MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Fractions

Guidance on the teaching of fractions in Key Stage 1

Year 2

This short document provides guidance on the teaching of fractions in Key Stage 1.

As stated in the national curriculum, *'the principal focus of mathematics teaching in Key Stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value'*. As such, the majority of teaching should be focused on whole-number concepts and arithmetic, and the teaching of fractions in Key Stage 1 will, by necessity, be brief and light-touch. As fractions form such a significant part of the Key Stage 2 curriculum, the Year 3 *Mastery Professional Development Materials* start from first principles for the teaching and learning of fractions, so that firm foundations can be built for the rest of the primary programme of study.

These guidance notes outline a brief teaching progression which will cover the statutory requirements of the national curriculum in Key Stage 1. As stated in the national curriculum, within a Key Stage schools *'have the flexibility to introduce content earlier or later than set out in the programme of study.'* It is suggested that all the Key Stage 1 fractions content is covered in Year 2.

National curriculum for fractions in Key Stage 1

Children should be taught to:

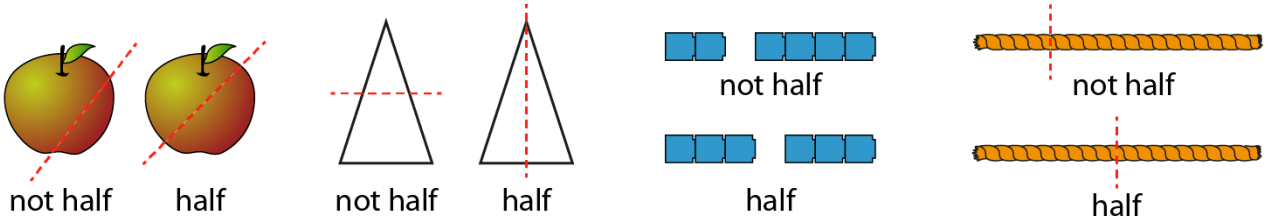
- recognise, find and name a half as one of two equal parts of an object, shape or quantity (Y1)
- recognise, find and name a quarter as one of four equal parts of an object, shape or quantity (Y1)
- recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity (Y2)
- write simple fractions, for example $\frac{1}{2}$ of 6 = 3, and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$ (Y2).

Suggested teaching progression for Key Stage 1 fractions content

1. Name the fractions 'one-half', 'one-quarter' and 'one-third' in relation to a fraction of a length, shape or set of objects

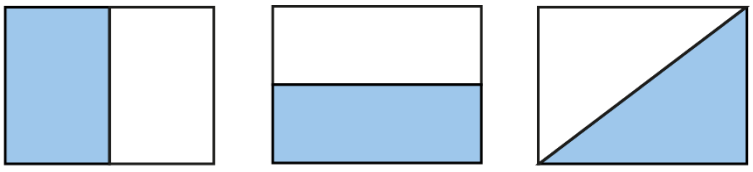
One-half

Throughout Key Stage 1, children have started to develop a good understanding of whole numbers up to 100. This includes having learnt about the numbers 0 and 1. However, sometimes we have more than zero but less than one, for example *'half a slice of toast'*. We can use numbers called fractions to tell us about these amounts. Children will have heard people talk about *'a half'* and *'a quarter'* in everyday contexts, such as when sharing out food or discussing the time. They will probably have used these terms themselves, for example, when comparing their age with other children. Although children are likely to be familiar with the word *'half'* and may well associate it with two parts, the importance of these halves being equal needs to be emphasised. Children may well say things like *'Can I have the bigger half?'*, when sharing food. As such, initial focus should be on understanding that *'a half'* of something is one of two *equal* parts. Discuss this using objects such as an apple or folded paper shapes, small sets of objects (such as a stick of Unifix® cubes or groups of counters), and lengths/linear representations (such as a length of ribbon or string).



Starting with practical objects and pictures, teach children to identify whether something has been split in half (*'It is split in half because it has been split into two equal parts.'*) or not (*'It is not split into halves because it has not been split into two equal parts.'*).

Once children can do this, move on to providing opportunities for them to demonstrate half of a whole. For example, they might hold up three cubes and say, '*The six cubes are my whole, so this is half of the whole.*' Alternatively, they might draw shapes and then attempt to split them into two equal parts in different ways, for example:



One-third and one-quarter

Once children know that a half of something means that it has been split into two equal parts, introduce the words *'one-quarter'* as meaning one of four equal parts, and *'one-third'* as one of three equal parts. As before, look at objects, paper shapes, small sets and linear representations, and identify whether or not they are split into equal parts. Where they *are* split into equal parts, discuss what name we give to each equal part:

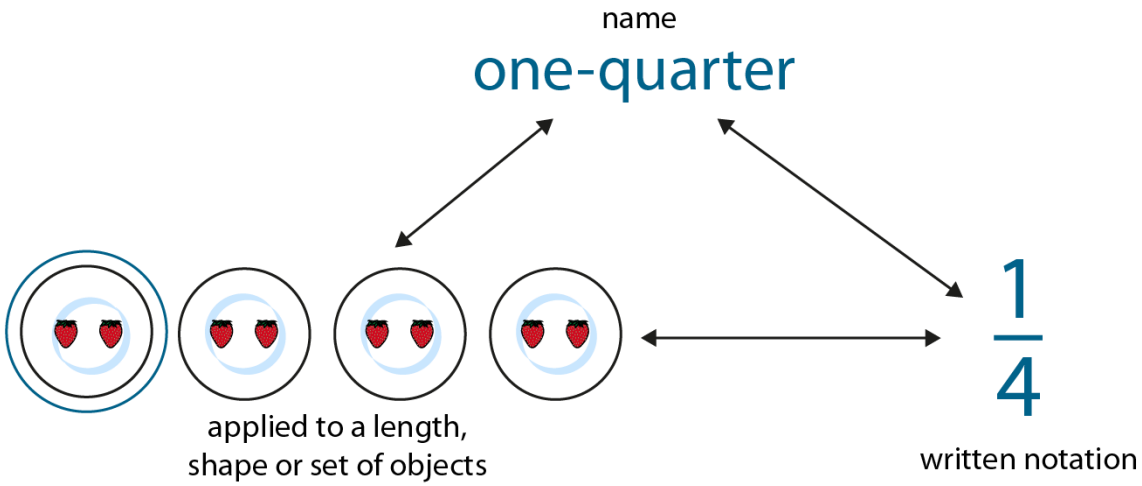
- *'The rectangle is split into three equal parts. Each part is one-third.'*
- *'The strawberries are split into four equal parts. Each part is one-quarter.'*

Once children can look at an image and identify whether it is split into halves, thirds, or quarters (or not), teach them how to find one-quarter and one-third. For example:

- *'Can you split it into three equal parts and give me one of the parts?'*
  'That's right – you have given me one-third.'
- *'Can you colour in one-quarter of the shape?'*
  'That's right – you need to split it into four equal parts, and colour one of the parts.'

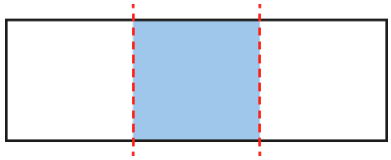
2. Read and write the fraction notation $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, and relate this to a fraction of a length, shape or set of objects

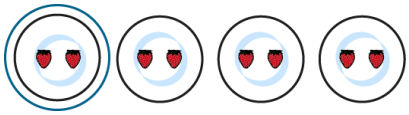
We have suggested starting from the names of the fractions (half, quarter and third) in this Year 2 teaching, as children may already be familiar with some of these. Children need to be able to link this to the written notation. The national curriculum for Key Stage 1 requires children to be able to make the links shown in the diagram below:



Look at a representation of each unit fraction with the children. Take each one in turn and begin by naming it, for example *'one-half'*. Then describe it, for example, *'We know this is one-half because the apple has been divided into two equal parts and we have one of them.'* Then show how to link this to the written notation, using the approach outlined in the tables below.

Model	Say	Write	Notation
 one-half	*'The apple has been divided…'*	Write the division bar.	$\frac{1}{2}$
	'…into 2 equal parts…'	Write '2' as the denominator.	
	'…and we have 1 of the parts.'	Write '1' as the numerator.	

Model	Say	Write	Notation
 one-third	*'The rectangle has been divided…'*	Write the division bar.	$\frac{1}{3}$
	'…into 3 equal parts…'	Write '3' as the denominator.	
	'…and 1 of the parts is shaded.'	Write '1' as the numerator.	

Model	Say	Write	Notation
 one-quarter	*'The strawberries have been divided…'*	Write the division bar.	$\frac{1}{4}$
	'…into 4 equal parts…'	Write '4' as the denominator.	
	'…and 1 of the parts is circled.'	Write '1' as the numerator.	

Children will need to learn the following:

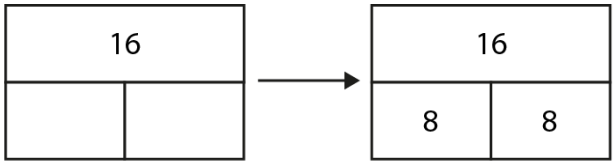
- How to write the fraction themselves.
  Give children lots of practice where they look at a picture of an object split into fractional parts. You then say the stem sentences aloud as they write.
- How to read the fraction.
  Ensure children use the name (e.g. *'one-third'*), rather than describing the written notation (*'one-over-three'*).
- How to link the name, written notation and a fraction of a length, shape, or set of objects.
  They need to be able to move between the rows of the tables above in any direction, switching between the model, the name and the written notation.

