#mathscpdchat 18 June 2019

Multiples, factors and prime numbers: teaching pupils to understand how they're connected and how they can be used

Hosted by Gerry McNally

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*

Some of the areas where discussion focussed were:

- when (at what stage(s) in their learning of mathematics) do (should) **pupils encounter the concepts of factor, multiple and prime number** … when, and in what depth, are their **properties and uses explored** … that in many schools pupils first start to explore these concepts in some depth early in Year 7;
- insights gained by pupils by trying to **find/sketch rectangles** in which the product of the dimensions is a given number;
- insights gained by pupils from **comparing and arranging Cuisenaire® rods** to show factors and multiples … for example, by fitting five pink rods alongside one orange rod, and also fitting two yellow rods alongside the orange rod pupils see and represent relationships such as 5 × 20 = 2 × 50 = 100, 5 × 1 = 2 × 2.5 = 5, 5 × ½ = 2

× 1¼ = 2½ … and so on … pupils making 'walls' by creating rows of same-length rods, one-upon-the-other, with the rod-length increasing systematically from row to row… extending such 'walls' to rod-lengths beyond those of Cuisenaire rods, and using the walls to find/show highest common factors;

- insights gained from drawing **regular polygons with all their corners on points on circles** in which points are spaced equally around the circle-edges … other explorations in which equally-spaced points on circles are joined with straight-line segments;
- using **Venn diagrams** to aid pupils in gaining insights into factors, multiples, common factors, common multiples, HCFs, LCMs, and relationships between them;
- designing and using **'non-textbook' approaches** … for example, using **games**, including **'people-games'** in which, for example, pupils place themselves, and move about on large grids chalked or painted on the ground … using a wide range of representations … **investigating conjectured relationships** (such as that 'the exponent, x, in the representation of the product of all the factors of n as n^{x}' is half the number of factors of n') (Inquiry Maths link provided below) … explorations that provide **opportunities for pupils to make conjectures** (eg while investigating relationships between a whole number N and LCM(A, B) where A and B are whole numbers such that A + B = N);
- **pupils 'seeing' facts and relationships**, and **exploiting those relationships in order to simplify calculations** … e.g. using the fact that all the common factors of two numbers are necessarily factors of their difference … deriving and using the relationship: LCM(p, q) × HCF(p, q) = p × q;
- that exploration of HCFs and LCMs (possibly using geometric images) can provide good **opportunities for pupils to reason mathematically** … for example HCF(12, 20) = 4, 12 = 3 × 4 and 20 = 5 × 4, 3 and 5 have no common factors, LCM(12, 20) = 3 × 5 × 4, **therefore** HCF(12, 20) × LCM(12, 20) = 12 × 20;
- using **UKMT Junior Olympiad problem**s (link provided below);
- pupils knowing that **the number 1 is not a prime number** … enabling pupils to 'see' by reasoning what would be undesirable about the consequences of regarding 1 as a prime number;
- thinking of **prime-factorisation** as revealing **'the DNA of each number'**;
- pupils learning to **use prime factorising to find Lowest Common Multiples (LCMs)** … eg from $p = ab^2cd$ and $q = a^2bd^2e$ it follows that LCM(p, q) = $a^2b^2cd^2e$ … that prime factorisation 'comes-into-its-own' as an efficient way to find the LCM(p, q) **when p and q are both large numbers** … that prime factorisation is also useful in

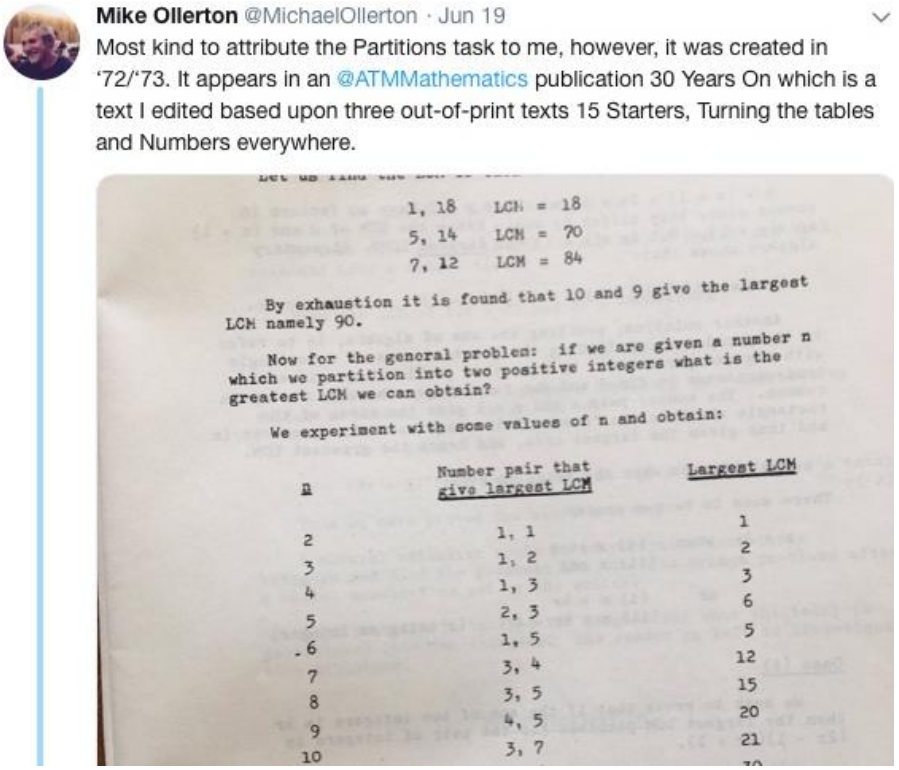
other ‘situations’, such as when simplifying surds and when solving quadratic equations;

