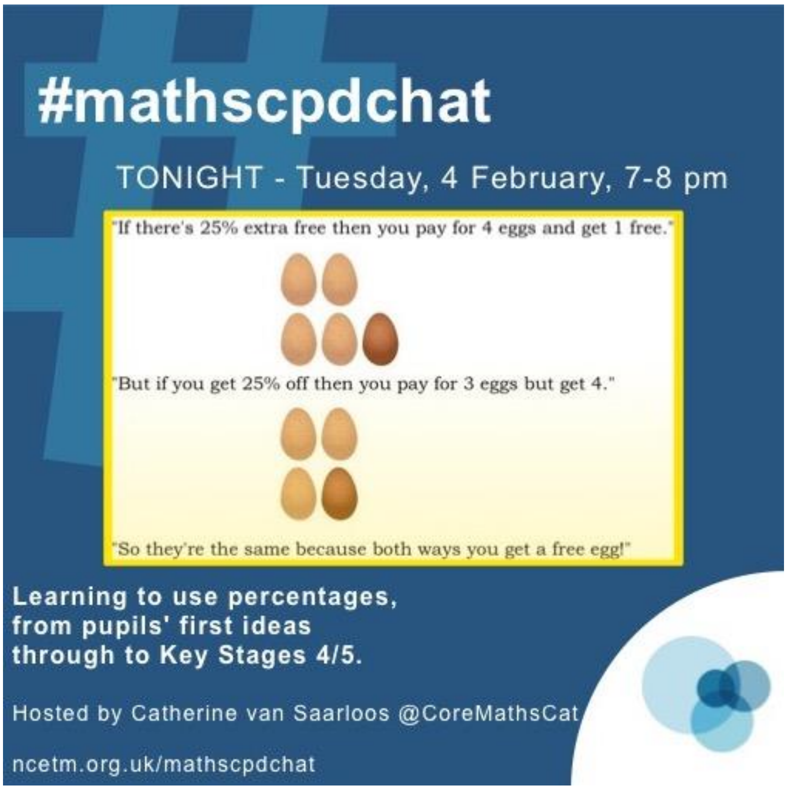
National Centre for Excellence in the Teaching of Mathematics

#mathscpdchat 4 February 2020

Learning to use percentages, from pupils' first ideas through to Key Stages 4/5

Hosted by [Catherine van Saarloos]

*This is a brief summary of the discussion – to see all the tweets, follow the hashtag **#mathscpdchat** in Twitter*



Some of the areas where discussion focussed were:

how the experiences of some participants in the chat had **prompted them to look into students' difficulties** when learning to understand and use percentages:

- that working with GCSE Resit students prompted a teacher to think deeply about **what might be causing those students to struggle** with percentages;

- that teaching Year 5/6 students gave a teacher lots of **opportunities to see their misconceptions** … for example some students were thinking that 'because you find 10% of an amount by dividing by 10, to find 5% you must divide by 5';

participants' comments in response to seeing **questions about percentages (**such as '36% of 450 = …' and '35% of 320 = …') **from the 2019 Key Stage 2 SATs**:

- that a **sixth form college teacher** would encourage students to use a **'build-up' method,** to calculate 36% of N (10% of N + 25% of N + 1% of N) … another teacher would expect students to know that 36% of N = 0.36 × N … building up/down from 10% … enabling students to understand **how 'build-up' and 'multiplier' methods are connected**, for example that multiplying N by 1.25 gives you 'one and a quarter of N' or increases N by 25% … that such understanding (of how 'multiplier', 'build-up' and 'ordinary thinking' methods can 'do the same thing') **depends (draws) on many previously acquired understandings**;
- that it is important for students to be able to **estimate the results of percentage calculations**, for example knowing that 36% of N is less than half of N would be a good start;
- that a **Year 6 teacher encourages the use of 'fact boxes'**, for example to work out any percentage of 450, start from 100% (of 450) = 450, then 50% = 225, 10% = 45, 1% = 4.5, and 5% = 22.5 … to work out 99% of N, calculate 1% of N and subtract it from N;
- that **calculations involving percentages provide opportunities for** students to experience and understand **alternative ways of solving the same problem** … that (generally) learning is supported better by 'solving one problem five ways rather than solving five problems the same way';
- that students' learning-path towards being able to 'calculate-a-given-percentage-of-an-amount' ought to **lead eventually to 'multiplying-by-a-decimal'** … that students **sharing and discussing their different methods is valuable** for their learning;
- that being challenged to do particular calculations, such as 8% of 25, helps students see that the **commutativity of '% of" calculations (that P% of Q = Q% of P)** can sometimes enable them to simplify calculations … that it is **important for students to know and understand that generally P% of Q = Q% of P**;
- the value of pupils and teachers **comparing and discussing the efficiency of alternative ways of finding a percentage of a quantity** … for example finding 51% of 900 as '0.51 × 900' or as 'half of 900 + one-hundredth of 900 (450 + 9)' or by calculating 10% of 900, 1% of 900 and building up from there … that exploring different methods deepens pupils' understanding, for example of why it is sometimes sensible to add in a context which appears to be multiplicative;

