Mastery Professional Development

Mathematical representations

Place-value charts

Guidance document | Key Stage 3

Trillions			Billions			Millions			Thousands					
100s	10s	1s	100s	10s	1s	100s	10s	1s	100s	10s	1s	100s	10s	1s

10 000s	1 000s	100s	10s	1s		0.1s	0.01s
3	2	1	0	9	•	8	7

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

What they are

Place-value charts are tables with column headings indicating the value of the digits of any number written in the base-ten (decimal) system. All the headings are powers of ten, with the power increasing by one (making the value ten times bigger) as you move one column to the left (i.e. ones = 10^0, tens = 10^1, hundreds = 10^2 and so on).

100s	10s	1s

The basic place-value chart is likely to consist of three column headings: hundreds, tens and ones*. However, it can be extended to include columns to the left, as well as to the right, to allow for decimal numbers to be represented (with a decimal point between the ones and tenths columns). (*N.B. the term 'ones' is preferred to 'units' because we can think of any value as the 'unit'.)

1 000 000s	100 000s	10 000s	1 000s	100s	10s	1s

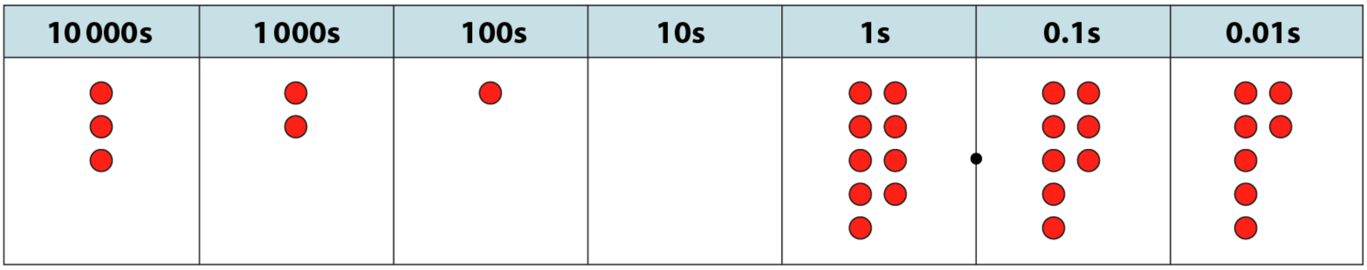
100s	10s	1s	0.1s	0.01s	0.001s
		•			

The columns in a place-value chart are sometimes grouped in threes to support an understanding of the structure of, and for ease of saying, large numbers, with each period containing the three column subheadings: 100s, 10s and 1s.

Trillions			Billions			Millions			Thousands					
100s	10s	1s	100s	10s	1s	100s	10s	1s	100s	10s	1s	100s	10s	1s

Such a structure is often conveyed by writing numbers using commas or spaces to separate out these clusters of three digits. For example, 'two million, three hundred and forty-five thousand, six hundred and seven' may be written as 2,345,607 or 2 345 607.

Dots can be placed in the columns of a place-value chart to represent a given number, and this may be the way that place-value charts were first introduced to students at primary school. The number 32 109.87, for example, can be represented like this:



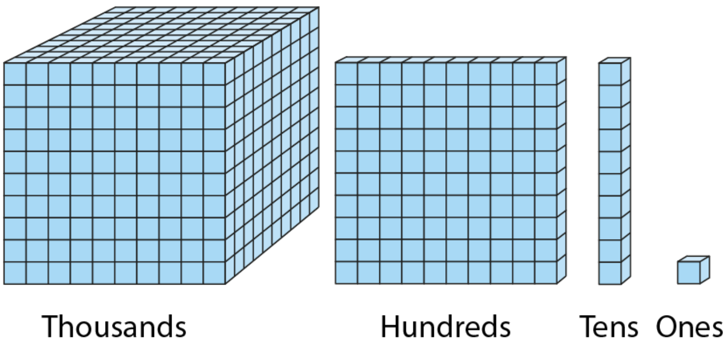
It is common practice to arrange the dots in each column into vertical groups of five. The use of dots can support students in understanding the idea of exchange (which is important in developing fluency with standard algorithms for calculation) as groups of ten dots in any column can be exchanged for one dot in the column to the left. Similarly, a dot in any column can be exchanged for ten dots in the column to the right (for example, to facilitate a subtraction).

