



Welcome to autumn, and to the realisation that half of a half-term has already passed us by. What happened to September? Anyway, this month we have much for you to chew on while the leaves start to fall. The mix includes imagining geometric rotations, factorising cubics, teaching trigonometry without resorting to SOHCAHTOA, and trying to re-shape the diet offered to GCSE re-sit students. As always, your comments and feedback are very welcome, either at the bottom of the page, by email to [info@ncetm.org.uk](mailto:info@ncetm.org.uk), or [@NCETMsecondary](https://twitter.com/NCETMsecondary) on Twitter.

## Contents

### [Heads Up](#)

Here you will find a checklist of some of the recent, or still current, mathematical events featured in the news, by the media or on the internet: if you want a “heads up” on what to read, watch or do in the next couple of weeks or so, it’s here. This month there is news about two BBC programmes with a mathematical theme, and still available on iPlayer, a free workshop for maths teachers at the Royal Institution, a few places left on our free courses developing PD Leads specialising in A level maths, and a suggestion that you look at our new materials to assess primary (yes, primary) pupils’ depth of mathematical understanding.

### [Building Bridges](#)

A bridge of a slightly different sort this month. With many of you sure to be handling Year 12, or FE College, re-sit GCSE classes, we present some ideas of how to build a bridge from pre to post-16 study of GCSE maths.

### [Sixth Sense](#)

One of the key skills we need to help Year 12 pure mathematicians acquire is factorising cubic expressions. We suggest a step-by-step approach to this teaching and learning task.

### [From the Library](#)

Want to draw on maths research in your teaching but don’t have time to hunker down in the library? Don’t worry, we’ve hunkered for you: in this issue, we draw together research relating to the teaching (conceptual and procedural) of trigonometry.

### [It Stands to Reason](#)

Developing pupils’ ability to imagine rotations, and other transformations, not only increases their chances of tackling geometry questions. It can strengthen their all-round powers of mathematical reasoning as well.

### [Eyes Down](#)

A picture to give you an idea: this month, three classroom snapshots to demonstrate how the simplest and cheapest of resources we have lying around our homes or classrooms can often prove powerful aids to understanding. And if this reminds you of *Blue Peter*, you’re definitely not the baby of the department!

#### Image credit

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## Heads Up

*Whilst not wanting to define this article as mathematical gossip, it does come close! We've brought together news and current mathematical affairs, all in one place. We do hope it will interest you.*



In November, 68 teachers from Shanghai are coming to secondary schools across England to share with us how they teach mathematics in Years 7 and 8. All the host schools will be organising events for teachers from their and local schools to observe lessons and take part in discussions: these will be coordinated by your [Maths Hub Lead School](#), so get in contact to make sure you're informed about and invited to these unique PD opportunities.



If you're looking for ways of measuring your pupils' deep understanding of mathematical concepts, we're sure you'll find the NCETM's recently published [assessment materials](#) useful. Although ostensibly aimed at primary school teachers, the materials (look at Year 6 in particular) include copious examples that will challenge most Key Stage 3 (and some KS4/5!) pupils, especially the questions in the *Greater Depth* column, on the right-hand side of every booklet.



There are a few places left on the latest round of courses to develop professional development leads specialising in advanced level maths, jointly run by the NCETM and the Further Mathematics Support Programme (FMSP). Application details (under *How Do I Apply to Take Part?*) are on [this page](#).



Two recent BBC programmes to flex your mathematical synapses. The [first](#), on television, features Marcus du Sautoy explaining what he calls the hidden world of algorithms. The [second](#) is a recent episode of BBC Radio Four's *Mind Changers* series, which investigates the concept of mindset and relates it to the learning of maths.



Secondary mathematics teachers are the target audience for an [evening workshop](#) at the Royal Institution in London on 12 October. The subject is how to assess problem-solving and mathematical thinking.



There a chance to win yourself a £20 voucher, simply by sending us a photograph from your classroom that prompts thoughts about how maths can be taught and learnt. See the [Eyes Down](#) section of this month's magazine.

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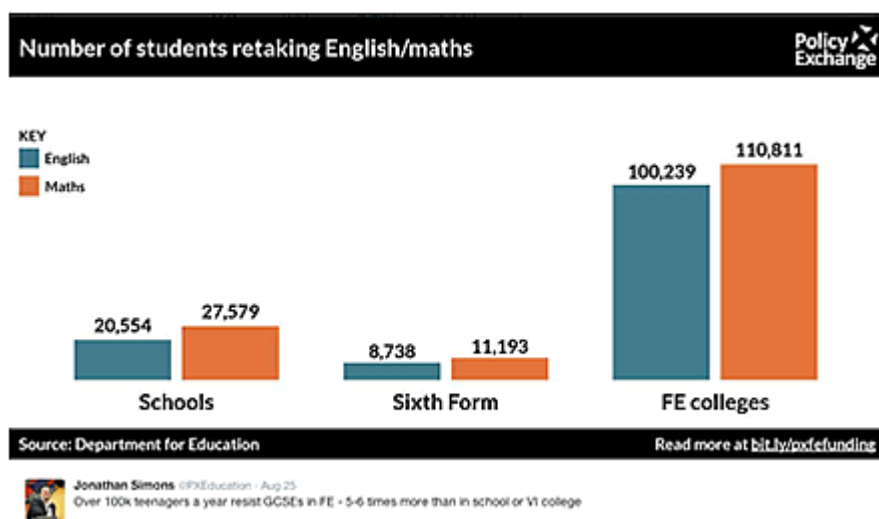
## Building Bridges

*There's been much in the media over the summer and the last few weeks about GCSE Maths in FE colleges, and the challenges of teaching FE students who now are supposed to re-sit Maths and English if they haven't achieved a grade C. With this in mind, this month's article considers how to build a bridge from pre- to post-16 study of GCSE Maths.*

When winter's shadowy fingers first pursue you down the street and your boots no longer lie about the cold around your feet, do you spare a thought for your Scheme of Work, and does it reflect what we know to be true in relation to GCSE maths resits? (with huge apologies to Lindisfarne!)

