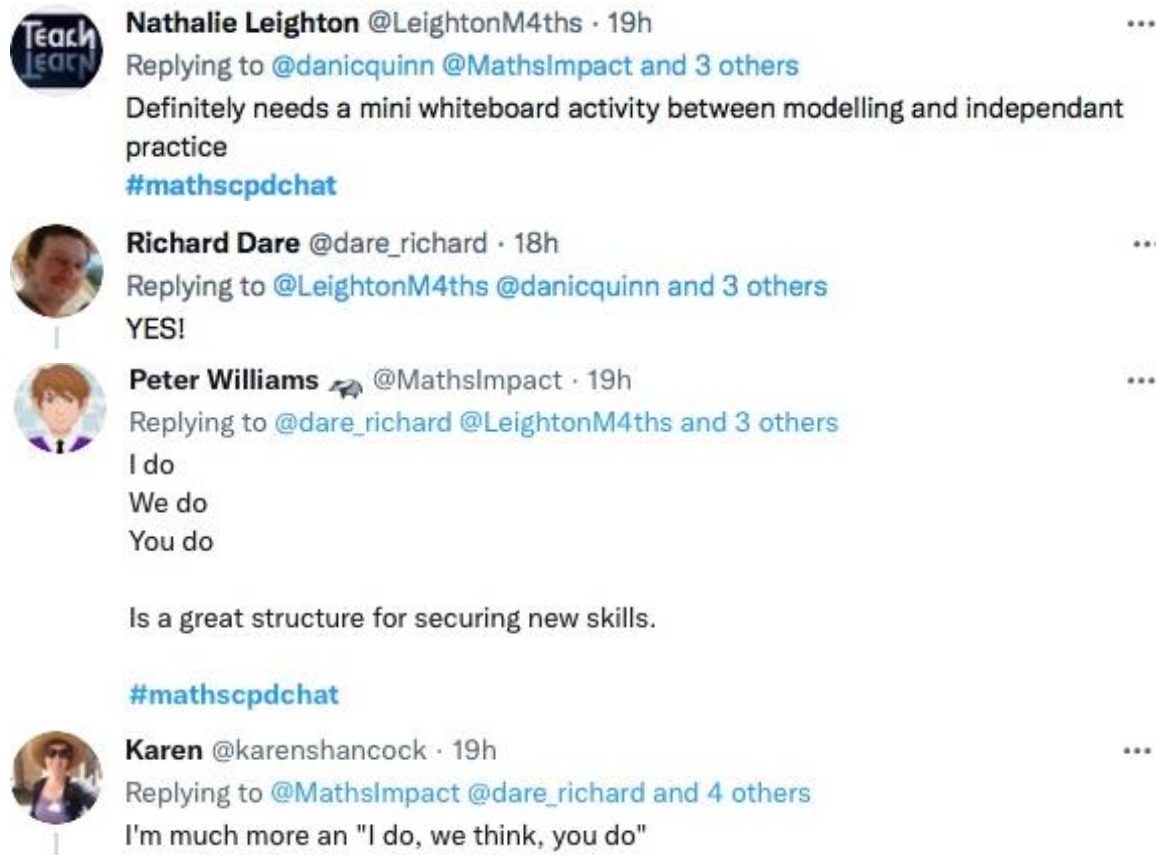
#mathscpdchat

Nathalie Leighton @LeightonM4ths · 19h
Replying to @Edwards08C @dare_richard and 3 others
I agree the pens are a pain,
but so worth it to see that they can solve the questions (not necessarily why 😳)
and also to make sure that they all show their workings every time and that they present their work "my way"

...teaching solving linear equ to Y8 this week 😏

Nathalie Leighton @LeightonM4ths · 19h
And rubbing your work out is what allows students to not be shy about making mistakes, I think.
I bought a2 size whiteboard for my Y12/13 classes and they work on those all the times in lesson - great for sketching .
#mathscpdchat

this conversation was prompted by an exchange between Nathalie Leighton and Richard Dare which was part of another conversation shown above, and included tweets from Peter Williams, Karen Hancock and Catherine Edwards:



Generating Sequences

Date:

Worked Example

Alice has answered this question correctly

Write down the first four terms and the 100th term of

$u_n = 3n + 8$

$u_1 = 3(1) + 8 = 3 + 8 = 11$
$u_2 = 3(2) + 8 = 6 + 8 = 14$
$u_3 = 3(3) + 8 = 9 + 8 = 17$
$u_4 = 3(4) + 8 = 12 + 8 = 20$

$u_{100} = 3(100) + 8 = 300 + 8 = 308$

Study the solution carefully and answer these questions

(1) Alice thinks that she had just double the 100th term to find the 200th term.
Explain why Alice is wrong.