Whilst students are likely to be familiar with using dots to complete place-value charts, they are more likely to use digits, especially by the time the charts are being used at Key Stage 3.

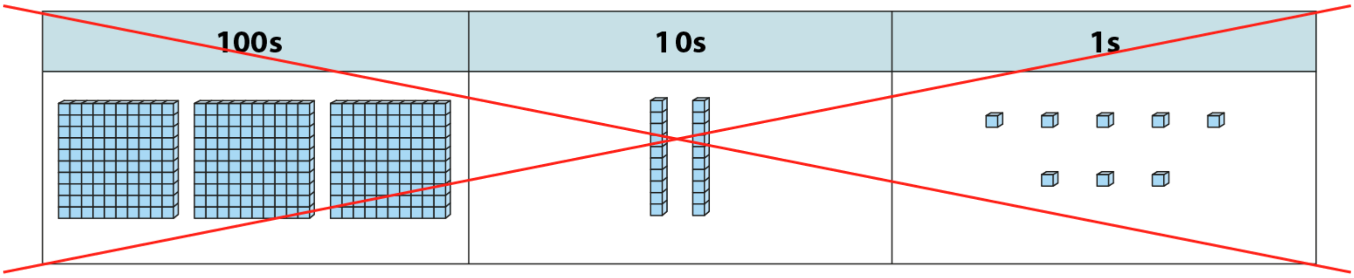
10 000s	1 000s	100s	10s	1s	0.1s	0.01s
3	2	1	0	9 •	8	7

The use of digits is a more efficient way of representing the constituent parts of a number. However, while the place-value chart may not be the first representation used when introducing the exchange of tens and ones, the idea that when ten is reached in any particular column it can then be written as one in the column to the left, and that a one in any column can be exchanged for ten in the column to the right, is crucial in supporting students' mastery of written calculation strategies.

This notion of exchange will also have been met when using manipulatives such as Dienes, but it is important that students appreciate the difference between using Dienes to represent a number, and using the digits (or dots) in a place-value chart. Dienes do not rely on the need for column headings because the size of each block directly relates to its value.



It is, therefore, not correct to use these representations within a place-value chart itself.



Instead of representing three in the 100s column and two in the 10s column, the incorrect example given above would represent 300 in the 100s column (a value of 30 000) and 20 in the 10s column (a value of 200).

When students are familiar with the column headings in a place-value chart and have a firm grasp of the place-value system, the process of writing numbers in a vertical format (as in standard algorithms), where digits with the same value need to be lined up, can be performed with understanding.

Why they are important

Place-value charts show the structure of the column headings in our base-ten place-value system and support students in understanding that any digit can have a number of different values, depending on its position.

The decimal place-value system is based on four properties:

- Position – the quantities represented by individual digits are determined by the position they hold in the number
- Base-ten – the values of the position headings are all powers of ten and the powers increase by one as you move from right to left
- Multiplicative – the value of any individual digit is found by multiplying the value of the digit by the value assigned to its position
- Additive – the overall quantity represented is the sum of the values represented by the individual digits, e.g.:

$$\begin{aligned} 483 &= 4 \text{ hundreds} + 8 \text{ tens} + 3 \text{ ones} \\ &= 400 + 80 + 3 \\ &= 4 \times 100 + 8 \times 10 + 3 \times 1 \\ &= 4 \times 10^2 + 8 \times 10^1 + 3 \times 10^0 \end{aligned}$$

Place-value charts can support students in exploring these properties and provide a framework in which the structure of the place-value system can be understood. When adding and subtracting, the place-value chart supports standard columnar procedures, allowing students to lay out calculations in a way that makes the place value of numbers clear.