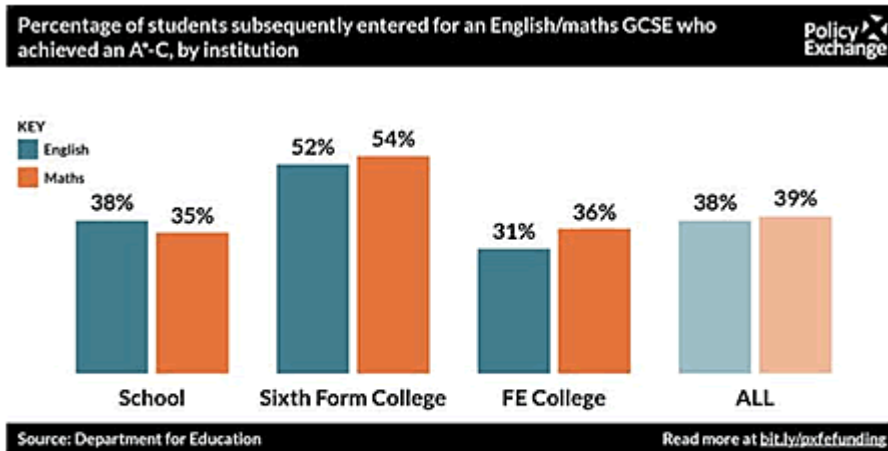
A GCSE maths resit 16-year old student is a wily beast. He will have a plethora of reasons as to why he didn't get the Willy Wonka golden ticket of a C grade that will pave the way to a great future full of fame and fortune. Not one will be the fact that he didn't work hard enough. It's always someone else's fault: you will hear a multitude of excuses - supply teachers, rubbish teachers, no teachers, hadn't covered the topics, gave us the wrong coloured post-its - you will hear it all. Recognising - and overcoming - the distancing and disengaging defence mechanisms of the species is crucial if he is to avoid exam-tinction next June.

There are thousands of these students, and to be honest, we let most of them down at present: look at the pitiful success rates in resit GCSE. As Jonathan Simons from the Policy Exchange tells us, over five times as many students retake GCSE in an FE setting rather than remaining at school: 110 811 students in fact.



With pass rates at an average of 36% in FE that means we haven't got this right for a mind-numbing 75 000 students, though it's not much better in schools either. Note the comment below that this is not the full cohort of D grade students - but from this year they are mandated to resit, so the number of students re-taking will increase enormously.

There is a multitude of reasons why the pass rates are so low, but one of them is definitely the Scheme of Work, and that's one that is under our control. It has to look, feel and sound different to the regime that these students have followed (have endured, they feel) before. There is no new content that will surprise them - they've seen it, they can do some of it, they can't do the rest of it - so we have to think about the presentation and the teaching: old wine in new bottles.



**Jonathan Simons** @PXEducation · Aug 25

Although pass rates are better than thought, still low, and too many students don't do GCSE even when achieving a D

A resit student has seen all the maths needed for GCSE: she cannot do all of it, but she can do some of it, that's why she has a D grade already. The skill in teaching resit students is uncovering what they can't do fully and tackle that, while sustaining everything else so that they don't forget what they do know – and remember that they will not have done any maths since June, so what they did know then is already fading. The clock's ticking.

Preparing for a resit can be split into the following "Five R's" (inflation from the three when I was at school...), each feature being an element of your SoW:

## RECALL

There are 48 "killer" facts for a C grade and 80 "killer" facts for an A\*: well, so says the Mathematical Association with their [revision postcard sets](#). Use these as the basis of your fast and furious, fluent, regular recall with questions such as:

- name the first 20 prime numbers
- what are the six types of quadrilateral? And sketch them
- how do you calculate mean, median and mode?
- draw and name the parts of a circle.

## ROUTINE

The effort of practice, practice, practice is imperative if students are to be successful in a resit. Elements such as Corbettmaths [5-a-day](#) and JustMaths [Bread & Butter](#) are good routines to set up from day one. If you can encourage students to keep up their 5-a-day through half term and over Christmas (six, then, if you include the sprouts), then even better. Routinely going over misconceptions such as those presented as [Challenge 2 in the C/D borderline page](#) on m4ths.com will bring rewards. [studymaths.co.uk](https://www.studymaths.co.uk) and [Form Time Ideas](#) are further favourites for practice opportunities. The 30 second challenges in the Daily Mail are good, and there are some of these types of activities at [mathematicshed.com](https://www.mathematicshed.com). A fun regular element is playing "You can't do simple maths under pressure". Doing maths makes people more successful at maths; routine hard work and practice translates into success.

## REVISE

There are key topics that straddle the D/C borderline and always present difficulties to students. Topics such as finding an expression for the value of the  $n$ th term of a sequence, manipulating fractions, index laws and Pythagoras' Theorem always raise a painted eyebrow, and so these are the topics that are worth teaching in full and in depth with all students. Just Maths lists its [Top 40 topics](#), m4ths.com has a really good [Help Guide Sheet](#) highlighting all of the C grade topics, while [Hegarty Maths](#) and [Corbett Maths](#) provide some excellent self-study videos too. With around 40 revision topics to focus upon on the borderline, that is roughly one per week in an FE timetable, which is manageable. Using an Entrance/Exit ticket to ascertain prior knowledge and progress made would be an informative and worthwhile addition to your lesson.

