Maths HUBS Resources

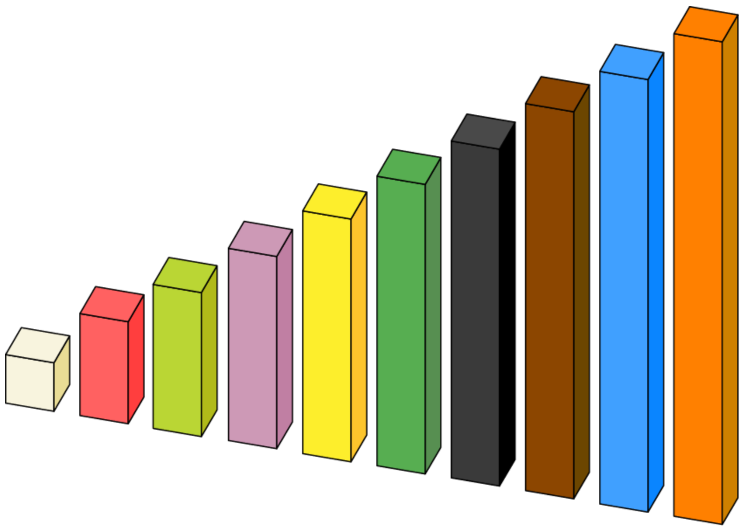
National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Mathematical representations

Cuisenaire® rods

Guidance document | Key Stage 3

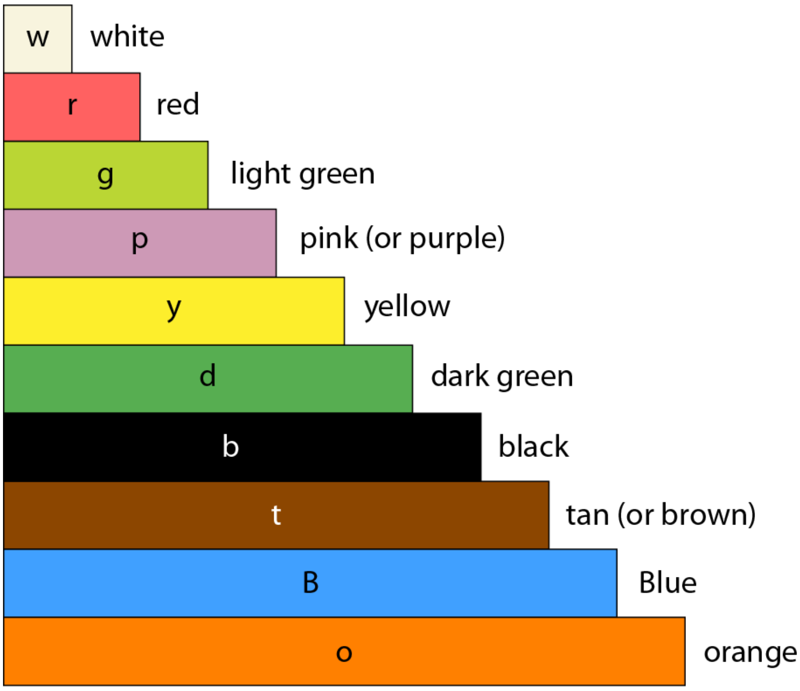


Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

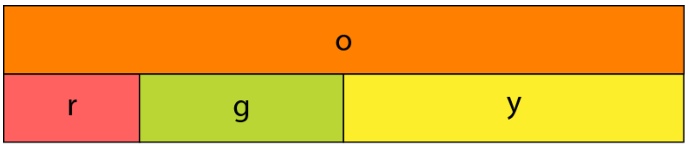
What they are

Cuisenaire® rods are proportional number sticks, ranging from 1 cm to 10 cm in length, in increments of 1 cm. Each length is represented by a different colour. No rod has any value in itself; rather, because there are no numeral markings, you can choose any sized rod as the 'unit'. For example, if white represents one, then orange represents ten; if red is one, then orange is five.

To aid teachers and students with colour-vision deficiency, we have used the standard rod names and labelled each rod with its colour abbreviation, as shown below.



You can create a 'train' by placing two or more rods end to end:



'Red plus light green plus yellow is equal to orange.'