The extension of the place-value system from integers to decimals is not always straightforward, and place-value charts can be used to support students in making links between decimals and fractions. For example, by labelling the tenths column as $\frac{1}{10}$s as well as 0.1, students can see that a four in the tenths column represents $\frac{4}{10}$ and, therefore, that $0.4 = \frac{4}{10}$.

10 000s	1 000s	100s	10s	1s	$\frac{1}{10}$s	$\frac{1}{100}$s

Place-value charts can be particularly useful at secondary level when working with numbers expressed in standard form, where powers of ten can be used to label the column headings.

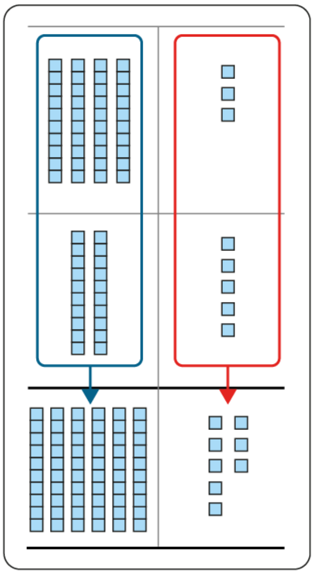
10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}

Labelling the column headings in a place-value chart in this way can support students in exploring the different ways in which numbers can be expressed and, in particular, how any number can be expressed in the form $A \times 10^n$ (where $1 \leq A < 10$ and n is an integer); e.g. $4000 = 4 \times 10^3$; $4200 = 4.2 \times 10^3$; $4207 = 4.207 \times 10^3$.

How they might be used

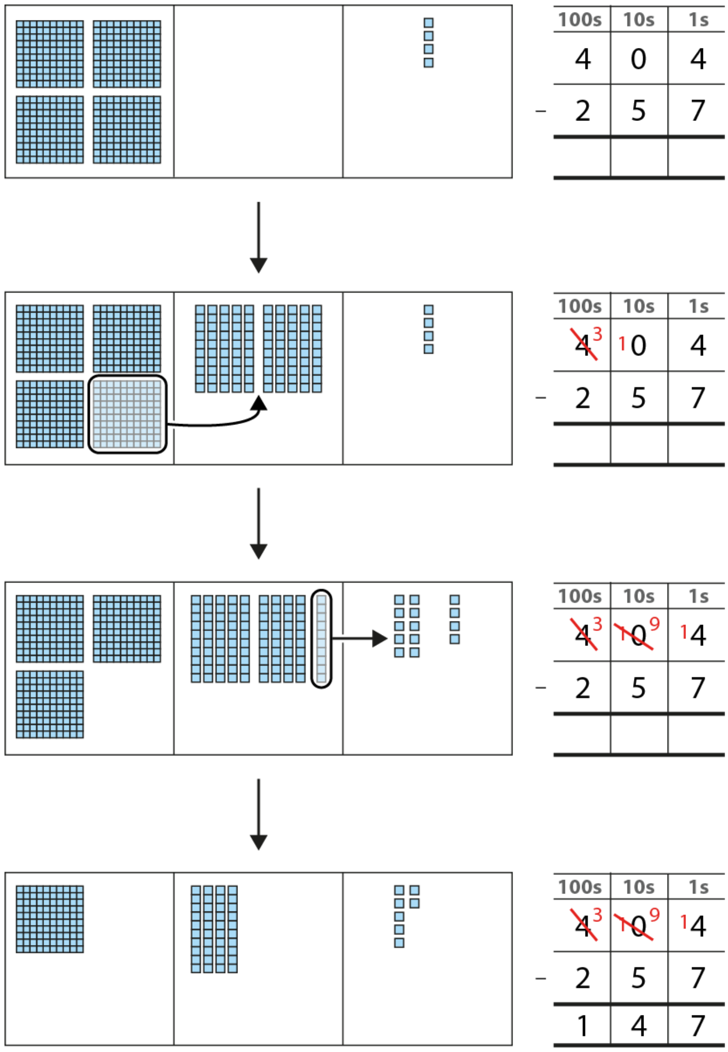
Column addition and subtraction including decimals

Place-value charts can be used to support students in moving from the use of quantity-value representations of place value (like Dienes), for additive calculations, to a column representation of place value. Students are likely to have used place-value charts alongside Dienes at primary school to support column addition and subtraction.



