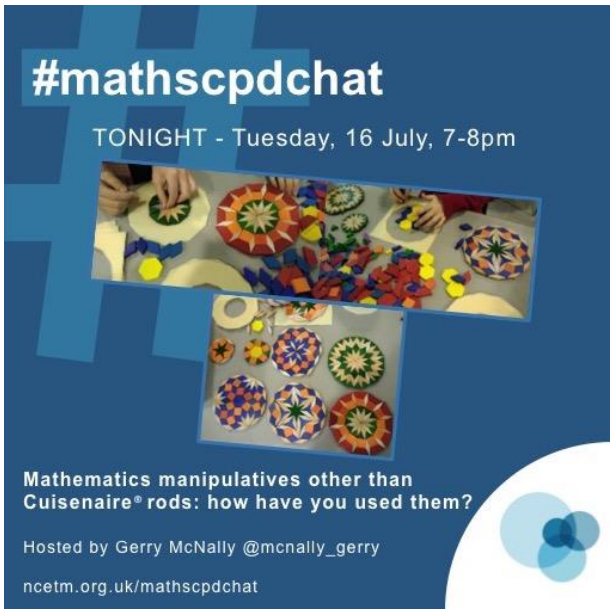
#mathscpdchat 16 July 2019

Mathematics manipulatives other than Cuisenaire® rods: how have you used them?

Hosted by Gerry McNally

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



Cuisenaire rods are one of the most useful and valuable manipulatives. But they were not discussed during this chat because their use had been the only topic of a recent previous #mathscpdchat, on 7 May 2019.

Some of the areas where discussion focussed were:

- **that which 'comes-to-mind' on hearing the words 'mathematical manipulatives'** … anything tactile that can be moved around to explain mathematics, such as counters, base 10 blocks, multilink cubes … resources that are manipulated to deepen understanding of mathematical concepts … all the practical apparatus that

we use in our classrooms, such as counters, bead strings, hundred squares, digit-cards … dice, unusual dice such as non-transitive dice (link provided below) … interlocking cubes … real fruit pieces and seeds, learners' own bodies;

- whether **a person's hands** are regarded as mathematical manipulatives … that the word 'manipulate' is strongly associated with the word 'hand' … pupils using their fingers to count on … that hands may be used to measure distances approximately;
- using **geoboards**, for example to develop a deep conceptual understanding of area and fractions, or while investigating relationships between angles that feature in circle theorems (link provided below);
- using **paper**, for example in folding and cutting to create particular shapes … tasks that involve and invoke mathematical thinking across the Y1 to Y11 age-range … expressing algebraically relationships between lengths and areas in/of shapes created by folding and cutting paper … making and investigating mÖbius strips (link provided below);
- using **pegboards** to generate mathematical thought … for example in creating and exploring number sequences (link provided below);
- using **dominoes** in a great variety of tasks, including in searching for general numerical relationships, and in puzzles (links provided below);
- using real or virtual **pentominoes** … that 'play' is an important aspect of doing mathematics (links provided below);
- various ways of using **pattern blocks** at all (st)ages, for example to make similar shapes to explore in order to aid understanding of trigonometry, or in order to explore the addition of simple fractions (link provided below);
- using **double-sided counters** (link provided below);
- using **pieces of fruit** as mathematical manipulatives … for example, slicing or cutting through a cucumber or an apple in various ways, sharing apples (eg 'How many apples do we need so that everyone gets three-quarters of an apple?') … relationships leading towards understanding ratio and proportion (eg when making fruit-salad, the number of grapes to the number of peaches that results in the nicest taste);
- that **imagination is the only limit** to what may be used as a mathematical manipulative … for example using matchsticks, straws, newspapers-rolled-around-knitting-needles, toothpicks … 'maths-in-the-forest' with pine-cones, acorns, leaves, sticks, etc;
- that improved understanding may happen as a result of using **algebra tiles**, for example when solving equations and understanding the process of completing-the-square (link provided below);