- learning from exploring (consequences of) **representing whole numbers as ‘strings’ of their prime factors** … that is: 1, 2, 3, 22, 5, 23, 7, 222, 33, 25, 11, 223, 13, 27, 35, 2222, 17, 233, 19, 225, … ;
- how/when to make use of the **prime factorisation button on a calculator** (link provided below);
- **algebraic factorising** … pupils looking at/for, and discussing, different factorised forms of algebraic expressions (‘seeing’ various different common factors);
- arranging for pupils to view the **2006 Royal Institution Christmas Lectures by Marcus du Sautoy** … pupils appreciating surprising uses of prime numbers (link provided below).

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is part of a ‘conversation’ of tweets, about a task involving HCFs and LCMs that provides opportunities for pupils to make and test conjectures. The conversation was generated by this tweet from Siobhan McKenna:

including these from Mike Ollerton, Ms Dolan and Siobhan McKenna:

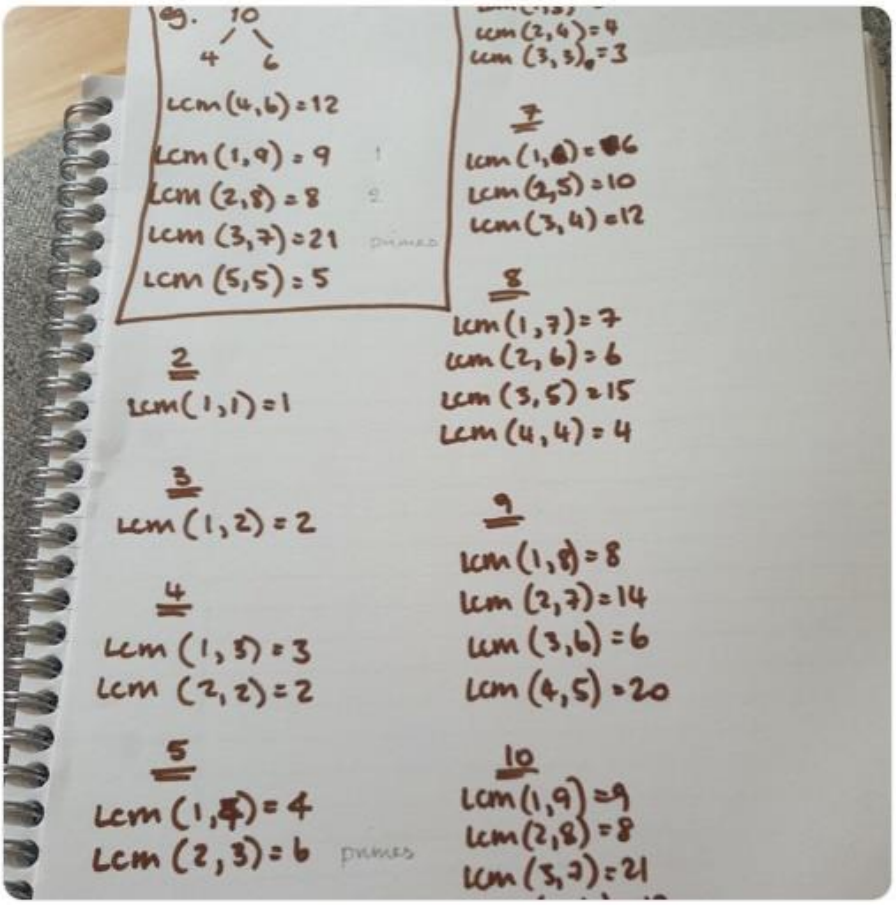


Ms Dolan @ASHMD16 · Jun 18

This looks great, must have a go.