$r + g + y = o$

Note that there are a few things to be aware of when using Cuisenaire® rods:

- Use of the rods is dependent on colour recognition. Consider any students who may be colour-vision deficient and adjust accordingly (such as using the standard rod names and labelling them with symbols denoting the colours, as here). Take care not to inadvertently ascribe errors made by students to a lack of mathematical understanding in cases where they are actually due to colour-vision deficiency.
- Initially, avoid describing (or encouraging students to see) the rods as having a specific number value; for example, instead of saying *'Two ones and a three is equivalent to a five'*, ensure you say *'Two whites and a light green is equal to a yellow'* (or $2w + g = y$). When later assigning values to rods for specific examples, it can be helpful to use numbers that are not equal to the lengths of the rods, to avoid students always associating, for example, white as one, red as two, and so on.

Why they are important

Whilst the use of Cuisenaire® rods fits comfortably within the primary classroom, students can also use the rods to think about higher level mathematics, including topics that are explored at A level. One such example is the binomial theorem, where students can investigate permutations of the rods to aid their understanding of the mathematical structure of the binomial coefficients / Pascal's triangle:

Number of rods	**1**	**2**	**3**	**4**
Number of permutations	w 1			
	r 1	w w 2		
	g 1	w r r w 2	w w w 1	
	p 1	w g g w r r 3	w w r w r w r w w 3	w w w w 1

The versatility of Cuisenaire® rods in supporting advanced level – as well as primary – mathematics makes them a powerful representation to support students' understanding at Key Stage 3 also.

It is important to recognise that Cuisenaire® rods are about length, and it is the relationships between the rods, rather than any particular values they have been assigned, where the most power lies (for example, red is always two-thirds of light green). By allowing the value of any particular rod to vary, multiplicative relationships can be explored.

How they might be used

Comparing lengths

You can use Cuisenaire® rods to support students in conceptualising number as length, by paying attention to and comparing the lengths of the rods. This is not a trivial task, and activities that students may have done at primary school can be revisited to gain a familiarity with which rods are longer or shorter than others. Students are likely to be familiar with making a staircase with the rods (as shown above) and it is important that they have a clear image of how the rods are sequenced and their names.

To help students get a physical sense of the rods, as well as a visual familiarity, ask them to hold, for example, the white, red, light green and pink rods behind their back, then challenge them to pull out, without looking, the red rod or the light green rod, and so on. Follow-up tasks and questions could include:

- *'Find a rod that is longer than the yellow.'*
- *'Find a rod that is shorter than the light green.'*

- *'If we call the white rod "one", find a rod that is one longer than the yellow. Now find a rod that is one shorter than the pink.'*
- *'Which rod is the same length as two reds?'*
- *'Which two rods make the same length as the orange? Can you find all the possibilities?'*

Modelling additive and multiplicative relationships

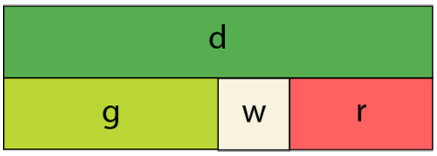
Once students are confident about interpreting rods in terms of their length, they can place rods end to end to model additive and multiplicative relationships, corresponding in a similar way to addition on a number line. Recall:

- A relationship is *additive* if the quantities are related through combining, partitioning or direct comparison, and involves the operation of addition or subtraction.
- A relationship is *multiplicative* if the quantities are related in a proportional sense and involves the operation of multiplication or division.

Multiplication can be thought of as repeated addition as well as scaling. In the third representation below, the train of red rods can be seen as either $r + r + r + r$ or $4 \times r$.

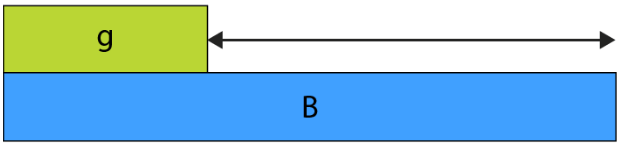
Students can use the rods to model subtraction by searching for a missing addend, as in the second representation below.