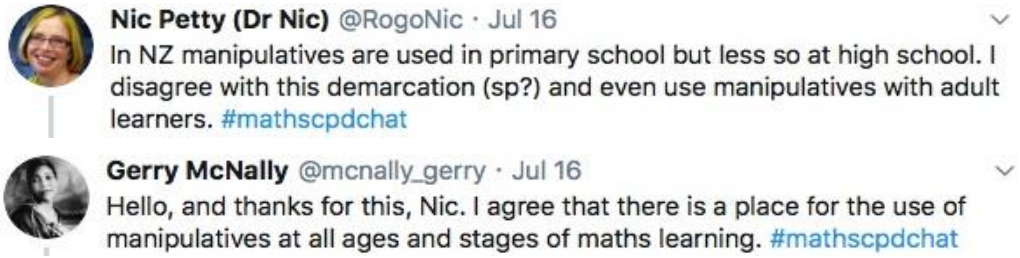
- **whether it is helpful to distinguish between manipulatives and game pieces**, for example whether/when we regard dominoes as manipulatives or as game pieces;
- whether to regard **number lines and place-value charts as manipulatives or as diagrams** … that you can move counters about on a number line, and move digits around on a place-value chart;
- using **online virtual manipulatives** … creating and exploring virtual 3D models using *GeoGebra 3D* (link provided below);
- that **Bruner's 'enactive-iconic-symbolic' distinction has been misinterpreted**, being crudely interpreted as a 'one-way learning street' summarised by 'concrete-pictorial-abstract' … that concrete materials are not only well-used at the 'start' of a new topic, but may also be returned to as required or desired … that coming back to the concrete can help pupils to see the nuance of any change or complication;
- that manipulatives are associated with 'play', and that an unfortunate consequence of this is that **the use of manipulatives in mathematics teaching may be primarily associated with early-years-teaching** … that mathematical 'play' is an important aspect of mathematical learning at any (st)age … that when pupils no longer need to depend on particular manipulatives to support their learning, they may nevertheless not move-on completely from their use … that many very good adult mathematicians use 'something concrete' to model their thinking … that it is a **teacher's responsibility to remove any age-stigma associated with the use of manipulatives**;
- that in New Zealand manipulatives are used in primary schools, but less so in secondary schools … and in Sweden **the older pupils are, the less that manipulatives are used**;
- that, and how, secondary teachers can **build on** the use of mathematical manipulatives in primary schools;
- that the **nature of the manipulatives used may change** as the stage-of-learning changes;
- that **pupils of any age should have free reign in choosing** from a wide range of learning 'aids', including manipulatives;
- **'off-loading' onto the manipulation of objects, operations carried out during computation** (eg when manipulatives such as blocks or fingers are used as a support);
- that sometimes it is **effective for all pupils in a class to explore the potential of just one particular manipulative** in order to deepen their understanding of mathematical ideas and relations;

- **using manipulatives in 'demonstrations'**, e.g. using double-sided counters to generate productive whole-class discussion about operating with and on negative numbers;
- **teachers** themselves learning mathematics **using manipulatives**;
- avoiding **'pitfalls' in the use of manipulatives** … e.g. avoiding using manipulatives as a reward for good behaviour rather than when they will be useful to support learning … avoiding using manipulatives ONLY at the end of term, as a 'treat';
- whether manipulatives are presently a significant feature of western cultures … or whether the use of manipulatives in maths teaching and learning has **'fallen-out-of-favour' in recent years** … teachers 'making do' with online 'virtual' manipulatives … that virtual resources are 'one step removed from concrete resources, and another step on the way to symbolic representation';
- the **history of the Abacus** … that originally round pebbles were moved along grooves … that the Latin word for 'pebble' is 'calculus' … that the 'abax', or sand table, was the origin of our abacus (link to Martin Gardner source provided below);
- that there is presently a felt need for **PD provision that focusses on the effective use of manipulatives in mathematics education**.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is part of a 'conversation' of tweets, about how Jerome Bruner's 'enactive-iconic-symbolic' distinction has been misinterpreted as a 'concrete-pictorial-abstract' progression for all mathematical learning, **thus sending the false message that concrete materials should be used ONLY at the start of learning some new mathematics**, and then 'put away'. The conversation was generated by this tweet from Gerry McNally:

including these from Nic Pretty (Dr Nic) and Gerry McNally:



these from Andrew Jeffrey and Dr. Helen J Williams:

and these from Andrew Jeffrey and Kathryn Darwin:

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Manipulatives in the Primary Classroom which is an article on NRICH written by Jenni Black in 2013. Although it was written six years ago this article is still very relevant and useful today; Jenni explores the use of manipulatives in primary schools and offers suggestions about how using manipulatives can support children's mathematical thinking, reasoning and problem solving. It was shared by Lisa and Gerry McNally