	10s	1s
	4	3
+	2	5
	6	8

Here, the manipulatives act as an aid to laying out the calculation correctly, and students should be encouraged to use the abstract representation as early as possible in Key Stage 3.



As students are encouraged to work more efficiently, and increasingly using abstract symbolism, they can begin to use place-value charts without column headings. With the extension to decimals, the decimal point marks the separation of the integer headings to the left and the decimal or fractional headings to the right.

Students should already be familiar with adding and subtracting decimal numbers from Key Stage 2, and understand that they can apply the same strategies when working with digits to the right of the decimal point, as those to the left. However, the extension to decimals can be challenging for students who do not fully understand the structures that underpin the standard column algorithms. When working with natural numbers, the numbers being operated on can be aligned 'from the right'. With the inclusion of decimals, students need to generalise this to the idea of aligning digits of the same place value and, therefore, the need to align decimal points.

When working with decimals, students may need to revisit the idea of zero as a placeholder and, specifically, the effect of putting a zero at the end of a number. When working with integers, the idea of 'adding a zero' (producing a number ten times bigger) may well be second nature, and there is a danger that this is generalised to decimals and students thinking (mistakenly) that, for example, 2.340 is ten times bigger than 2.34.

Care will need to be taken to raise students' awareness of the key differences between what is happening with integers and decimals, namely:

- that the placement of a zero at the end of an integer introduces the zero as a value in the 'ones' column, resulting in all the other digits moving to the left and, therefore, raising their value by a factor of ten

1 000	**100**	**10**	**1**
	3	7	2
3	7	2	0

$372 \times 10 = 3\,720$

- that the placement of a zero at the end of a decimal introduces the zero into an extra column to the right and, therefore, has the effect of adding zero lots of this column's value (i.e. not changing its value at all).

10	**1** •	$\frac{1}{10}$ **0.1**	$\frac{1}{100}$ **0.01**
3	7 •	2	
3	7 •	2	0

$37.2 = 37.20$

This understanding is important when students are calculating with decimals and need to realise that for some calculations it is helpful to add a trailing zero to one of the numbers without changing its value. For example, when subtracting 7.62 from 23.4, the minuend has no hundredths digit, but the subtrahend has.

$$\begin{array}{rcccc} & 2 & 3 & . & 4 & \\ - & & 7 & . & 6 & 2 \\ \hline & & & . & & \\ \hline \end{array}$$

$$\begin{array}{rccccc} & 2 & 3 & . & \cancel{4}^{3} & {}^{1}0 \\ - & & 7 & . & 6 & 2 \\ \hline & & & . & & \\ \hline \end{array}$$

The addition of a trailing zero in the minuend highlights the need to exchange one tenth for ten hundredths, enabling the subtraction of two hundredths to take place.

It is important that place-value charts are used to reveal the structure of the place-value system, and not just as a tool for 'finding the answer' to calculations. Once students are confident with calculations that include decimals and where multiple exchanges are required, the presentation of reverse problems can be used to encourage deeper thinking about the structures that underpin the algorithm.

	3	1	7	.	□	□	□
−	1	1	9	.	1	5	1
	□	□	□	.	8	1	1

	□	□	□	.	2	5	3
−	1	4	9	.	1	5	1
	2	2	9	.	□	□	□

	4	0	3	.	1	□	□
−	3	1	□	.	□	2	5
	□	□	8	.	9	9	6

When completing the missing digits in these calculations, students are required to analyse the exchanging that has taken place in order to reach a solution.

Comparing decimals

A common misconception when decimals are first introduced in Key Stage 2 is to think, for example, that 1.69 is greater than 1.7, because 69 is greater than 7. Reading the decimal 1.69 as 'one-point-sixty-nine', rather than 'one-point-six-nine' reinforces this misunderstanding and should be avoided. The place-value chart can be a helpful tool in addressing this misconception.