## REPEAT

Once revised, each topic has to be repeated: that means exam questions. [Corbett Maths](#) provides sets of exam questions and textbook questions. Combined with the use of a maths passport such as the ones from [Miss B's Resources](#), you now have a mechanism by which you can check knowledge and understanding of the key C grade topics in a measured way. [Pret homeworks](#) is another excellent source of repeat and challenge questions, as is Miss B's [Quick Wits](#).

## READY

Exam-ready and prepared. A good strategy for final preparations is to mark a past paper as it is and award the actual marks and grade it would get; then mark it again as though no avoidable errors had been made, but instead award these marks back to the student. This really highlights the impact on grading, and these are the easiest pitfalls to avoid in exam technique: simple notes-to-self such as "remember units" and "check results by estimating" have a big impact.

Some colleges routinely set a past paper per week from February, and this is generally a higher paper. Past paper "walkthroughs" are effective, as are "group papers" where everyone contributes and peer marking occurs with the mark scheme. Another successful technique is to offer exam questions with no grading attached. Students are not really interested in the grade F and E type questions: they want to know what they need to do to get a C grade. Some teachers are clever and can present questions up to a B grade without mentioning it until completed, which hugely raises confidence.

If you can include these five elements of recall, routine, revise, repeat, ready within every session, then you have the basis of a SoW which should lead to more successful outcomes. The finer detail on this suggested FE GCSE Maths Resit SoW is at [Making the grade D to C](#), which has a number of elements of exemplary practices from FE Colleges that seem to have addressed at least some of the challenges this group of students presents.

## Further reading

- the NCETM's [Developing a Scheme of Work](#) is well worth a read. It was written in response to the Ofsted report, [Mathematics: Understanding the Score](#), which stated that a good SoW is rare in secondary schools. There are few if any bespoke FE GCSE resit SoWs out there: everything focuses upon GCSE in Year 11, but the FE SoW needs to be slightly different as we have discussed
- [Kangaroo Maths](#) is favoured by many and links to numerous NRICH resources as well as their own. It covers the full range of key stages including AS and A2

- each awarding body produces a SoW that runs alongside its syllabus. These tend to be specific to the exam board style, although they are written by experts and informed by practising teachers. Cambridge University Press has been über-generous with their free editable [new curriculum SoWs](#) for AQA, OCR and Edexcel, with one-year, two-year and three-year schemes combining to form a five-year scheme through KS3 into KS4
- the inimitable Craig Barton has a great series on his [blog](#), with 19 articles and a hand-holding guide through the production of a tailored SoW
- [We Teach Maths](#) has posted a variety of SoWs on the TES, written for the new specification and designed for two- or three-year delivery.

But what if you're looking for a tried and tested and been-round-the-block-a-few-times Scheme of Work for a GCSE maths resit or a fast-track GCSE? Well, apart from a few commercially available (i.e. hand-in-your-pocket time) items, there's not much out there. We can change that: let's all share [@NCETMsecondary](#) the SoW we're following at the moment.

You can find previous *Building Bridges* features [here](#).

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## Sixth Sense

One of the key skills you'll soon be teaching your new year 12 Pure Maths group is how to factorise cubics (and higher order polynomials).

If you've already shown your students the "field method" (see [Building Bridges, issue 119](#)), so much the better – if not, now is the time to convince them of its value. They might well have seen in KS2 how to multiply 13 by 82:

×	10	3	
80	800	240	
2	20	6	
			1066

and then in KS3 developed the method to expand  $(2x + 3)(x - 5)$ :

×	$2x$	3	
$x$	$2x^2$	$3x$	
-5	$-10x$	-15	
			$2x^2 - 7x - 15$

and then in KS4 used the same model to factorise  $x^2 + 7x - 30$ :

FIRST ATTEMPT (picking two integers that multiply to equal -30)

×	$x$	-5	
$x$	$x^2$	$-5x$	
6	$6x$	-30	
			$x^2 + x - 30$

SECOND ATTEMPT (because that didn't quite work)

×	$x$	10	
$x$	$x^2$	$10x$	
-3	$-3x$	-30	
			$x^2 + 7x - 30$

GOT IT:  $x^2 + 7x - 30 \equiv (x + 10)(x - 3)$ .

Before attempting to factorise any cubics, it is worth using a field to expand the product of three factors.  
e.g. Expand and simplify  $(2x+3)(x-5)(x+2)$ .

Step 1 – expand the first two factors (see above)

Step 2 – multiply the answer by the third factors:

×	$2x^2$	$-7x$	$-15$	
$x$	$2x^3$	$-7x^2$	$-15x$	
$2$	$4x^2$	$-14x$	$-30$	
				$2x^3 - 3x^2 - 29x - 30$

Practice will help students become fluent with this procedure. Now let's look at how to use this to factorise a cubic when we are given one of the factors.

e.g. Factorise  $x^3 + x^2 - 16x + 20$  given that  $x - 2$  is a factor.

Step 1 – Present the known information

×	$?x^2$	$?x$	$?$	
$x$				
$-2$				
				$x^3 + x^2 - 16x + 20$

Step 2 – To have an  $x^3$  term, the first term of the quadratic must be just  $x^2$ , so:

×	$x^2$	$?x$	$?$	
$x$	$x^3$			
$-2$	$-2x^2$			
				$x^3 + x^2 - 16x + 20$

Step 3 – Consequently, we have a new term,  $-2x^2$ , but eventually we want just  $x^2$ , so we need to create another term,  $3x^2$ :

×	$x^2$	$?x$	$?$	
$x$	$x^3$	$3x^2$		
$-2$	$-2x^2$			
				$x^3 + x^2 - 16x + 20$

Step 4 – So the coefficient of the  $x$  term in the quadratic factor must be 3, and we can fill in another patch in the field:



×	$x^2$	$3x$	?	
$x$	$x^3$	$3x^2$		
-2	$-2x^2$	$-6x$		
				$x^3 + x^2 - 16x + 20$

Step 5 (like step 3) – we now have a new term,  $-6x$ , but ultimately we want the term to be  $-16x$ , so we have no choice on what to fill in next:

×	$x^2$	$3x$	?	
$x$	$x^3$	$3x^2$	$-10x$	
-2	$-2x^2$	$-6x$		
				$x^3 + x^2 - 16x + 20$

Step 6 – thus the final “?” needs to be “-10” to create the desired  $-10x$  term:

×	$x^2$	$3x$	-10	
$x$	$x^3$	$3x^2$	$-10x$	
-2	$-2x^2$	$-6x$	20	
				$x^3 + x^2 - 16x + 20$

This is very satisfying, because -10 multiplied by -2 is +20 so we must have factorised the cubic correctly. This is why I would discourage students from trying to “hit” the constant term first: if they get to the end and the numerical multiplication works then it is highly likely they’ve completed the process correctly; if it doesn’t, then they immediately know that they haven’t! So we now know that:

$$x^3 + x^2 - 16x + 20 \equiv (x - 2)(x^2 + 3x - 10)$$

In this example, we can factorise the quadratic factor, so:

$$x^3 + x^2 - 16x + 20 \equiv (x - 2)^2(x + 5)$$

This process looks long-winded in print or on a screen, but clearly on a whiteboard (classroom or handheld) the field only needs drawing once and judicious use of coloured pens can make the whole example compact and clear!

All that remains is to factorise a cubic when we are not given a factor initially. In my experience, students are easily convinced (informally) of the factor theorem given their prior experience of solving quadratic equations: they will, with perhaps a little prompting, recall that “if one of the factors is  $(x + 7)$  then one of the roots is  $x = -7$ , and this means that substituting  $x = -7$  into the original quadratic expression gives an output value of 0”.



Therefore, reversing the argument, when given a quadratic / cubic / higher order polynomial expression, if we can find a “magic number” that, when substituted, makes the expression equal 0, we have found a root and hence a factor. For example, to factorise  $2x^3 - 7x^2 + 11x - 10$  the students first need to determine that the cubic expression is equal to 0 when  $x = 2$ , and so one of the factors is  $(x - 2)$ . They now proceed with the field method. This is preferable to seeking a second “magic number” in general because (a) we may have a repeated factor as was the case above, and no simple (without calculus) way of seeing this; or (b) the other factor may be an irreducible polynomial, as is the case here.

Students’ conceptual understanding of the method can be deepened by asking them “what happens if we try and factorise  $x^3 - 4x^2 + 2x + 7$  using  $(x - 3)$  as our factor?” Let’s pick this up next month.

You can find previous *Sixth Sense* features [here](#).

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## From the Library

How best to teach trigonometry so that pupils acquire both procedural fluency and conceptual understanding (even in the basic cases of right-angled triangles) is not obvious; but, ironically, it is probably one of the least extensively researched topics – and the results that have been gathered sometimes appear, at least on the surface, contradictory. The identified complexity of the topic reflects that it “may be seen as the confluence of a number of streams of mathematical difficulty” ([Pritchard and Simpson, 1999](#), p1247): pupils need to have a confident grasp of angle, ratio, similarity and algebraic manipulation (and, later, functions), as well as being able to shift their perspective between trigonometric imagery, scale factors and algebraic symbolism. No wonder there is the temptation to resort to vacuous mnemonics such as SOHCAHTOA and procedurally dogmatic algebraic manipulation.

Conceptual understanding of function is essential for successful reasoning about sine and cosine at the higher GCSE tier and for solving equations and differentiating trigonometric functions at A level. But because trigonometry comprises multiple concepts (as described in [Key Ideas in Teaching Mathematics - Similarity, ratio and trigonometry in KS3](#) in Issue 115), including scale factor, ratio and function, how should one introduce the subject?

Craig Pournara (2001, University of the Witwatersrand) compared pupils’ conceptions of trigonometric ratio and function. Where pupils used calculators to produce a ratio in decimal form they were unable to relate it back to the sides of a triangle. But one pupil who treated the ratio as a scale factor was able to work with both trigonometric representations, as the output from a function and as a ratio between the sides of a triangle. Pournara’s unpublished Master’s dissertation is a significant contribution to the research, both in terms of its theoretical approach and its practical investigations; he can be contacted at the university.

Keith Weber ([Weber, 2005](#)) argues that trigonometry should be introduced via a function approach and that pupils need to understand a trigonometric function as a process rather than solely as the result of applying a set of rules. He proposes that trigonometric functions can be problematic for at least two reasons: not only are they early examples of functions which cannot be explicitly calculated, but they also map between different (to the pupils) types of mathematical objects: from angles to real numbers. Weber carried out research introducing the functions through use of the unit circle, constructing a variety of right-angled triangles on a Cartesian plane, measuring the lengths of the sides and calculating ratios. The concrete geometric approach, combined with time for reflection, was seen as essential to the conceptualisation of trigonometric function as a process. The lesson activities and follow up work are detailed [here](#). Appreciation of the process enabled pupils to reason about the functions: for example, when asked to approximate  $\cos 340^\circ$  and explain for what values of  $x$  the function  $\sin x$  is decreasing, and to justify their reasoning, a much higher proportion of pupils demonstrated the facility for successful reasoning.

However, the results from this research may be perceived as contradicting those of Kendal and Stacey ([Kendal and Stacey, 1996](#)). They considered the solving of right-angled triangle problems and compared the ratio approach (SOHCAHTOA) with the unit circle approach. They found that the right-angled triangle was method more effective for solving these types of problems, but they still recommended introducing the concepts of trigonometry via the unit circle.