Note that thus far fractions of quantities have not been introduced. Where children are working with sets of objects, keep the focus on visually dividing them into equal parts, rather than saying, for example *'We have eight strawberries, so one-quarter of the strawberries is two.'*

3. Find half of numbers

Finding half of numbers is taught in *Spine 2: Multiplication and Division*, segment *2.5 Commutativity (part 2), doubling and halving*. From this, children should already know that, for example, finding half of 16 means splitting 16 into two equal parts. They should have learnt half of all the even numbers to 24. Refer back to *Spine 2*, segment *2.5*, as necessary, to revise this prior learning. Now that children have learnt that one-half is written as $\frac{1}{2}$, instructions to find one-half of numbers can be presented with the fraction written in notation form, rather than as a word. For example, *'Find* $\frac{1}{2}$ *of 16'*.

Relate 'finding half' to a bar model split into two equal parts. Whilst children might not need this to support them in finding half of 16 (they should have learnt their halves of even numbers to 24 by now), it is still important to make this link in order to support their work with thirds and quarters.



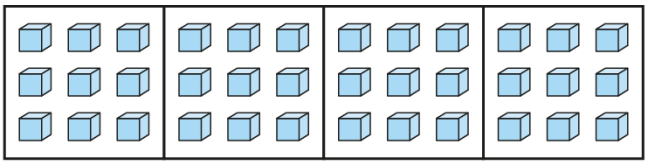
4. Find $\frac{1}{3}$ or $\frac{1}{4}$ of a number

The national curriculum for Key Stage 1 also requires children to find $\frac{1}{3}$ and $\frac{1}{4}$ of numbers. This needs some thought, as children have not yet learnt to divide by three or four (these are both in the Key Stage 2 curriculum). However, the children have learnt to divide shapes and lengths into three and four equal parts. This knowledge can be used to draw bar models as a scaffold for dividing quantities into three or four equal parts. Generally, the *Mastery Professional Development Materials* encourage you to build on what the children know already and to try *not* to rely on counting in ones to find an answer to a question. However, as the underpinning understanding of dividing by three and four is not yet in place, there is no alternative but to resort to counting in ones as a universal strategy to meet this objective. This approach is introduced first. Some examples are then highlighted where the children may be able to use an alternative and more efficient strategy.

Tell the children they are going to find one-quarter of a number of cubes. Ask them to draw a large bar divided into four equal parts and then to take a number of cubes (select a multiple of four up to 48, such as 12 cubes). Show the children how to share the cubes equally between the four parts: 'dealing out' the cubes one at a time, into each of the four parts. Once the cubes are split into the four parts, explain that we can count the cubes in each quarter to work out that $\frac{1}{4}$ of 12 = 3.

Note that for smaller amounts (such as 12 cubes), children may 'just see' that three cubes should be placed in each part. However, for larger quantities, for example 36 cubes, the vast majority of Year 2 children will need to use one-at-a-time 'dealing out' to divide them into four equal parts. It is a good idea to model an organised layout of the cubes in each part (as shown below), as this will help children to check visually that each part has the same amount.

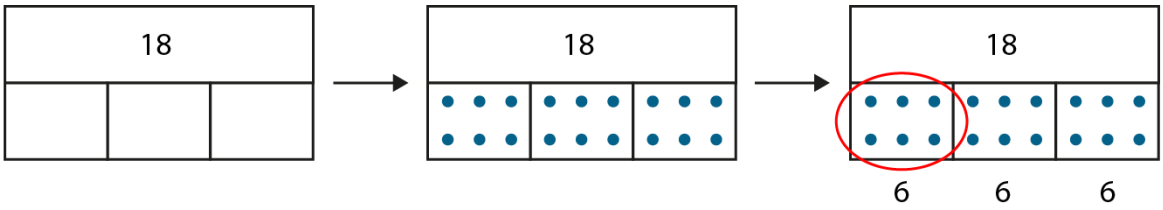
Using cubes to calculate $\frac{1}{4}$ of 36:



This whole sequence should be repeated for three equal parts (finding $\frac{1}{3}$), making sure that the numbers you ask the children to find one-third of are multiples of three (up to 36).

Once children are confident with this, repeat the process but drawing dots, so that the children are independently able to find $\frac{1}{4}$ or $\frac{1}{3}$ of an amount without needing practical resources. They should be able to produce diagrams like those provided below. Children will need to be taught to work left-to-right, as working forwards and backwards often results in more dots being drawn in the middle section(s).

Drawing dots to find $\frac{1}{3}$ of 18:



Draw a bar model, divided into 3 equal parts.