(2)What if the question had asked for the first four terms of the sequence
$u_n = 8 - 3n$

Date:

Generating Sequences from the nth term

Find the first 5 terms in each sequence:

1. $u_n = 2n$
2. $u_n = 3n$

10. $u_n = \frac{1}{2}n + 10$

Richard Dare @dare_richard · 19h

Replying to @karenshancock @MathsImpact and 4 others

Is there a bit of mini-whiteboard or multiple-choice questioning in there too? #mathscpdchat

Karen @karenshancock · 19h

Replying to @dare_richard @MathsImpact and 4 others

This one.
Read worked example. Explain to neighbour.
Cold call to explain to class.
Think, pair share on (1).
(2) on mini whiteboards, or direct to paper if they prefer.

Catherine Edwards @Edwards08C · 19h

Is it substantially different for A-level than KS3?

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Karen @karenshancock · 19h

A Level:

M07 Differentiation (part 2) Date:

Worked Example:

$$y = x^2 - \frac{16}{x}$$

has one local minimum and one root.

Find these two points and sketch the curve.

$y = x^2 - \frac{16}{x}$

$y = x^2 - 16x^{-1}$

$y' = 2x + 16x^{-2}$

At local minimum:

$y' = 0$

$2x + 16x^{-2} = 0$

$2x = -16x^{-2}$

$x^3 = -8$

$x = -2$

$y =$

Local minimum (,)

Root:

$y = 0$

$x^2 - \frac{16}{x} = 0$

$x^2 = \frac{16}{x}$

$x^3 = 16$

$x =$

M07 Differentiation (part 2) Date:

Curve:

Cuts x-axis at (, 0)

Local minimum at (,)

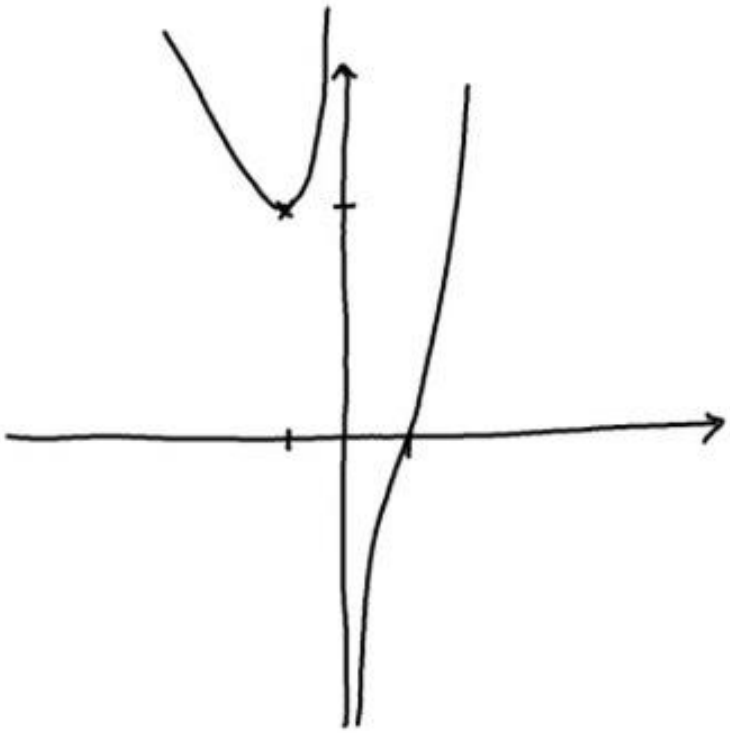
Vertical asymptote at $x = 0$

As $x \to \infty^+$ $y \to \infty^+$

As $x \to \infty^-$ $y \to \infty^+$

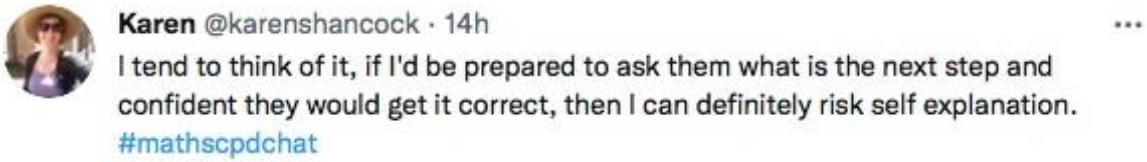
As $x \to 0^+$ $y \to \infty^-$

As $x \to 0^-$ $y \to \infty^+$



the following conversation developed from one of Dani Quinn's tweets in another conversation, and included tweets from RHMaths, Catherine Edwards and Peter Williams:

and finally this conversation was generated by a question from Bryony Black, and included tweets from Catherine Edwards and Mary Pardoe:



(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Replies to Catherine's second question …