1. Additive – combining or partitioning



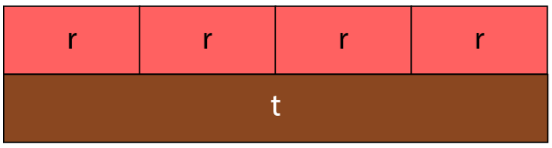
$$g + w + r = d$$
$$d = g + w + r$$

2. Additive – comparative



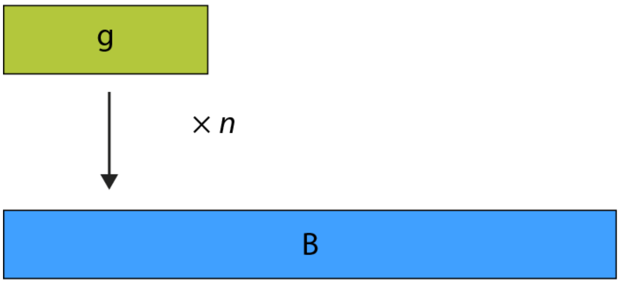
$$g + ? = B$$
$$B - g = ?$$

3. Multiplicative – repeated addition

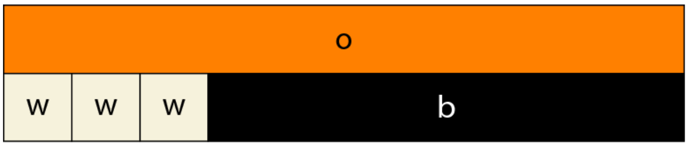


$$r + r + r + r = t$$
so
$$4 \times r = t \text{ or } 4r = t$$

4. Multiplicative – scaling



5. Additive and multiplicative – combined

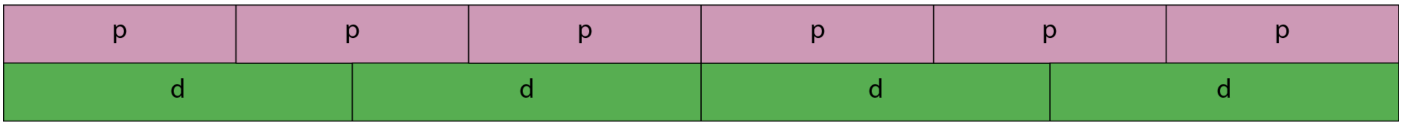


$$w + w + w + b = o$$

so

$$3w + b = o$$

When students are confident using Cuisenaire® rods, you may wish to assign values to the rods to explore commutativity. You need to assign the rods values equal to their lengths. For example, pink is four and dark green is six. Building trains shows that six pinks are equal to four dark greens:



$$p = 4$$

$$d = 6$$

$$6p = 4d$$

so

$$6 \times 4 = 4 \times 6$$

Use of the rods in this way helps to show that multiplication is commutative.

Modelling decimals and fractions

Because of their different yet related lengths, Cuisenaire® rods allow you to assign a value of your choice to any rod, and then identify the values of the other rods by using the relationships between them. In this way, the rods make effective models for decimals and fractions.

For example, if you designate the orange rod as having a value of one, then the white, red and light green rods have values of 0.1, 0.2 and 0.3 respectively.

Students can then model decimal calculations using the rods with these assigned values, for example:

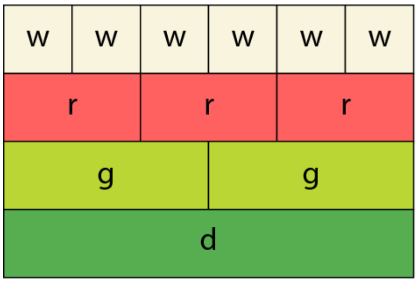
$$4g = o + r$$

so

$$4 \times 0.3 = 1 + 0.2 = 1.2$$

Alternatively, asking students to find the value of all the other rods in the box, if the red rod has a value of $\frac{1}{3}$, forces them to think multiplicatively and to become familiar with the language, notation and meaning of fractions.

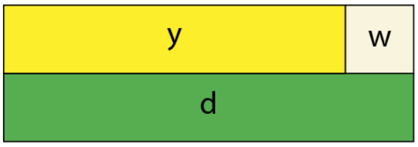
They can then use the rods to show equivalent fractions:



$$\text{If } r = \frac{1}{3}\text{, then } w = \frac{1}{6}, g = \frac{1}{2} \text{ and } d = 1$$

$$\text{and } 2r = 4w\text{, so } \frac{2}{3} = \frac{4}{6}$$

Students can also model fractions calculations using the rods with these assigned values, for example:



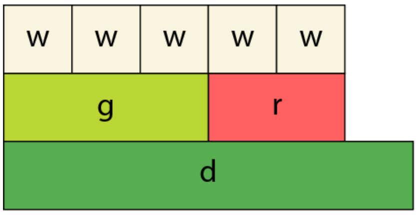
$$y = d - w$$

$$\text{so } \frac{5}{6} = 1 - \frac{1}{6}$$

The rods are particularly useful for highlighting common misconceptions. For example:

$$\frac{1}{2} + \frac{1}{3} = \frac{2}{5} \; ✗$$

If we continue to use the values assigned above, $\frac{1}{2} + \frac{1}{3}$ can be represented as $g + r$. The rods clearly show that the sum is just less than d (1); in fact, it is equal to 5w ($5 \times \frac{1}{6} = \frac{5}{6}$):

