National Centre
for Excellence in the
Teaching of Mathematics

#mathscpdchat 19 March 2019

Helping pupils turn word-phrases into expressions and statements into equations.
Hosted by Kathryn Darwin

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*

Some of the areas where discussion focussed were:

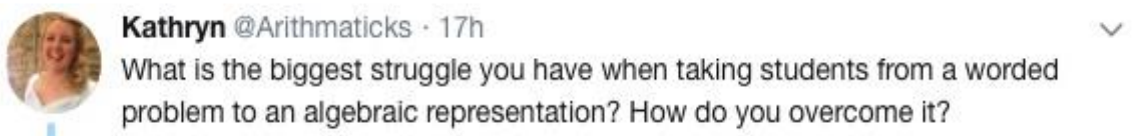
- **how pupils come to know what an algebraic expression is** … pupils **extending their knowledge** from knowing that word-phrases involving numbers, such as 'nine less than ten', can be written as **arithmetical expressions**, such as '10 − 9', to knowing that (more general) word-phrases involving **unspecified or variable numbers**, such as 'nine less than any number' or 'the sum of any two different numbers', can be written as **algebraic expressions** such as '$n - 9$' or '$a + b$';
- **approaching the teaching of algebra via the representation in a general way of arithmetical structures** that pupils already understand and can use, or that they suddenly 'see' … for example writing $(a + b) + (a - b) = a + a = 2a$ to represent the arithmetical structure exemplified (and used) in 75 + 23 + 75 − 23 = 75 + 75 = 150 … or seeing a common structure in 5 × 7 = 6 × 6 − 1, 9 × 11 = 10 × 10 − 1, 2 × 4 = 3 × 3 − 1, and expressing that structure generally as $(n - 1)(n + 1) = n \times n - 1$;

- **‘nudging’ pupils towards natural use of algebra by suggesting ‘algebraic shorthand’** when pupils are making general mathematical statements … for example the teacher asking “Do you mind if I call it ‘*n*’ rather than ‘the number of rows’?”;
- **useful tasks to support pupils’ first steps** in learning to write word-phrases as algebraic expressions, and to ‘read’ simple algebraic expressions … for example, pairing-up given algebraic expressions with given word-phrases;
- **pupils writing their own algebraic expressions** in which they have to **choose the variable**, and then **use relationships** between unspecified numbers or quantities, in order to derive the expression … for example, writing an expression to represent the perimeter of a rectangle in which one side is twice as long as another;
- **pupils’ struggles with writing algebraic expressions** to represent things in **contexts involving money** … for example in contexts used in GCSE questions;
- that **pupils may come up with the right expression, but not for the right reason** … for example (thanks to ProfSmudge for this example) given the information that ‘bananas **cost b pence each** and coconuts **cost c pence each**’, and being challenged to ‘write an algebraic expression for the total price of 5 bananas and 2 coconuts’, pupils who write ‘5b + 2c’ might simply be expressing their observation that it is 5 **b**ananas and 2 **c**oconuts that are being purchased!
- pupils **falling into other ‘fruit salad algebra’ traps** … the temptation to think of a letter, such as ‘a’, as being shorthand for an object (such as an apple) rather than as representing a number (possibly of apples!) or as a quantity (such as the mass of one apple, or the price of one apple);
- pupils **interpreting given algebraic expressions** in given contexts … information is provided via words and images, letters are assigned to items that the information is about, and students are then challenged to write a word-phrase to fit (show how to interpret) a given algebraic expression involving those letters … for example ‘2p + 5n’ represents ‘the price of p pins and n needles’ and does NOT represent ‘two pins plus five needles’;
- whether pupils learning to write algebraic expressions for word-expressions is better facilitated by focussing on **algebraic conventions before or after exploring some examples**;
- presenting pupils with **definitions of the words ‘variable’, ‘term’, and ‘expression’,** then challenging them to identify (create?) examples of each … challenging them to distinguish between examples of each in a collection of mixed-type examples … pupils distinguishing between, and understanding the meanings of, algebraic expressions that are commonly confused, such as ‘x^2’ and ‘2x’;

- pupils first encountering algebraic notation via **'missing-number' problems** … eg moving from '3 + ☐ = 5' to '3 + *n* = 5' … that the introduction of a letter in place of a 'gap' is a learning obstacle for some pupils;
- **pupils gradually learning to construct equations to represent facts derived from information given in diagrams** … eg if pupils can write expressions involving addition they can write equations to express facts (that are evident in diagrams) about perimeters of shapes;
- which **order in teaching and learning** is most effective: thinking about a situation in a context and then using algebra to facilitate reasoning about it, or learning algebraic conventions and trying to become fluent with procedures before applying those skills to contextual situations?
- **forming simple equations from given grid-puzzles-involving-shapes** by using letter symbols to represent the shapes;
- that pupils need to be able to 'translate' simple word-expressions (such as 'five less than n') into algebraic expressions, and vice-versa, **in order to understand the substitution of values into algebraic expressions;**
- e**xpressing simple relationships algebraically 'the wrong way round'** … for example if '3 green rods are the same length as one blue rod, and g green rods are the same length as b blue rods', showing the relationship between g and b by writing '3g = b' instead of 'g = 3b'.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