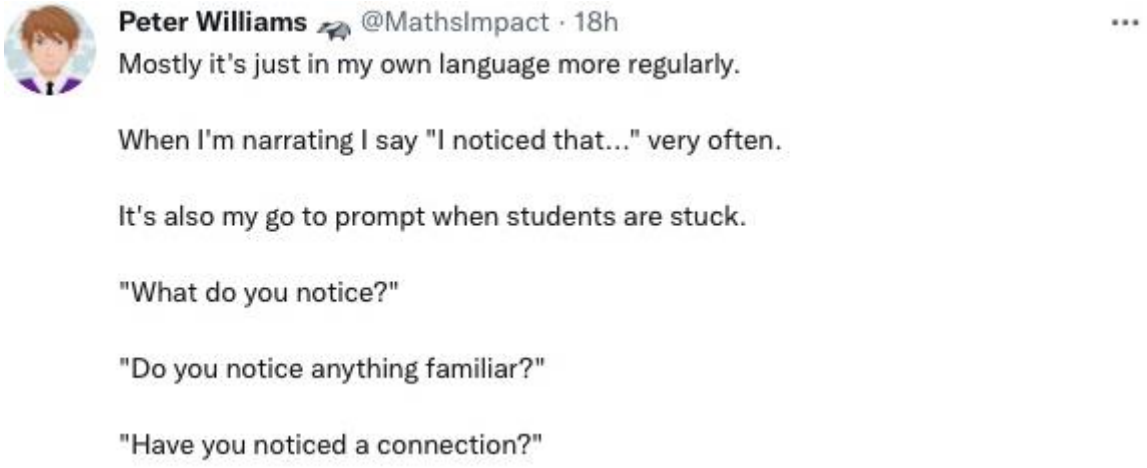
… included:

- a reminder to think about possible misconceptions before creating a worked example;
- a suggestion that it is effective to provide a 'model solution' and challenge students to 'annotate' it;
- a comment that it is advisable to pre-plan the questions (about/related-to a worked example) that you intend to ask students, for example …

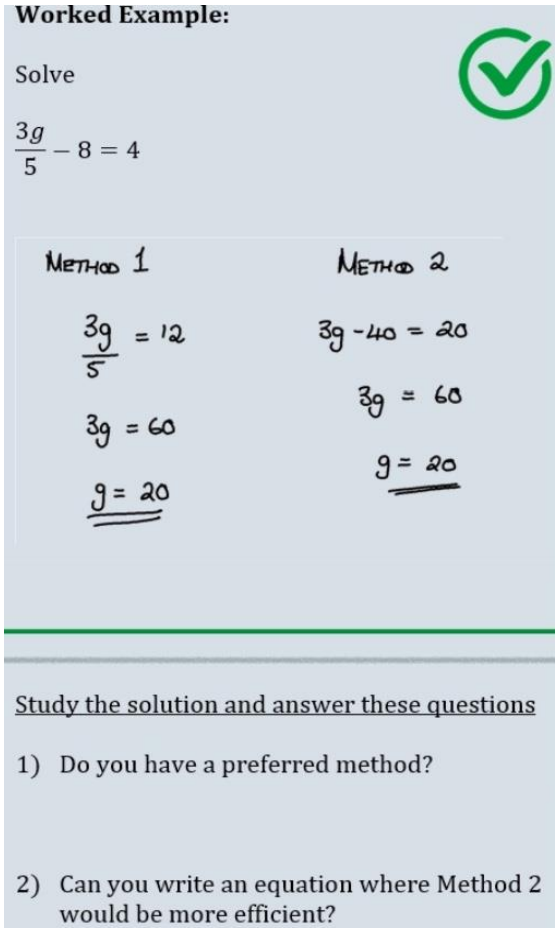
Suggestions made in response to Catherine's third question …

… included:

- show part of a solution and ask students ‘What do you notice?’ ‘What could/would you do next?’;
- the teacher might ‘speak-internal-reasoning-and-questioning’ out-loud as she works through a problem in front of the class … ‘I model my thought process’;
- talking to students about noticing:



- creating ‘model solutions’ for ‘minimally different questions’ and ‘same surface, different depth’ questions;
- challenging students to create their own examples, for example …



Catherine Edwards @Edwards08C · 19h

Replying to @karenshancock

I love the question about write a question where this method would be better.

I do find students get wedded to the "right" way of doing it

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- challenge students to work (possibly in pairs/small groups) on method selection without actually solving the problems;

Comments in response to Catherine's last question …

Catherine Edwards @Edwards08C · 19h

Question 4

How can we best match worked example types and structures to different groups and subjects?

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… included:

- using ‘probing’ questions (possibly to find out if students have ‘understood a ‘compressed step’ in a worked example’) … to which students might respond by writing/drawing on individual whiteboards;
- always creating fresh worked examples for a new class … that getting to know students (‘read the class’) is important, but it cannot be rushed!