10 000s	1 000s	100s	10s	1s	0.1s	0.01s
				1	• 6	9
				1	• 7	

When using a place-value chart to compare 1.69 and 1.7, we can see that the digits in the ones column are the same in both decimal numbers and the first place-value column that contains different digits is the tenths column. As there are more tenths in 'one-point-seven' than there are in 'one-point-six-nine', we know that 'one-point-six-nine' must be smaller than 'one-point-seven'. It is important that students recognise that once the largest place-value column containing different digits has been identified and a comparison made, the decimals can be ordered based on their value, with no further columns needing to be compared.

Standard index form

Standard index form is first introduced at Key Stage 3, and the place-value chart provides a familiar representation that can be used to support the introduction of this new concept. Standard form is an efficient way of writing very large and very small numbers, by considering multiplication and division by powers of ten. Students need to recognise that numbers can be written in multiple ways and be confident with interpreting and comparing numbers written in this form. For example, students will likely recognise that:

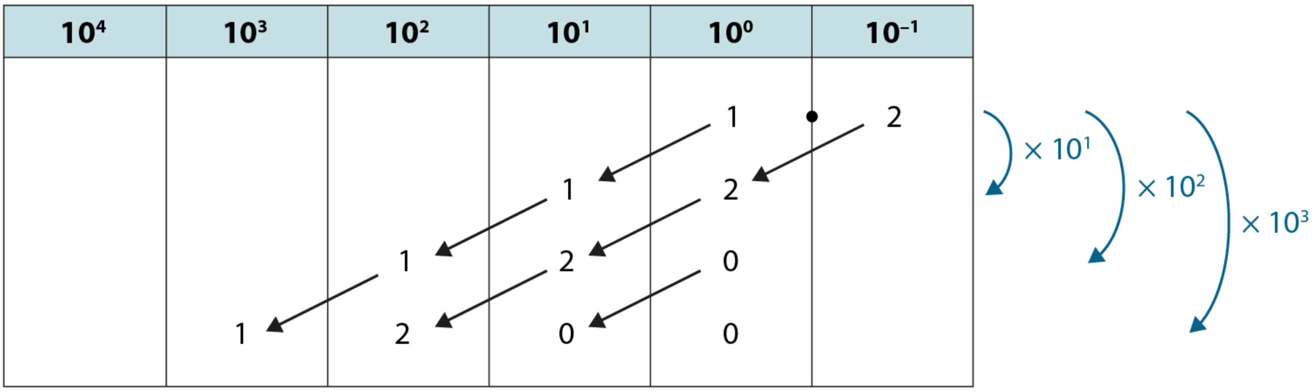
$$2.3456 \times 100 = 23.456 \times 10 = 234.56 = 2345.6 \div 10 = 23\,456 \div 100$$

However, the use of a power notation, such as 2.3456×10^2, to represent this will be new to students.

Students need to be able to interpret the value of numbers written in standard form (of the form $A \times 10^n$, where $1 \le A < 10$ and n is an integer) and a place-value chart can support them in doing this. For example, to interpret 1.2×10^3 it is helpful to label the column headings as powers of ten. The decimal 1.2 can then be placed in the chart.

10^4	10^3	10^2	10^1	10^0	10^{-1}
				1 •	2

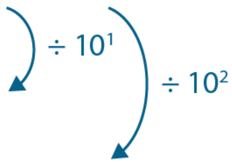
The digits are moved three columns to the left (corresponding to a multiplication of 10^3) until the digit in the ones column is in the 10^3 column. Reminding students that the decimal point is fixed and it is the digits that are moving is important, and the place-value chart should help to reinforce this.



Students need to recognise the need to fill all columns between the last non-zero digit and the decimal point with zeros. Once these placeholders have been inserted, the conversion from standard form is complete and the value of the number is identified as 1 200.

When n is negative, e.g. 4.32×10^{-2}, the digits will move to the right, because multiplying by 10^{-2} is equivalent to multiplying by $\frac{1}{10^2}$ or dividing by 10^2.