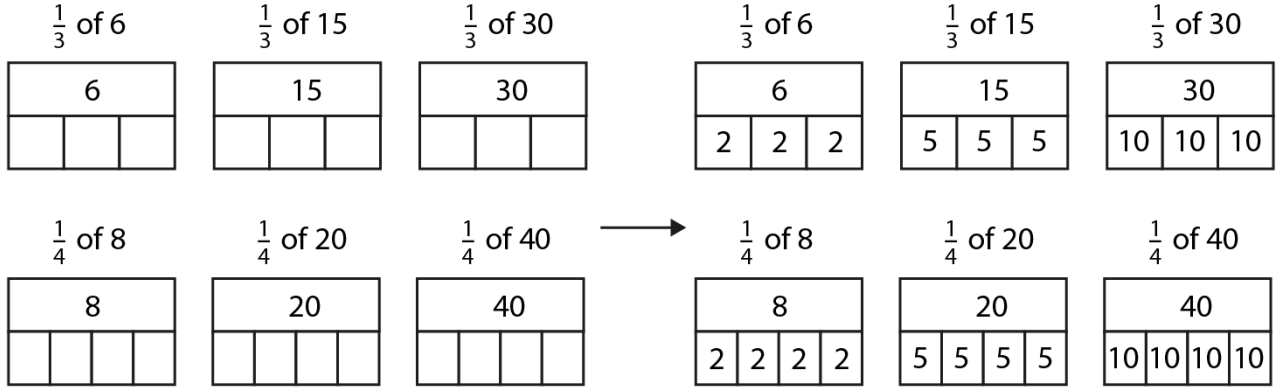
Share out 18, by drawing dots.

Write the number of dots you have in each part. Then, circle the number of parts you need.

There are a couple of ways the children might be able to work more efficiently, without drawing dots:

- You could teach children that to find $\frac{1}{4}$, we can halve and then halve again. This can support them in finding an answer for numbers that are simple to halve, for example $\frac{1}{4}$ of 28. Be aware that halving some numbers, such as 32, is hard for Year 2 children, so it is unlikely this strategy can be universally applied.
- For groups of two, five and ten, children have learnt about division as grouping. Where the size of the parts results in two, five or ten, some children may be able to spot the link once they have drawn a bar model. They may be able to take the total and the number of parts, and then just write the size of each part onto the bar model, without needing to share out dots first.

For example:



Whichever method they use, children need to be able to apply their understanding to both the following types of scenario:

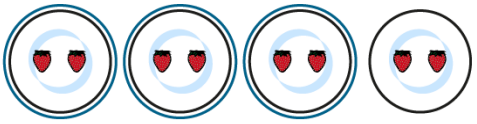
- simple, everyday contextual problems, for example, *'James had 20 seeds and planted $\frac{1}{4}$ of them. How many seeds did he plant?'*
- non-contextual problems, for example, *'Find $\frac{1}{3}$ of 21'.*

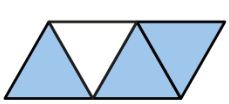
5. Find $\frac{2}{4}$ and $\frac{3}{4}$ of an object, shape, set of objects, length or quantity; recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$

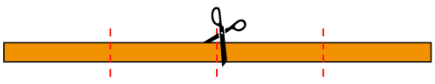
Once children have a firm understanding of how to find $\frac{1}{4}$ of an object, shape, set of objects, length or quantity, they need to extend this to $\frac{2}{4}$ and $\frac{3}{4}$. Look back at some of the 'quarters' images you used to introduce the spoken and written forms (*'one-quarter'* and $\frac{1}{4}$) of the unit fraction. Use these to introduce the idea that if we have two or three of the quarters, we can say that we have *'two-quarters'* or *'three-quarters'*.

Introduce the written notation of $\frac{2}{4}$ and $\frac{3}{4}$, using the stem sentence: ***'The whole is divided into four equal parts and we have ___ of them.'***

If children get into the habit of saying this every time they see written fraction notation, it will really help them with interpreting the fraction and relating it back to fractions of sets and objects.





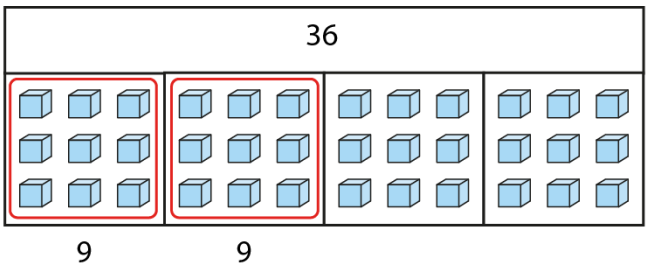


Children need to be able to:

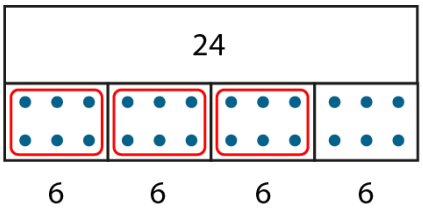
- Write (using fraction notation), or say, the fraction indicated on a picture of an object, shape, set of objects or length.
- Shade or circle the relevant share of a picture to indicate a fraction presented in written form. For example, if asked to shade $\frac{3}{4}$ of a shape, they know they need to shade three of four equal parts.