What role can dynamic geometry tools play in getting a feel for a trigonometric function as a process? Steer *et al* argue for the benefits of a dynamic tool over a protractor and ruler ([Steer, de Vila, Eaton part 1](#)), combining the ratio and unit circle approach. Steer *et al* are closer in approach to Weber in that “the

investment here is in introducing students to a complete picture of trigonometry - ratio and function". Detailed descriptions of lessons and activities are provided in [Steer, de Vila, Eaton, 2009 part 2](#).

It is important to recognise the different aims of these research articles: Weber was looking to address the issue of pupils' conceptual understanding of trigonometric functions, but Kendal and Stacey were comparing ways of learning to solve right-angled triangle problems. As Pournara concludes, "Based on the interview data, it seems that learners' conceptions of trigonometric ratio are closely tied to the methods they use". Different aims need different strategies; our responsibility is to read the research and draw on its findings to match our context to our chosen approach as closely as we can. Let us know what you decide to do – perhaps you could contribute a picture to a future *Eyes Down* article.

### References:

Pritchard, L. and Simpson, A. (1999) [The role of pictorial images in trigonometry problems](#), Zaslaysky, O, ed., Proceedings of the Conference of the International Group for the Psychology of Mathematics Education (23rd, Haifa, Israel, July 25-30, 1999) Vol 4, p. 81.

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Kendal, M. and Stacey, K. (1996) [Trigonometry: Comparing Ratio and Unit Circle Methods](#), Technology in Mathematics Education: Proceedings of the 19th Annual Conference of the Mathematics Education Research Group of Australasia, 322 – 329.

Steer, J., de Vila, M. and Eaton, J. (2009) [Trigonometry with Year 8: Part 1](#), Mathematics Teaching, 214, 42 – 44.

Steer, J., de Vila, M. and Eaton, J. (2009) [Trigonometry with Year 8: Part 2](#), Mathematics Teaching, 214i.

Pournara, C. (2001) *An investigation into learners' thinking in trigonometry at Grade 10 level*, a research report submitted as part of a degree in Master of Science, School of Science Education in the Faculty of Science, University of the Witwatersrand

You can find previous *From the Library* features [here](#).

### Image credit

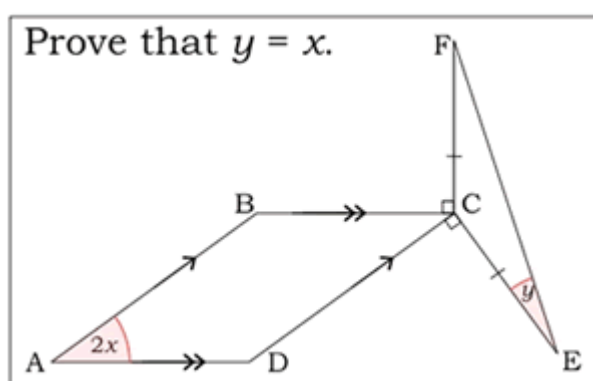
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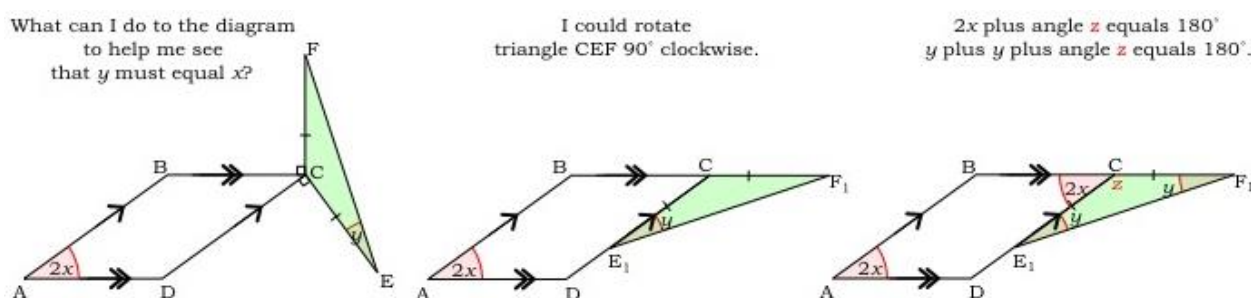
## It Stands to Reason

Both the use and study of rotations provide opportunities for our pupils (and we teachers) to develop their (our!) natural powers of imagining and of expressing to others what they (and we) are imagining. By working over time to refine the skills of 'picturing in the mind', all pupils can acquire a personal problem-solving resource to use widely.

Recently, in trials of sample assessment materials, many Higher Level pupils were unable to answer the following GCSE question:



It is possible that, had they been given more opportunities throughout their whole mathematics learning to practice visualising adaptations to, and movements in, images, they might have considered rotating triangle CEF and then been able to construct a valid argument:



The diagram on the right shows the position of triangle CEF (coloured green) after a clockwise rotation of  $90^\circ$  about C. Triangle  $CE_1F_1$  (the image of triangle CEF after the rotation) is isosceles with equal angles, of size  $y$ , at  $E_1$  and  $F_1$ . In the parallelogram ABCD, angle BCD is opposite (and so equal) to the angle of size  $2x$ . As a result of the rotation, we can see that the two equal angles in triangle  $CE_1F_1$  (each of size  $y$ ) are opposite to the exterior angle of size  $2x$ . Because any exterior angle of any triangle equals the sum of the opposite interior angles,  $2x = y + y = 2y$ . Therefore  $y = x$ .

What are some effective tasks that will provide opportunities for pupils to picture rotations in their minds? What teaching strategies will help them visualize rotations, and think hard about what they are visualizing? What kinds of questioning will develop and extend their ability to imagine rotations so that they can use that skill to reason with confidence and fluency in geometrical situations?