- that, whereas **Year 6 students were observed using efficient methods** of calculation, **A level students were seen 'sticking to' (less efficient) remembered procedures** … for example, Year 6 students calculated 99% of 200 as '200 – 2 = 198', whereas the A level students wrote '200 × 99/100 = 2 × 99 = 2 × 9 + 2 × 90 = 18 + 180 = 198';

using **visual images** to represent (and aid) percentage calculations, thus enabling students to 'access' problems involving them:

- **using a 100-square** … but that providing students with reasons for dividing-up a whole into 100 equal parts is not 'straightforward' … that, for example, **the 'hundredness' may get in the way of seeing what happens** when a quantity is increased by a fraction of itself and then that quantity decreased by the same fraction of itself … that **a simpler image (not involving 100 equal parts) may be a better aid** … that many adults struggle to understand how to find the final result of applying to a quantity a percentage increase followed by a percentage decrease;
- using **'bar-models'** to represent operations involving percentages;
- using a **variety of images and manipulatives**, such as 'percentage-walls', centimetre cubes, circles, rectangles, pentagons, 'post-it' squares, … that students seeing **what is the same in different representations can deepen understanding** … that prompting students to create and compare their own individual representations of percentage operations can support their learning, reveal their thinking, and possibly avoid their teacher undervaluing their understanding;
- **contexts, other than those involving percentages, in which 'bar-models' are helpful aids** … that images of bars can be used effectively by students in Year 1 onwards … that such images 'come into their own' in learning about fractions, percentages, ratio and proportion;
- that images of bars are particularly useful when trying to do **'reverse percentage' calculations** … they help students solve problems such as 'After a 20% discount a coat costs £96. What was the original price?';
- strategies that enable students to understand and use the **equivalence of fractions, decimals and percentages**;
- when, or whether, it is helpful for students to use the **'% button' on a calculator**;

the **learning progression** that is indicated by comparing a percentages question from a **Foundation level GCSE exam paper** with a percentages question from a **Key Stage 2 SATs paper**:

- that in Key Stage 4 (but not in Key Stage 2) students are expected to calculate with **percentages in contexts** … that it may be harder to 'find the maths' when the problem is posed in a context;

- that in Key Stage 4 (but not in Key Stage 2) students are expected to understand and use **percentages greater than 100%**;
- that the **ability to reason that is displayed by students in Key Stage 2** may (to some extent) become lost when the same students are faced with ‘much new mathematical content’ in Key Stages 3 and 4;

ways of **engaging Foundation level GCSE Resit students** in mathematics involving percentages:

- that older students can be ‘hooked-in’ by **‘facts about adult-living’**, such as facts about income tax, interest you have to pay on loans, interest gained on savings, interest accumulated through investments, take-home pay after tax has been deducted;
- that effective ‘hooks’ for learning **depend on the students’ interests** … for example some students following ‘building-construction’ courses were hooked in by percentage calculations in the context of bricklaying;
- that **calculations about chocolate(s)** often effectively engage students;
- that **Sports contexts** can engage students … for example some students like to compare their own fastest time to run 100 metres with those of Olympic champions;

the reasons why students struggled with **a particular challenging Higher level GCSE exam question involving percentages**:

- that students found the question **hard to ‘decode’** … that students may not have been able to interpret the phrase ‘value for money’;
- students were required to reason about quantities that were not specified … that teachers would advise students to ‘unlock their reasoning powers’ by substituting their own values for the unspecified amounts;

other discussion points included:

- whether (how often) teachers pose problems, or ask questions involving percentages when the **main focus is on a different mathematical topic** … that percentages may arise naturally in statistical contexts, for example when interpreting scatter-diagrams and describing types of correlation;
- that the idea of **percentage increase/decrease cannot be applied to changes in temperature** … because any temperature-measurement scale is an interval scale (not a ratio scale) and does not have a ‘true’ zero point, it is not possible to make statements about how many times higher one temperature is than another … it makes no sense to say ‘it’s twice as hot today as it was yesterday’!