Extend their learning to finding $\frac{2}{4}$ and $\frac{3}{4}$ of a quantity. Refer back to the bar model images used to find $\frac{1}{4}$ in point 4 above. Children have already used these models to find the value of each one-quarter. They can now circle the number of quarters they need and add them together using repeated addition. Again, this can be done practically with cubes, before working on paper with dots.

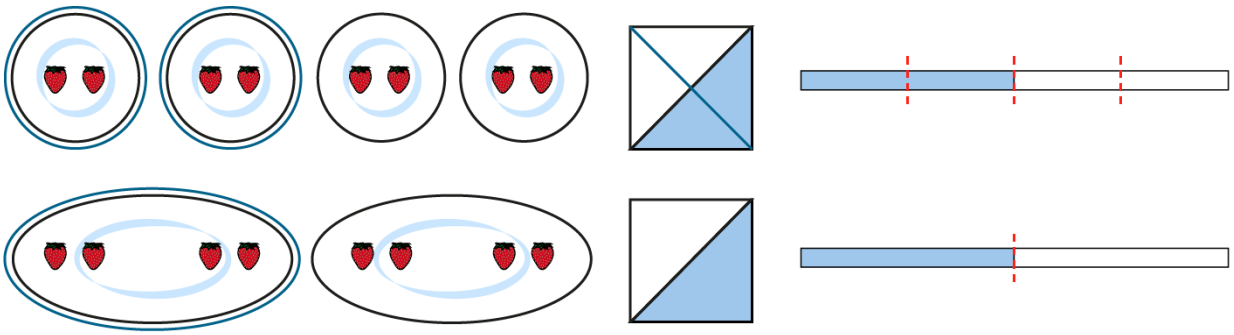
Using cubes to calculate $\frac{2}{4}$ of 36:



Drawing dots to calculate $\frac{3}{4}$ of 24:



Finally, the national curriculum requires children to recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$. This is probably most clearly demonstrated through a practical activity, such as sharing out food. In this type of activity, it will be fairly easy for children to see that if they eat, for example, $\frac{2}{4}$ of an apple, or $\frac{2}{4}$ of a four-fingered chocolate biscuit, they will have eaten the same amount as if they had eaten half of the apple or biscuit. Support this with discussion around pairs of images that are split into two equal parts and four equal parts. These images offer another way to demonstrate and discuss that $\frac{2}{4}$ and $\frac{1}{2}$ both indicate the same part of a whole.