When is a Symbol Symbolic? which is an article by John Mason. From page 10 onwards he clarifies meanings in Bruner's (J.S. Bruner, *Towards a Theory of Instruction*, New York: Norton, 1968) distinction between 'Enactive', 'Iconic', and 'Symbolic' modes of learners' thinking. Mason explains how typically these modes of thinking are employed in a 'spiral' way, being returned to again and again in new learning, rather than each being a stages-of-thinking that a person passes through once only. He points out that some teachers have even wrongly identified 'Enactive' with primary school, 'Iconic' with middle school, and 'Symbolic' with upper school. It was shared by Andrew Jeffrey and Mary Pardoe

Exploring Area and Fractions with Square Geoboards which is a book and download written by Geoff Faux, from the ATM. It consists of a sequence of tasks that develop a deep conceptual understanding of area and fractions. It was shared by Mary Pardoe

An Introduction to Algebra Tiles for Teaching Mathematics which is a detailed and useful blog by Mark McCourt in which he shows, with clear illustrations, how Algebra Tiles can be used to facilitate pupils' use and understanding of algebra. It was shared by Heather Scott

How I teach ... using manipulatives which is a video in which Simon Gregg shows how he and his primary-school-aged pupils use manipulatives to support mathematics learning and deepen understanding. It was shared by Mary Pardoe

Focus on ... dominoes which is an article in the NCETM Secondary Magazine Issue 72. It shows, with illustrations, how explorations of arrangements, combinations and structures of dominoes and domino sets provide opportunities for pupils to encounter various mathematical ideas, and to act mathematically. It was shared by Mary Pardoe

Mathematical Dominoes activity booklet which is an original, interesting and useful resource by Heather Scott. It was shared by Heather Scott

Focus on ... learning with pegboards which is an article in the NCETM Secondary Magazine Issue 67. It shows, with illustrations, how pupils can learn about numerical and algebraic

ideas and relationships by creating and exploring arrangements of coloured pegs placed in pegboards. It was shared by Mary Pardoe

Focus on ... paper folding which is an article in the NCETM Secondary Magazine Issue 79. It shows, with illustrations, how pupils can work mathematically, and learn some new mathematics by folding and cutting paper. It was shared by Mary Pardoe

Wind and Mr. Ug which is a recommended 'cautionary tale' in the form of a Mobius video! (It is related to using paper as a mathematical manipulative!). It was shared by Kathryn Darwin

Focus on ... non-transitive dice which is an article in the NCETM Secondary Magazine Issue 82. It explains and illustrates explorations of interesting phenomena that occur when two people each throw a die (of a special kind) and the 'winner' is the person with the highest score. It was shared by Mary Pardoe

An idea for the classroom - clusters of counters which is an article in the NCETM Secondary Magazine Issue 69. Students reduce ratios to their simplest forms in order to solve these puzzles, rather than for no apparent reason as in many textbook exercises. It was shared by Mary Pardoe

Cuisenaire - from early Years to Adult which is a full colour Cuisenaire book from the ATM by Mike Ollerton, Dr. Helen J Williams and Simon Gregg, with 35 whiteboard slides. It was shared by Dr. Helen J Williams

Manipulatives which is a lovely source of many virtual manipulatives, including counters, geoboard, pentominoes, Cuisenaire rods, pattern blocks, double-sided counters, Rekenrek, and much more! It was shared by Kathryn Darwin

Manipulatives Maths which is a collection of virtual manipulatives, including Double Sided Counters. It was shared by Phil Bruce

Two-Color Counters which are double-sided yellow/red counters, one inch in diameter, available on Amazon. It was shared by Phil Bruce

Creative Maths which is a social enterprise that creates and shares innovative resources for all learners and teachers, to grow a world of mathematicians. It was shared by Nic Pretty (Dr Nic)

Mathematical Circus which is a book by Martin Gardner that includes interesting mathematical material involving manipulatives. It was shared by Gerry McNally

A Meta-Analysis of the Efficacy of Teaching Mathematics with Concrete Manipulatives which is an article that examines the evidence regarding the use of manipulatives during mathematics instruction. It was shared by Kieran Mackie

Manipulatives - the start of the journey which is a recent short blog by Cav in which he writes about his preparation to start using manipulatives with Year 7 pupils next term. It was shared by Cav

GeoGebra 3D with AR (Google): Explorations & Lesson Ideas which is an online introduction, lesson ideas, and various means through which maths teachers can use GeoGebra Augmented Reality to create dynamic, student-centred exploratory 3D learning environments. It was shared by Tim Brzezinski

Sand table which is the Wikipedia entry for 'Sand table'. An Abax was the predecessor to the abacus. It was shared by Simon Gregg