In what follows, click on any screenshot-of-a-tweet to go to that actual tweet on Twitter.

This is a part of a conversation about using efficient methods, in particular about making use of the commutativity of the operation ‘percent of’; P ‘percent of’ Q = Q ‘percent of’ P. The conversation was generated by this tweet from Catherine van Saarloos:

Catherine van Saarloos @CoreMathsCat · Feb 4
I thought it might be interesting to start by looking at the % questions that appeared in the 2019 SATs. These are all from Paper 1 (arithmetic) #mathscpdchat How would you expect your students to approach these problems? Please state in responses which phase/year group.

33	36% of 450 =	29	51% of 900 =
27	35% of 320 =	18	20% of 3,000 =

and included these from Nikki Stix and Lou H-S:

Nikki Stix @tanglytortoise · Feb 4
Replying to @CoreMathsCat
In Year 6 we teach fact boxes for % and division so for example 36% of 450. Chn would do 100% = 450 50% = 225 10% = 45 1% = 4.5 and 5% = 22.5. They could then make any amount. We teach 99% find 1% and take away. Fact boxes have been such a help for our children.

Lou H-S @LouiseHStaples · Feb 4
I use a bar model to provide a visual. Particularly helpful for when we know a percentage part e.g price in sale and want to know full price.

these from Rachael Brown, Lou H-S and Lee Overy:

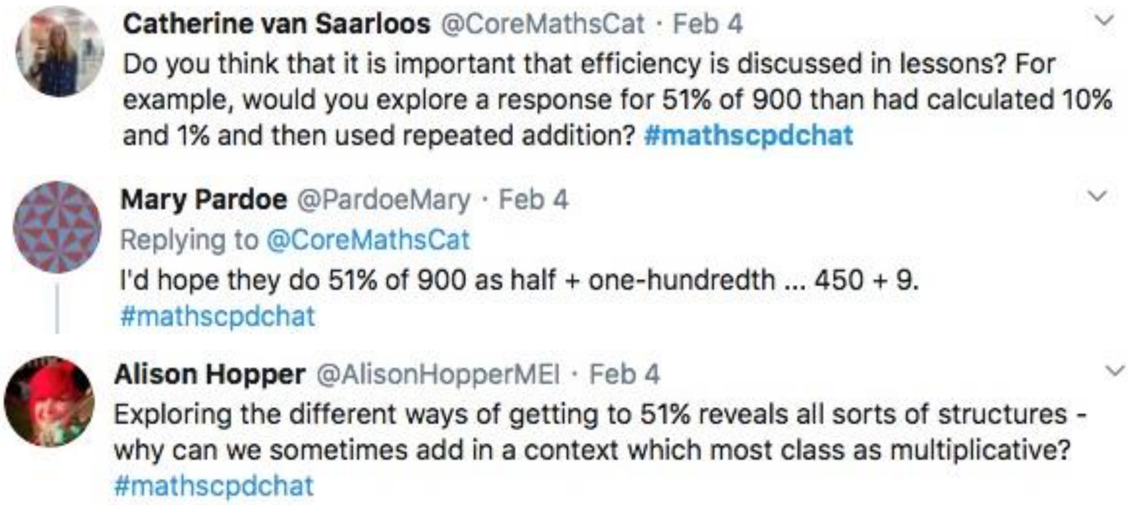
Rachael Brown @RachaelBMaths · Feb 4
Replying to @CoreMathsCat
Questions such as 98% of 200 show the importance of understanding commutativity, which I didn’t realise until I completed MaST training as primary teacher. The number of error points in adding 9 groups of 10% and 8 groups of 1% is huge. #mathscpschat

Lou H-S @LouiseHStaples · Feb 4
I used to love showing my ITE students 25% of 16 and 16% of 25. Some of them even got the same answer.