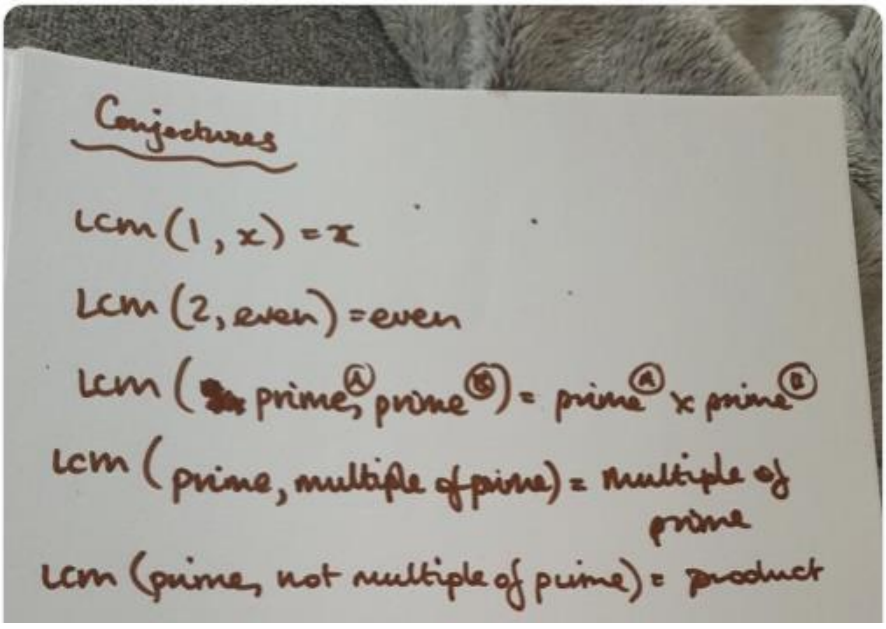
Siobhán McKenna @ShivMcKenna55 · Jun 18

Replying to @ASHMD16

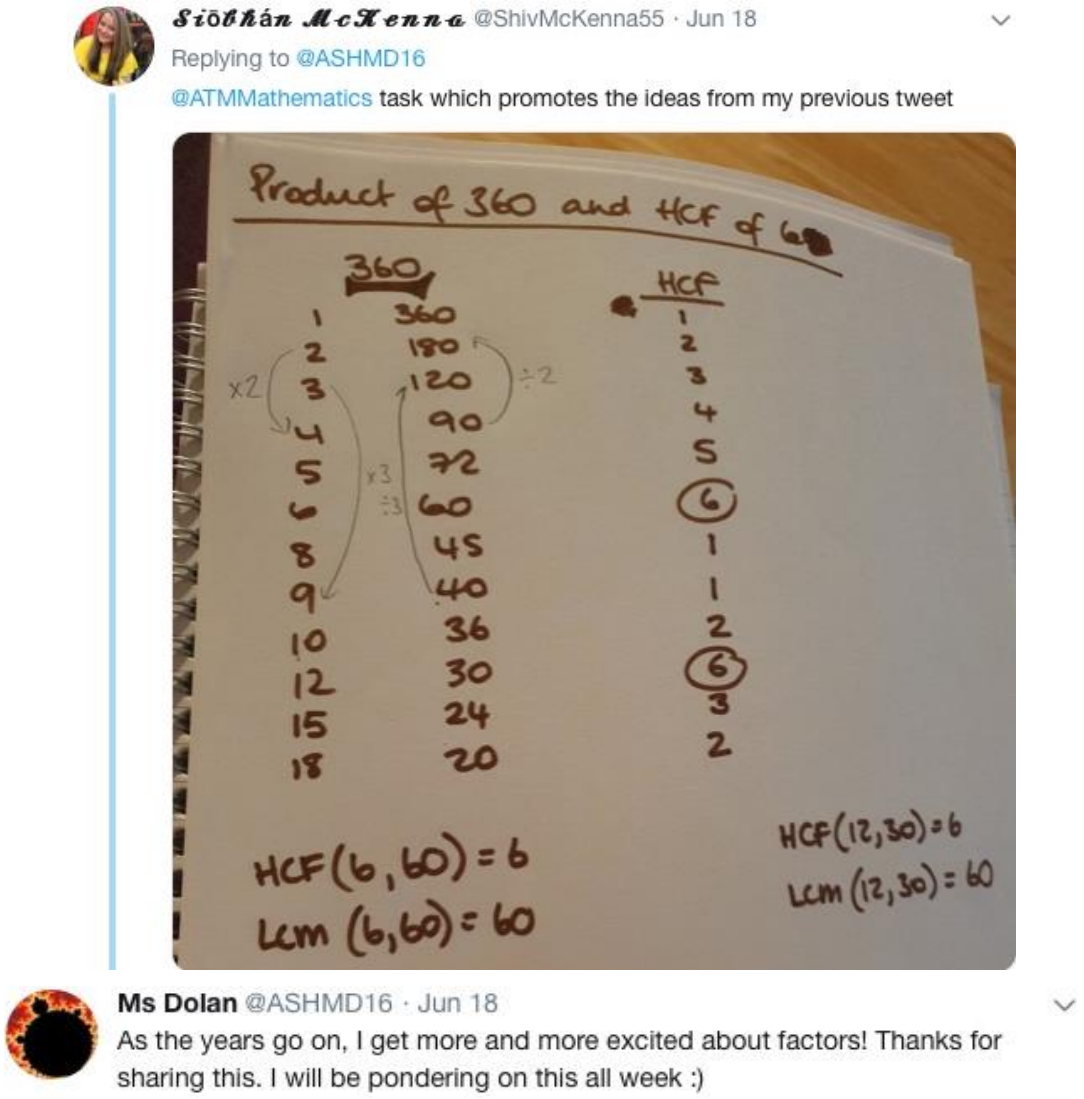


Siobhán McKenna @ShivMcKenna55 · Jun 18

Replying to @ASHMD16



and these from Siobhan McKenna and Ms Dolan:



(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Discussion, conjectures, noticing... which is a blog (12 December 2018) by Siobhan McKenna about a S1 (Year 7) lesson about factors. It was a lesson in which the teacher and pupils enjoyed doing mathematics together, making and testing conjectures, searching for examples and counterexamples, and being surprised! It was shared by Siobhan McKenna

Factors inquiry is a mathematical inquiry for which the starting point is the statement, 'The product of the factors of 24 equals 24^4'. It was shared by Ms Dolan

The sieve of Eratosthenes: revisited which is an article by Gerry McNally which was published on the website of Chalkdust magazine on 24 January 2019. The article addresses the question 'How can we teach people about the sieve in a way that helps them best understand prime numbers?' It was shared by Gerry McNally

Clock Flowers which is a delightful (try it!) Scratch program designed by Simon Gregg with which you can draw 'petals' for the flower; petals that are created by jumps of between 1 and 22 units. By asking yourself what you notice and wonder you may soon start to think about factors and multiples! It was shared by Simon Gregg

30 Years On which is an ATM (Association of Teachers of Mathematics) publication containing interesting ideas for teaching and learning mathematics that were originally published in the ATM booklets, *Numbers Everywhere*, *Turning the Tables* and *15 Starters* that are now out of print. It was shared by Mike Ollerton

Drawing Stars & Building Polyhedra which is a book by Christopher Freeman that teaches students how to draw stars with seven, eight or more points. It includes a useful investigation involving factors and prime numbers. It was shared by Gerry McNally