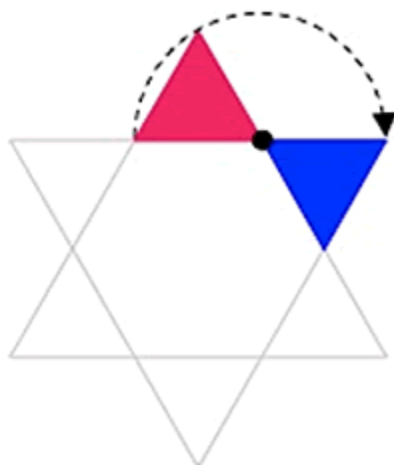
### Say What You See

Prompting pupils simply to look at images and to imagine parts moving makes use of pupils' natural powers to imagine and express what they are imagining, and helps those powers to develop. For example, you could ask pupils to say what rotations of one shape onto another they see in an image such as:

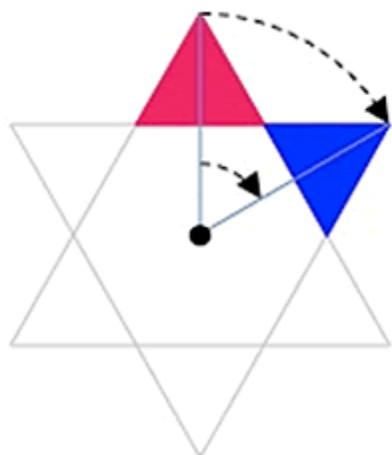


Different pupils will usually focus on different aspects – they will see different things, or see things differently. When a pupil describes something that she sees, the attention of other pupils may be directed to features or ideas that they would not otherwise have noticed. So it is valuable for pupils to hear what others have to say.

Suppose pupil A visualizes...



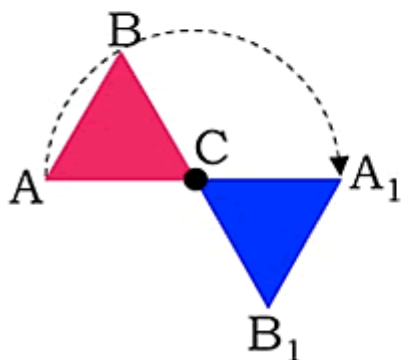
...and says "the red triangle at the top can be rotated through  $180^\circ$  onto the blue triangle on the right." Then pupil B might respond (correctly) "I think the angle of rotation is 60 degrees", because that pupil is visualizing this...



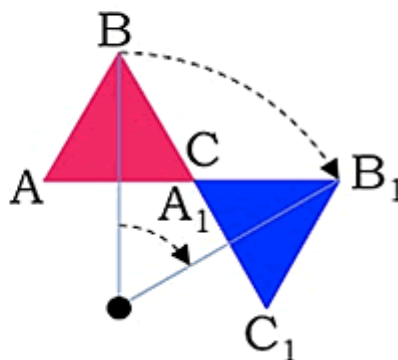
It may be tempting to tell the class that both pupils are right. But, in order to encourage reasoning, a better strategy would be to ask an open question such as “Why don’t pupils A and B agree about the angle of rotation?”. This is likely to draw other pupils into the discussion while motivating pupils A and B to refine (re-state or add to) their initial statements, in particular by explaining where in their particular mental images the centre of rotation is.

Some pupils may point out that after pupil B’s rotation the orientation of the triangle is different to its orientation after pupil A’s rotation...

What pupil A sees



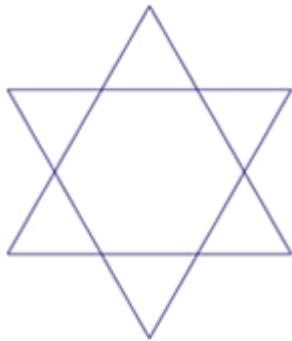
What pupil B sees



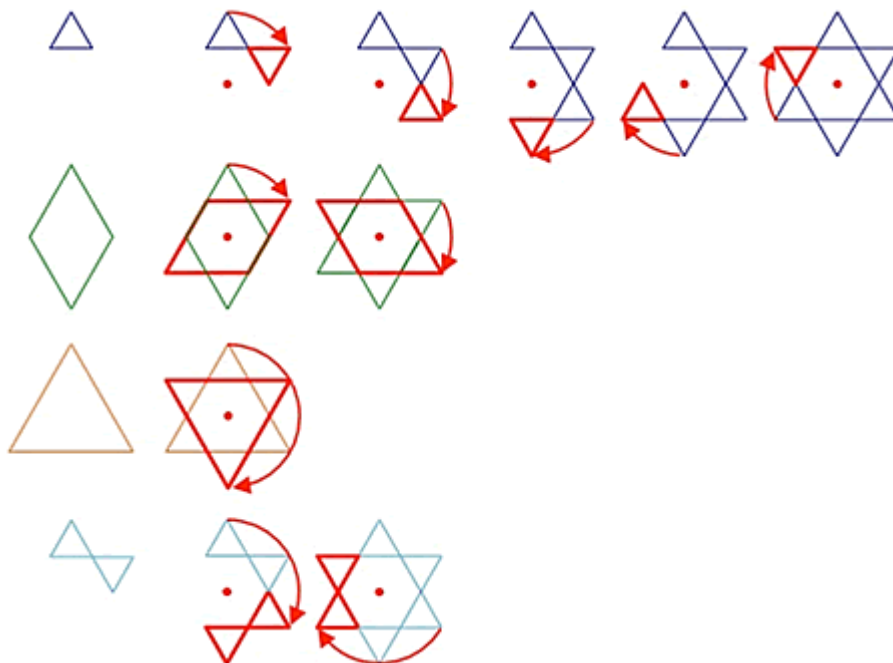
If no pupils show this awareness it is well worth prompting discussion in that direction.

It is important that pupils understand that you are not asking for ‘right answers that you already know’. Also, pupils respond best in an atmosphere in which tentative statements are valued and may be offered as starting points for further modification by the same pupil or by other pupils.