An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

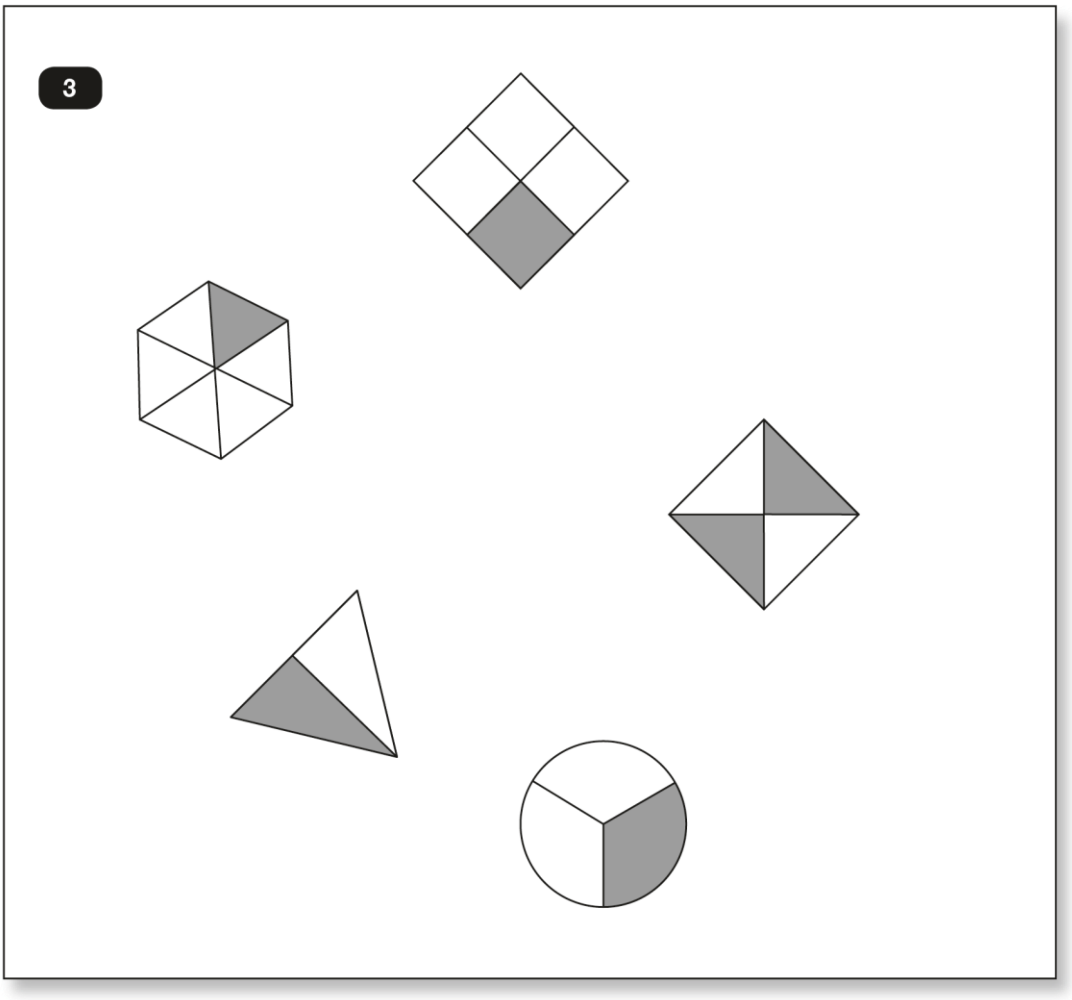
Appendix

The appendix to this document shows all the national curriculum Key Stage 1 Standards & Testing Agency's past questions on fractions from the 2016–2019 mathematics papers. It includes questions from both the arithmetic papers, the reasoning papers, and the 2016 sample papers. Consider how the strategies outlined in this document could be applied to these questions.

2019 reasoning paper

The teacher reads out the following:

- *'Look at the five* shapes.'
- *'Tick all of the shapes that have half shaded.'*



11 Complete the number sentences.

One is done for you.

$\frac{1}{2}$ of $\boxed{8} = 4$

$\frac{1}{2}$ of $\square = 3$

30 A shop has 12 bags of crisps.

It sells $\frac{1}{4}$ of the bags.

How many bags of crisps did the shop **sell**?