An interesting 'conversation' of tweets, about pupils' struggles to express relationships between two variables algebraically and the 'right-way-round', followed from this tweet by [Kathryn]():



including these from [Vanessa Moreland]() and [Kathryn]():

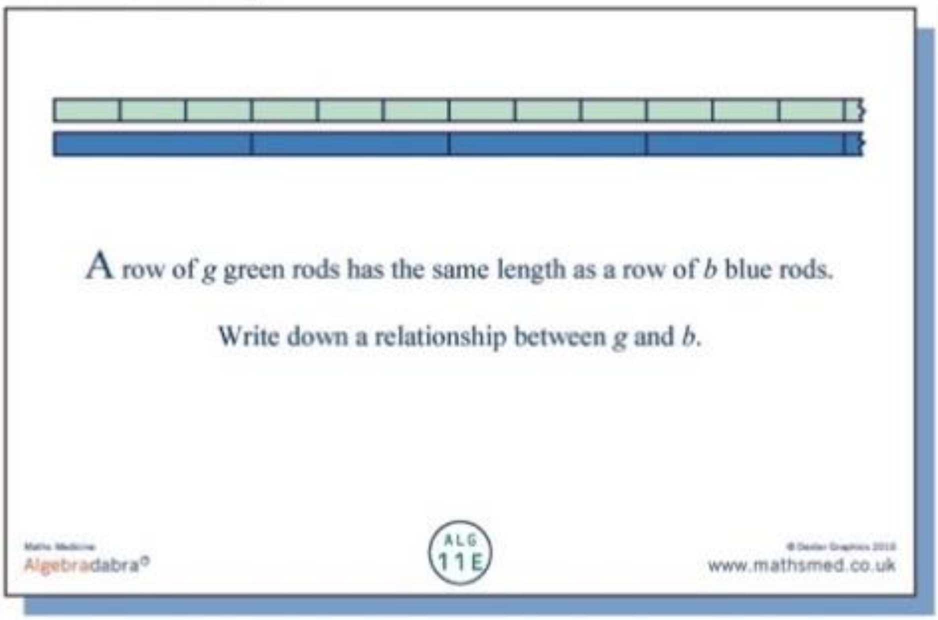
Kathryn @Arithmaticks · 17h
LOVE this idea. The more than and less than thing really get students... the amount of times they put 10 - x and not x - 10!!! #mathscpdchat

and these from [Vanessa Moreland]() and [Mary Pardoe]():

Vanessa Moreland @VanessaM_S · 17h
Yep! It's one I keep coming back to in the early stage of algebra and why I often link that to numbers and using a thermometer like 10 less than 5 which way would you go? #mathscpdchat

Mary Pardoe @PardoeMary · 17h
Replying to @VanessaM_S @Arithmaticks
Yes ... and this kind of 'pull' ... (thanks to @ProfSmudge again) ... #mathscpdchat

FRIDAY: Here we meet the same rows of green rods and blue rods, but this time we relate the number of green rods and blue rods, not their lengths.



In this task, we can feel a strong pull towards writing $3g = b$ again, on the basis that *3 green rods make a blue rod*, or *there are 3 green rods for every blue rod*. However, we need to fix firmly on the fact that in this version of the rods task, g and b are defined as *numbers of rods*, for example 3 and 1, or 6 and 2, or 9 and 3, etc. So g is 3 times b, ie $g = 3b$. This can feel counter-intuitive as it doesn't map onto our verbal descriptions of the situation.

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

[Algebradabra]() which is a very useful blog by [Professor Smudge](). It consists of many beautifully presented algebra problems each with a clear commentary suggesting teaching approaches and describing possible misconceptions that pupils' responses to the problems may reveal. It was shared by [Mary Pardoe]()

Maths Medicine which is Professor Smudge's website with pages devoted to his *Maths Medicine* pocketbooks, to links to some of his articles published in *Mathematics Teaching* (from ATM, the Association of Teachers of Mathematics), and to challenges and geometry tasks. It was shared by Mary Pardoe

Teaching Mathematics at Secondary Level by Anthony D Gardiner (first published in the De Morgan Gazette 6 no.1 (2014), 1-215) which contains clear and detailed advice about teaching mathematics at secondary level, with many excellent mathematical examples. It was shared by Mary Pardoe

Equations:angles/perimeter which is a Corbettmaths resource. It contains lots of examples in which pupils form and solve simple equations using given information, and there is a link to the related Corbettmaths video 114. It was shared by Vanessa Moreland

Fruit Salad Algebra - A Fiji Experience which is an article by Paul T Hathaway and Kamlesh Prasad. The first sentence is: 'Algebra is frequently introduced with variables representing objects such as *a* for apples and *b* for bananas.' It goes on to consider why algebra was at the time of writing often poorly understood and performed in Fiji. It was shared by Kathryn Darwin