Pupils ‘saying what they see’, perhaps just to themselves, is sometimes an important aspect of pupils’ activity (even) when the task is not specifically to ‘say what you see.’ For example the task might be to create a given image by repeatedly rotating a polygon. Suppose the given image is an ‘outline-only’ version of the previous image...

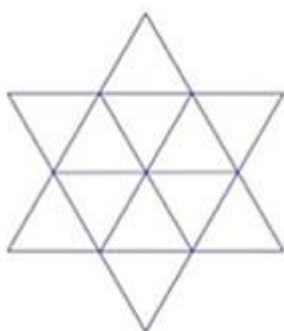


These are examples of polygons and sequences of rotations that create the whole image...



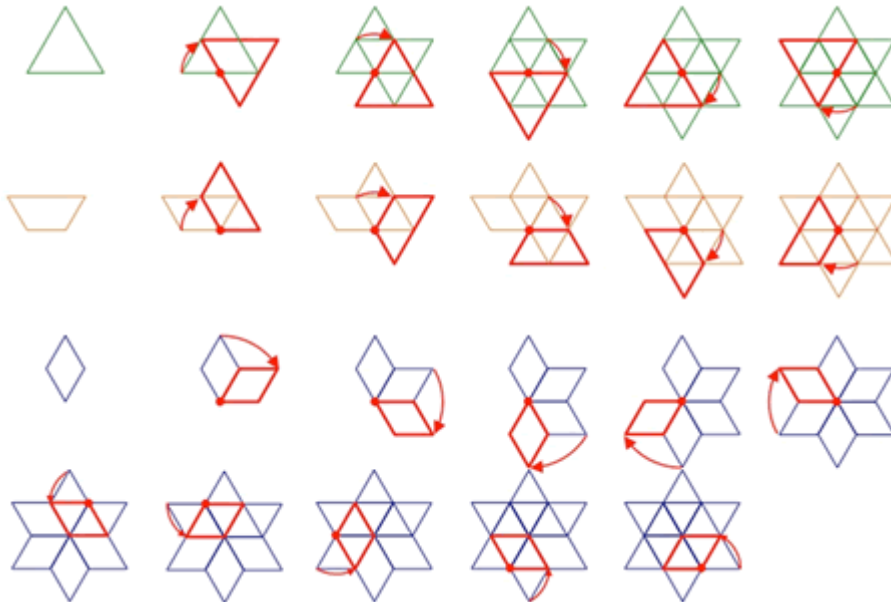
The different polygons and sequences of rotations shown here correspond to particular ways of seeing the whole image. That is, it can be seen as two overlapping triangles, as three overlapping rhombuses, as six touching triangles, or as three crossed polygons. Pupils need to learn to look flexibly at images, to be able to switch between alternative ways of seeing. This is an important skill that they can make good use of in problem-solving, particularly when they are stuck!

The same task on this modification of the whole image...



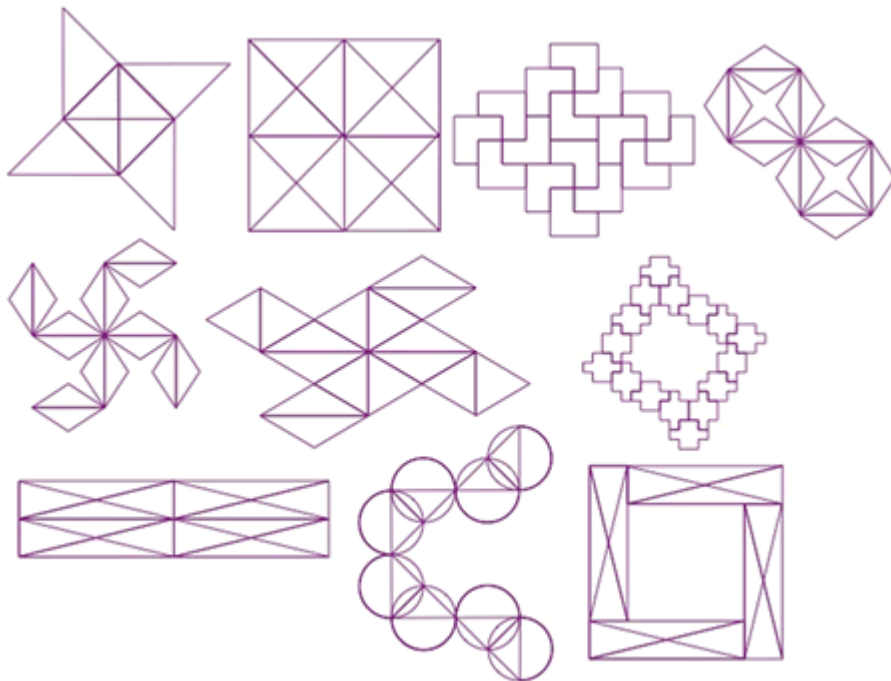


...provides more opportunities to see an image in different ways...



You and your pupils may see other possibilities!

Other images of which pupils can say what rotations they see, or can describe ways of making them by repeatedly rotating a single polygon, are not difficult to create. Here are some more examples...

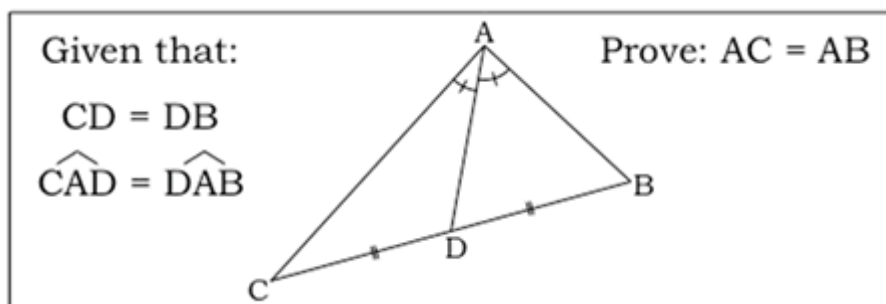


The excellent tasks presented in [Attractive Rotations](#) from NRICH are likely to interest and involve pupils. The page "provides a simple starting point for creating attractive patterns using rotations, with the potential to go much further". The images given, and pupils' own creations, would make good subjects for 'Say what you see' sessions.