bags

2019 arithmetic paper

20 $\frac{1}{4}$ of 8 = ☐

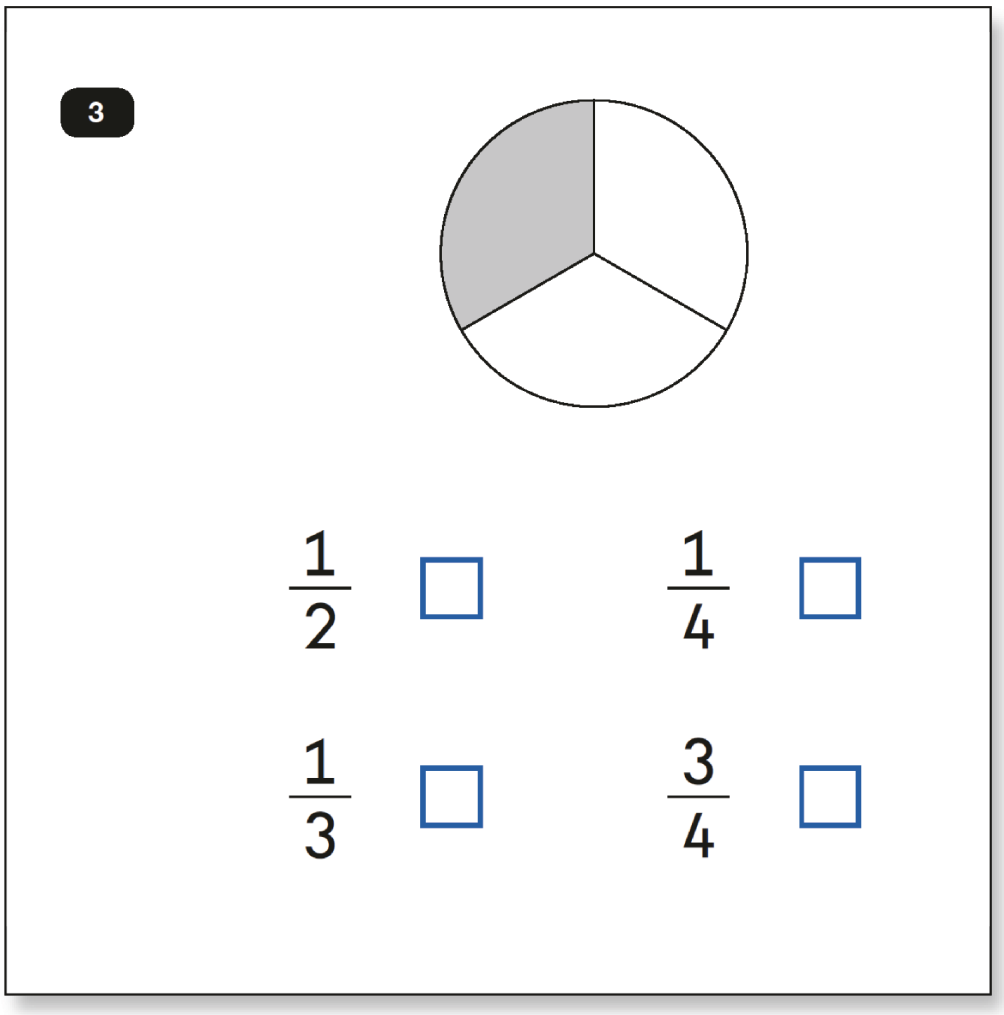
21 $\frac{1}{2}$ of 90 = ☐

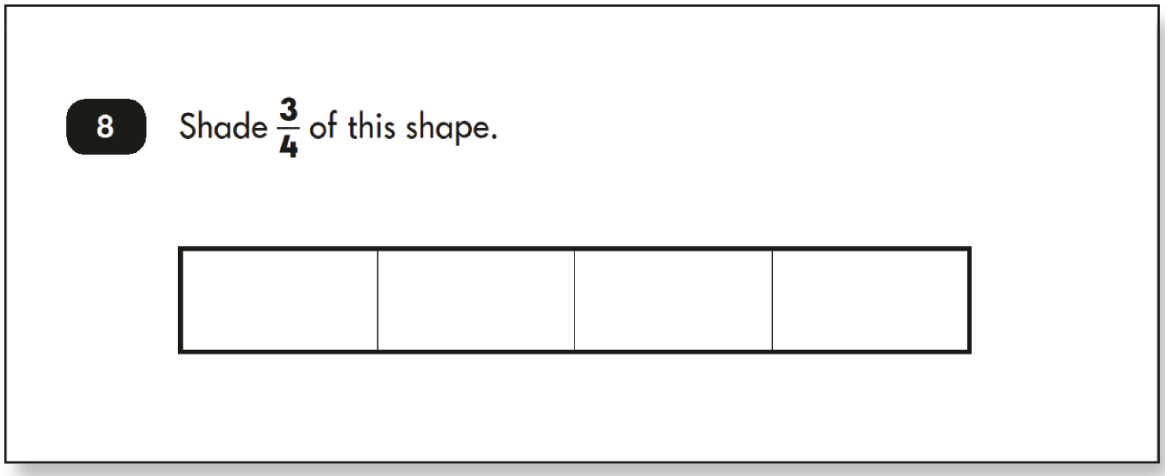
23 $\frac{2}{4}$ of 36 = ☐

2018 reasoning paper

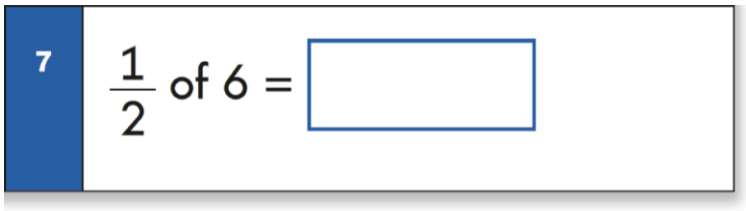
The teacher reads out the following:

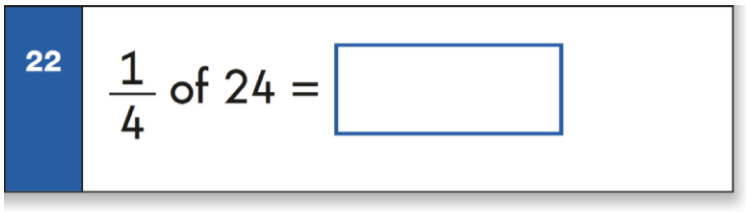
- *'Look at the circle.'*
- *'Part of the circle is shaded.'*
- *'Tick the fraction below that shows the <u>shaded</u> part of the circle.'*
- *'Put your tick in the box by the correct fraction.'*