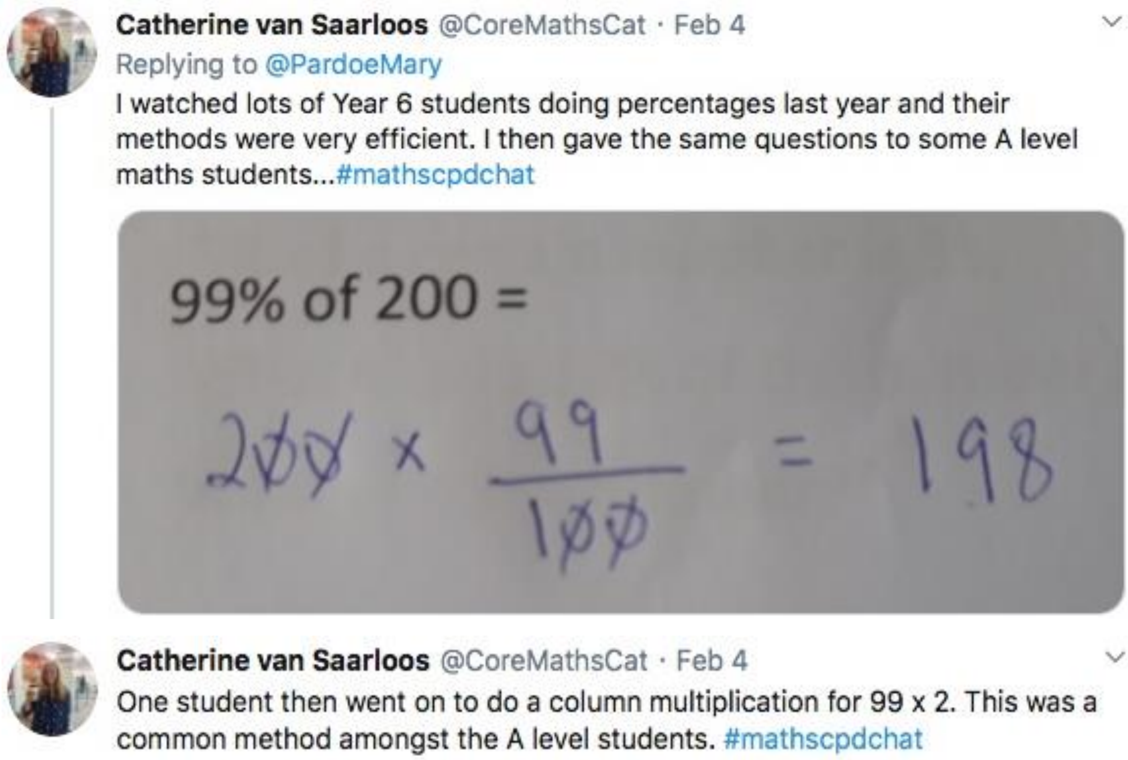
Lee Overy @Lwdajo · Feb 4
Replying to @PardoeMary @RachaelBMaths and @CoreMathsCat
Jim Al-Khalili tweeted recently that he had not realised this, I seem to recall. #mathscpdchat.

Rachael Brown @RachaelBMaths · Feb 4
Replying to @PardoeMary and @CoreMathsCat
I remember being equally wowed and appalled I’d never fully realised the implications. I had made basic links at school eg 30% of 50 and 50% of 30 but not as it being commutative and certainly not as a strategy to improve efficiency like the 8% of 25 example.

these from Catherine van Saarloos, Mary Pardoe and Alison Hopper:



and these from [Catherine van Saarloos](#):



(to read the discussion-sequence generated by any tweet look at the ‘replies’ to that tweet)

Among the links shared were:

[Percentage increase/decrease questions](#) which is a blog from Don Steward containing lovely examples of how bar-model images can support percentage calculations. It was shared by [Catherine van Saarloos](#)

Income Tax rates and Personal Allowances which is a UK Government website showing current income tax rates and allowances. It was shared by Catherine van Saarloos

Multiplicative Reasoning - The teaching Units which is part of an NCETM microsite providing comprehensive guidance for teaching multiplicative reasoning. It was shared by Alison Hopper

Analysis of 2019 KS2 Maths SATs Arithmetic paper (Part 1) which is a blog from *Herts for Learning* in which Year 6 teachers pull together some of what they learned from the 2019 KS2 SATs papers. It was shared by Rachael Brown

Teaching to Mastery Mathematics Bar Modeling: A Problem-solving Tool which is a book by Yeap Ban Har. It was shared by Lee Overy

Nature's Ratio Scale of Fields which is a blog from *McMurmerings* which contains some explanatory notes about why it is not appropriate to compare temperatures multiplicatively. It was shared by Lee Mc James