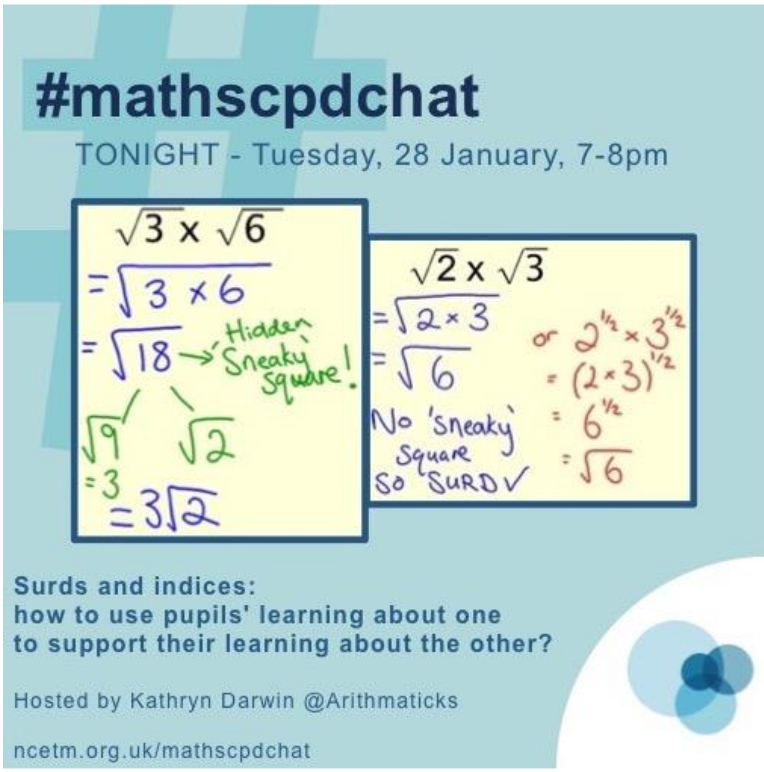
#mathscpdchat 28 January 2020

Surds and indices: how to use pupils' learning about one to support their learning about the other?

Hosted by [Kathryn Darwin]

*This is a brief summary of the discussion – to see all the tweets, follow the hashtag **#mathscpdchat** in Twitter*



Some of the areas where discussion focussed were:

whether pupils do, or should, encounter surds **before or after encountering indices**:

- an **A level scheme of learning has surds before indices** … indices are 'covered' just before logarithms;
- 'my **pupils meet surds in Year 7** in a unit on powers and roots', during which unit they also learn the basic **addition and subtraction index laws**;

- that **surds can be a way into fractional indices** … during a teaching-plan with the following topic-order: powers, roots, index laws restricted to whole-number indices, surds, index laws with fractional indices;
- that it's possible for pupils to learn effectively by 'meeting' surds and indices together … **pupils appreciating that a surd can be written as a number raised to a power … for example, $x^{(1/n)} = \sqrt[n]{x}$;**
- that surds might be introduced before indices **in the context of the general idea of an irrational number**;
- that when a number raised to the power ½ is multiplied by itself, representation as the equivalent surd **supports the 'makes-sense-aspect' of the addition index law** … for example $\sqrt{3} \times \sqrt{3} = 3$ and $3^{½} \times 3^{½} = 3^{(½ + ½)} = 3^1 = 3$;
- that **students' abilities to 'swap-between' surd and index notation is necessary** in order for them to make sense of some 'AS' calculus problems … so, for example, in the specification for Level 3 Advanced Subsidiary (AS) GCE, knowing the equivalence of $a^{(m/n)}$ and $\sqrt[n]{(a^m)}$ is 'early content';
- looking at indices again **before students explore mathematics that involves logarithms**;
- while teaching A level '**I flit between the two (surds and indices) all the time** … if we have an answer in one form I pretend to want the answer in the other form … so much is done on surds and indices for GCSE that when teaching A level, it's hard to know which has been taught first';
- that because **good use of powers is necessary throughout A level**, students cannot repeatedly focus on them too often;
- that providing opportunities for students in KS3 to develop deep understanding of, and fluency in, **simplifying expressions involving indices (such as $(b^{20}c^{14})^{½}$ or $(1000z^{15})^{⅓}$)** is good preparation for A level;
- that **surds should be taught as soon as Pythagoras' Theorem** … that unnecessary difficulties arise for students if their first encounters with irrational numbers are when they are using calculators during A level work;
- that **pupils meet simple indices in Key Stage 2, before they meet surds in KS3** … usually they first meet surds when they are calculating the side-lengths of right-angled triangles, and placing surds on number lines;

mathematical ideas that pupils need to **understand before they can build/develop understanding of indices and surds**;

- **in preparation for learning about surds**, prior learning should include: the laws of arithmetic, prime factor decomposition, application of Pythagoras' Theorem, and the ability to solve simple quadratic equations;

- **in preparation for learning about indices**, prior learning should include: the laws of arithmetic, order of operations, the ability to operate with/on fractions, and what the reciprocal of a fraction is;
- that some pupils in KS3 do not know/understand that m^n (where m and n are whole numbers) represents the number m multiplied by itself n times;

whether pupils learn to use and understand **numerical expressions involving indices** at the same time as, or at a time prior to when they start to work with **algebraic expressions involving indices**:

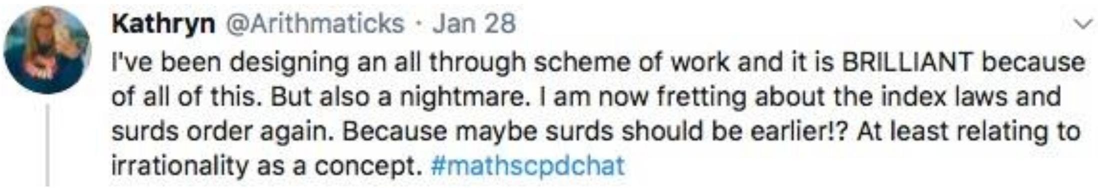
- in the past we taught them separately, but we are now **introducing algebra from the very beginning of Y7 … so numerical work on any operation is generalised** … for example 5 – 7 = -(7 – 5) is understood as ('seen as') a special case of the algebraically expressed generality p – q = -(q – p);

when students **first encounter surds 'as numbers in their own right'**:

- via **processes of equation-solution** … that all linear equations have solutions within the rational numbers, and that moving on to solving quadratic equations quickly requires 'a new set of numbers';
  that **working with Pythagoras' Theorem** enables pupils to locate surds on a number line using construction techniques … an example of pupils seeing and using links between number and geometry;
- pupils **learning something of the history of surds**;
- introducing **surds as side-lengths of squares with given areas** … and seeing that **$\sqrt{2}$ is the length of the diagonal of the unit square**.

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is a part of a conversation about whether, or not, in secondary-school teaching, to introduce surds and indices at the same time, and that it may be very beneficial for subsequent learning at A level for students to develop fluency in simplifying, and moving (converting) between, expressions involving surds and expressions involving indices. The conversation was generated by this tweet from [Kathryn Darwin](#):


Kathryn @Arithmaticks · Jan 28
I've been designing an all through scheme of work and it is BRILLIANT because of all of this. But also a nightmare. I am now fretting about the index laws and surds order again. Because maybe surds should be earlier!? At least relating to irrationality as a concept. #mathscpdchat

and included these from [Simon Ball](#), [Kathryn Darwin](#) and [Alex J-Williams](#):

Simon Ball @ballyzero · Jan 28

Hard to answer that! It might be one of those things that you trial with a group or two, to see if it works better. I feel like index laws and surds could be reasonably separate topics, though... #mathscpdchat

Kathryn @Arithmaticks · Jan 28

I think that is traditionally how it is done. I know it was for me. Until it came to needing index notation in differentiation/integration problems with surds at A Level! #mathscpdchat

Alex J-Williams @Trudgeteacher · Jan 28

Replying to @Arithmaticks

Don't seem them as a linked topic, any more than doing polynomials etc. Indices is about a form of notation, surds handling a gap in the number line? Why are they taught together (not that there is a reason not to either)?

these from [Kathryn Darwin] and [Alex J-Williams]:

Kathryn @Arithmaticks · Jan 28

For me the link was through fractional indices. Some students prefer one representation over another. It reinforces the power of a half as a square root, or that root(2) x root(2) = 2 because its actually 2^0.5 x 2^0.5 = 2^1 etc. #mathscpdchat

Alex J-Williams @Trudgeteacher · Jan 28

Yes happy with that proof but is it not just a justification for the fractional index notation rather than a direct link with surds? Indices are really important because of the link to calculus of polynomial/power functions? Don't see a pedagogic link between the two?

Kathryn @Arithmaticks · Jan 28

Ah but liguisitcally there is. We have special names for squares and cubes, but if you exclude those we have 'fourth root' for a power of 1/4, a 'fifth root' for a power of 1/5, etc. I think there are actually many links. They are just different representations imo. #mathscpdchat

Alex J-Williams @Trudgeteacher · Jan 28

Don't bring linguistics into this! That's true but we don't really explore surds in terns of cube or 4th roots or do we?? #mathscpdchat

Kathryn @Arithmaticks · Jan 28

Haha I already did!!!! Oh I definitley would! Would you never ask 8^(1/3)? #mathscpdchat

and these from [Kathryn Darwin] and [Alex J-Williams]:

Kathryn @Arithmaticks · Jan 28

I would also put that in my lessons, and have done. Just to see what students do with it! #mathscpdchat

Write as integers or surds:	Write as powers in their simplest form or surds:	
$144^{\frac{1}{2}}$	$x^{\frac{1}{2}}$	$(36z^8)^{\frac{1}{2}}$
$27^{\frac{1}{3}}$	$y^{\frac{1}{3}}$	$(35z^8)^{\frac{1}{2}}$
$81^{\frac{1}{3}}$	$z^{\frac{1}{10}}$	$(1000z^{15})^{\frac{1}{3}}$
$16^{\frac{1}{4}}$	$(x^{10})^{\frac{1}{2}}$	$(5c)^{\frac{1}{4}}$
$12^{\frac{1}{5}}$	$(x^9)^{\frac{1}{2}}$	$(b^{20}c^{14})^{\frac{1}{7}}$

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Among the links shared were:

Departmental Workshops - Surds which is an NCETM resource written (in 2010) to provide mathematics teams with structured professional development that can be delivered in-house. The objectives include using surds in exact calculations, without a calculator, and rationalising denominators of expressions involving surds. It was shared by SteveLMMXX

Rationalising the Denominator which is an illustrated, and comprehensively explanatory, PDF document by Alex ~ VicMathsNotes. It was shared by Alex ~ VicMathsNotes

Simplifying Surds which is another illustrated, and comprehensively explanatory, PDF document by Alex ~ VicMathsNotes. It was shared by Alex ~ VicMathsNotes