2018 arithmetic paper





2017 reasoning paper

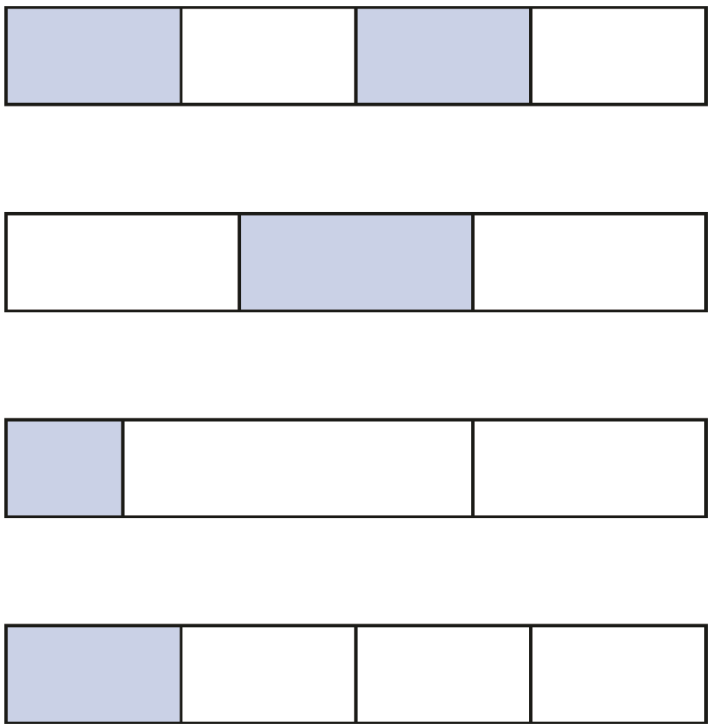
17 Ben ate half a pizza.

Which fraction shows the amount he ate?

Circle it.

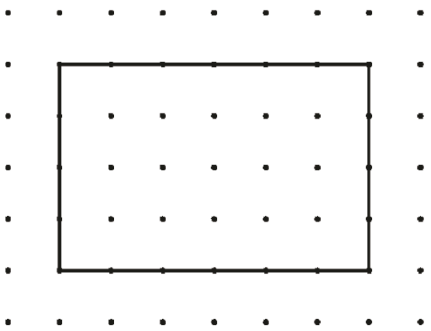
$\frac{1}{4}$ $\frac{1}{3}$ $\frac{2}{4}$ $\frac{3}{4}$

24 Tick the shape that has exactly $\frac{1}{3}$ shaded.



26 Draw lines to divide the rectangle into quarters.

Use the dots to help you.



2017 arithmetic paper

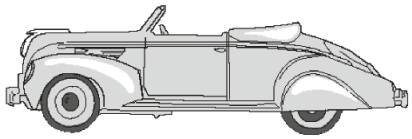
14 $\frac{1}{2}$ of 14 = ☐

24 $\frac{1}{3}$ of 12 = ☐

2016 reasoning paper

14 Abdul has some toy cars.

He gives half of them to Ben.



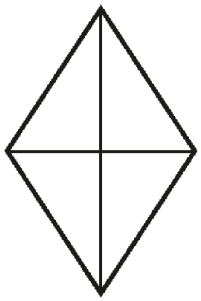
He has four toy cars left.

How many toy cars did Abdul start with?

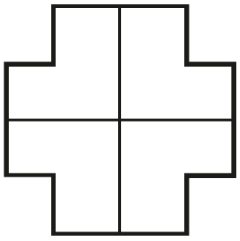
toy cars

27 Shade the fractions of the shapes.

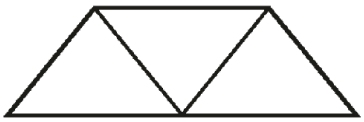
Shade $\frac{1}{2}$



Shade $\frac{3}{4}$



Shade $\frac{1}{3}$



2016 arithmetic paper

18 $\frac{1}{2}$ of 16 = ☐

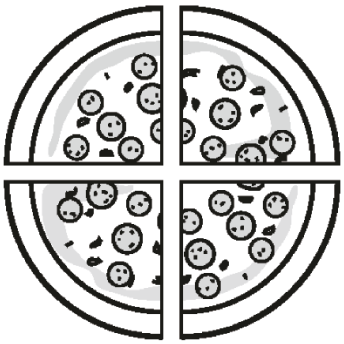
22 $\frac{1}{4}$ of 12 = ☐

24 $\frac{1}{3}$ of 30 = ☐

25 $\frac{3}{4}$ of 20 = ☐

2016 sample reasoning paper

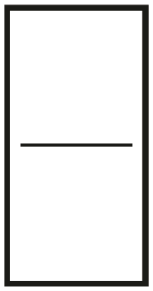
16



Sita cuts a pizza into four equal slices.

She eats one slice.

What fraction of the pizza does she eat?



30 Look at these fractions.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{4}$ $\frac{3}{4}$

Circle the **two** fractions that are **equal**.

2016 sample arithmetic paper

18 $\frac{1}{4}$ of 20 = ☐

21 $\frac{1}{2}$ of 30 = ☐

24 $\frac{1}{3}$ of 21 = ☐

22 $\frac{1}{4}$ of 24 = ☐