Stars is an interactivity from NRICH in which the user chooses a number of points to be evenly spaced around a circle, and then creates patterns by choosing a starting point and 'dragging' from one point to another. It could be used to explore factors of numbers. It was shared by James Etheridge

A Very Strange Game which is an interactive game from Transum.org. Students obey given instructions (such as 'fold your arms if it's a multiple of 4') when a new number appears in the centre of the screen. It was shared by Neil Tilson

The Royal Institution Christmas Lectures 2006 which are five popular mathematics lectures given by Marcus du Sautoy on various interesting topics, such as the roles that prime numbers may play in code making and breaking! It was shared by Sharon Malley

Teaching Multiples, Factors and Primes which is a document containing the presentation slides for a presentation by Gerry McNally given at this year's Scottish Mathematical Council's Annual Conference. It was shared by Gerry McNally

UKMT Junior Olympiad which is a source of questions (and solutions) from past papers of the Junior Mathematical Olympiad of the United Kingdom Mathematics Trust. It was shared by Gerry McNally

NCETM Secondary Magazine 127: It Stands to Reason which is the first of two articles that contain suggestions of interesting ways for pupils to use images in explorations of the HCF and LCM of two numbers. The suggested explorations provide opportunities for pupils to deduce for themselves the relationship between the HCF and LCM of any pair of numbers. It was shared by Mary Pardoe

NCETM Secondary Magazine 128: It Stands to Reason which is a continuation of the article described in the previous paragraph. It was shared by Mary Pardoe

The Calculator Guide: Prime Factor Decomposition which is a YouTube video showing how to use the FACT button on a calculator, for example to check if a number is prime. It was shared by The Calculator Guide