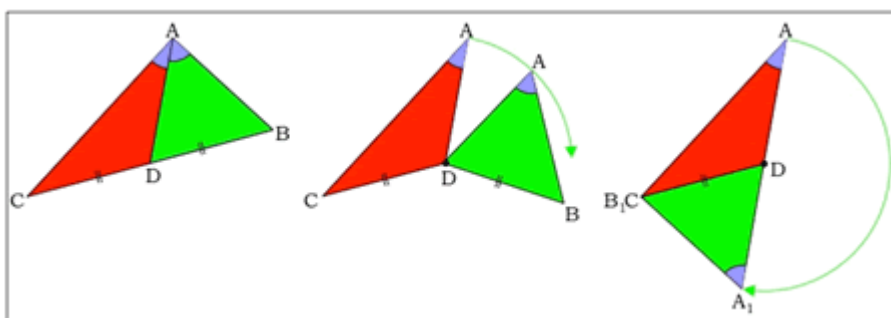
Pupils' mental and physical activity when they tackle, reflect on and discuss this kind of task will help them:

- visualize rotations;
- understand that two rotations with different centres and angles can put an unmarked symmetrical shape into the same position (so that it looks the same) although it is actually in a different orientation;
- use their developing facility with rotation to recognize relationships between parts of geometrical structures;
- use their developing facility with rotation to modify images in order to simplify the construction of valid chains of reasoning.

Not only do pupils gain insight by learning to look for different ways of seeing images, but they also become more skilful reasoners by exploring alternative problem-solving methods. Therefore it is useful that any problem-solution or proof by congruence can be translated into a proof by transformation, and vice versa. For example, how would you and your pupils construct a proof of the following fact?



This is a very simple proof by one rotation and use of the properties of an isosceles triangle:

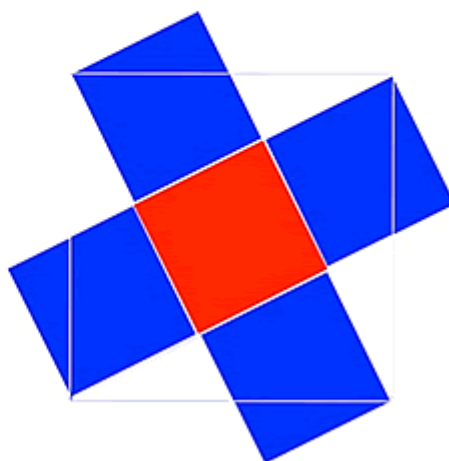
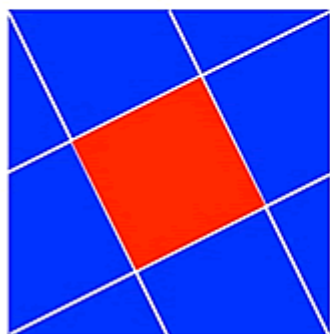


Many other mathematical facts can be proved visually using only rotation; thinking of such proofs is usually very satisfying, as is this one:



Prove that the area  
of the red square is  
 $\frac{1}{5}$  of the area  
of the whole square.

Rotating each of four  
right-angled triangles  
 $180^\circ$  about the mid-point of  
a side of the whole square  
constitutes a visual proof.



In the next issue we will look at some more problems in which thinking about rotation enables reasoning. In the meantime, do share similar activities that you know, or have used with your own pupils.

You can find previous [It Stands to Reason](#) features here

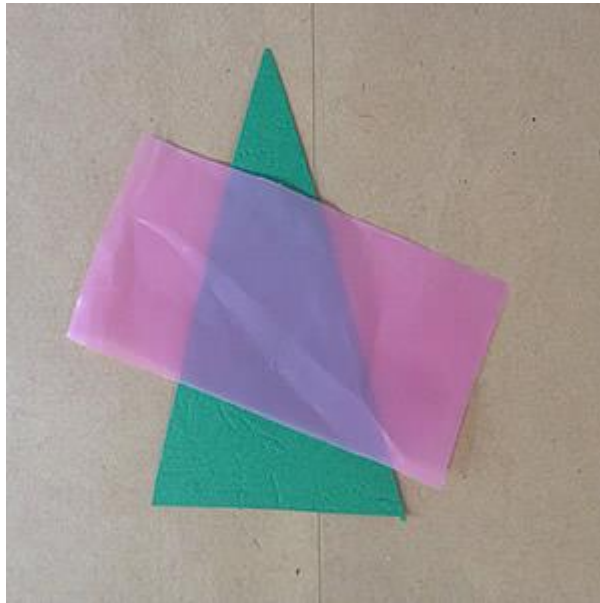
**Image credit**

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## Eyes Down...

New term: new resources, new textbooks, new catalogues of new teaching tools. Spend, spend, spend. But simple is so often better – a bit of card, a bit of see-through plastic, a bit of exploration ... and you have a rich, question-provoking, reasoning-stimulating set of trapezia. And it's consumed a micro-nibble of the budget.





What other “kitchen sink” resources do we have to hand? Share your ideas.

*If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at [info@ncetm.org.uk](mailto:info@ncetm.org.uk) with ‘Secondary Magazine Eyes Down’ in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we’ll send you a £20 voucher.*

Read previous *Eyes Down* features [here](#)

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