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
		4 .	3	2		
		0 .	4	3	2	
		0 .	0	4	3	2



It is important to emphasise that a trailing zero in the 10^{-5} column is not necessary and the conversion of the number expressed in standard form as 4.32×10^{-2} to a decimal is complete, and can be written as 0.0432.

A place-value chart with column headings labelled using powers of ten may also be useful when expressing numbers in standard form, helping students to think explicitly about the value of n. For example, if we want to express the numbers 6 573 and 0.0405 in standard form, we can first represent them in a place-value chart.

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
6	5	7	3 .				
			0 .	0	4	0	5

Determining the value of n needed to be able to express 6 573 in standard form is relatively straightforward by identifying the column in which the largest digit lies (e.g. the thousands column). Using the digits of the number to establish the value of A, a decimal greater than one and less than ten and then multiplying by the identified power of ten (n), 6 573 can be expressed in standard form as 6.753×10^3. For numbers less than one, it is the position of the first non-zero digit after the decimal point that is of interest when determining n, so for the decimal 0.0405 above, the first non-zero digit after the decimal point is 4 and this lies in the hundredths column. To determine A, we are interested in this digit and all digits to the right of this digit, so 4, 0 and 5. Using these digits to write a number between one and ten gives 4.05, and so 0.0405 can be written as 4.05×10^{-2}.

Further resources

When working with place-value charts, the ability to see digits moving to the left and right as a result of a multiplication or division by ten, can be helpful in reinforcing the fact that the decimal point is fixed and addressing the common misconception that, when multiplying 76.27 by ten to get 762.7, for example, the decimal point has moved one place to the right.

See, for example:

MathsBot.com

https://mathsbot.com/tools/placeValue

An interactive place-value chart, which can be customised with the desired number of rows (a maximum of ten) and columns (up to a maximum of ten either side of the decimal point). The column headings can be expressed in standard form by selecting the tick box. If left unselected, the headings in the columns to the right of the decimal point are expressed as fractions. Once the chart has been set up as required, the up and down arrows within each cell can be used to enter non-zero digits. This is not just restricted to natural numbers, and allows negative numbers as well as numbers greater than ten to be entered, (making it less appropriate for individual student use, and more appropriate as a demonstration tool). When a non-zero digit is entered within a cell, all cells to the right of that cell are automatically filled with zeros. Selecting the 'Leading Zeros' option places a zero in every cell within the chart that hasn't already got a digit entered.

Each row has an arrow at the start (pointing left) and an arrow at the end (pointing right). Clicking on the arrow at the start of the row multiplies the number in that row by ten, showing that the digits in the chart all move one place to the left. Similarly, the arrow at the end of each row results in a division by ten. The entries in any row can be copied to the row above and the row below, using the up and down arrows at the far right-hand side of the screen.

MathsFrame.co.uk

https://mathsframe.co.uk/en/resources/resource/60/itp-moving-digits

This place-value chart ranging from 10 000 to 0.001 (or '10 thousands' to '1/1000' depending on the choice of display using the HTU button) can be completed by clicking on and dragging down the number cards at the top of the screen. When a digit is placed in the top row, to the left of the decimal point, any vacant columns between the entry and the decimal point are auto-filled with zeros. These numbers can, however, be replaced by dragging an alternative digit from the number cards and placing it on top. The chart can be customised further to remove the column headings altogether, add and remove column divides, and add or remove the decimal point. Once the desired number has been entered into the top row of the chart, it can be multiplied and divided by 10 or 100 via the buttons at the bottom of the screen. Any number cards dragged to the second row of the chart stay in their original positions.

The Mathenæum: a counting dot machine

http://thewessens.net/ClassroomApps/Models/Dots/explodingdotscounting.html?topic=models&id=19

An animated representation of successive positive integers in a place-value chart using dots. Nine dots in a column merge together and become a dot in the column to the left as the counting continues. The animation can be played either 'fast' or 'slow' and can be paused, resumed and restarted via the buttons at the bottom of the screen. The 1–10 rule needs to be selected at the top of the screen for the place-value chart columns to be powers of ten.