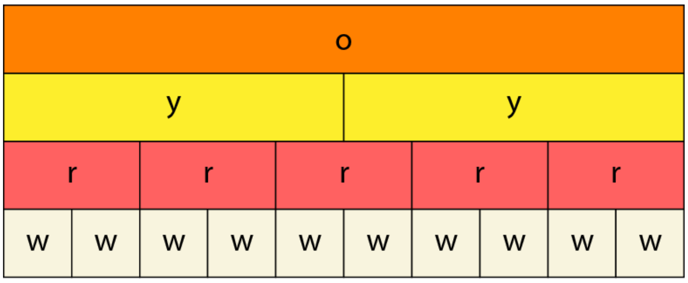


$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6} \; ✓$$

Exploring factors and primes

Cuisenaire® rods provide a concrete way to explore factors and prime numbers, by placing trains of the same colour rod underneath the number being explored. To do this, you must assign the rods values equal to their lengths, so white is one, red is two, and so on.

Looking at the orange rod, students can then explore the factors of ten:



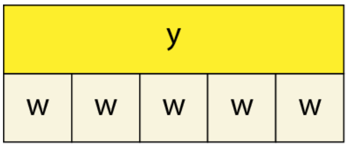
With $o = 10$, $y = 5$, $r = 2$ and $w = 1$, the rods show:

$1 \times 10 = 10$
$2 \times 5 = 10$
$5 \times 2 = 10$
$10 \times 1 = 10$

There are four equal trains and four factors of ten: 1, 2, 5 and 10.

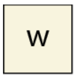
Ten is a composite number (not prime), because it has factors other than one and itself.

For the yellow rod with an assigned value of five, students will only be able to build an equal single-colour train using white rods (each with the value of one):



There are two equal trains, so five has exactly two factors (1 and 5). Therefore, five is prime.

Now consider the number one, represented here by white:



Only one train is possible, so one has only one factor. All positive integers are composite or prime, except for one.

Algebra

Algebra is concerned with relationships and use of Cuisenaire® rods, focusing on their colour names without assigning numerical values, can be valuable in supporting students' conceptual understanding of algebra and algebraic notation. Encouraging students to focus on relationships from the start and only later assigning values to rods supports their development of the idea of an unknown.

To support later work in algebra, it is important that students do not assume that the value of a rod is always the same as its length.

Further resources

You can find additional guidance on using Cuisenaire® rods to model mathematical structure in the NCETM primary mastery professional development materials:

Year 5: 1.28 Common structures and the part–part–whole relationship
https://www.ncetm.org.uk/resources/52610

While physical sets of Cuisenaire® rods are widely used in classrooms, there are several free online resources that enable the rods to be generated and arranged on screen.

See, for example:

Math Playground
https://www.mathplayground.com/mathbars.html
This weblink gives access to a 2-dimensional rod generator. Once created, the rods can be dragged and dropped into the work area. A square grid background can be applied to the work area or turned off, as required by the user. Clicking on a rod switches it from horizontal to vertical orientation, and vice versa. There is also the option to label the rods or not.

Math Toybox
http://mathtoybox.com/numblox/NumBlox.html#.XAld8C10d0s
This weblink gives access to 2-dimensional coloured rods, representing numbers 1 to 10, that can be dragged and dropped into a work area. Users have the option of applying one of two different square grid backgrounds to the work area or no square grid. The length of the rods is shown when the rods are clicked on. Clicking on a rod also changes its orientation from horizontal to vertical, and vice versa.

Modelling fractions with Cuisenaire® rods
http://pbs.panda-prod.cdn.s3.amazonaws.com/media/assets/wgbh/rttt12/rttt12_int_cuisenaire/index.html
Users can experiment with 3-dimensional rods, which display as 2-dimensional when dragged onto the square grid background. The resource includes eight fractions problems (each including a hint) but doesn't seem to have a way of submitting a solution.

One of the benefits of using an online resource, similar to the ones described above, is the ability to generate as many rods as are needed, rather than being restricted by the contents of a set of rods. The extent to which students can work together with the rods in the same way as with a physical set of rods may, however, be questioned. Furthermore, the ability of a teacher to watch students working with the rods online to ascertain a student's thinking, is something which may need to be considered when exploring online rod generators.

Examples of videos

NCETM secondary mastery professional development materials
https://www.ncetm.org.uk/resources/53609
Included in the secondary mastery professional development materials is a series of four videos on Cuisenaire® rods, covering and expanding on the contents of this guidance document.

Prime and composite numbers using Cuisenaire® rods
https://vimeo.com/educationunboxed/prime-composite
This video is an example showing the process of using Cuisenaire® rods to identify the numbers 1 to 10 as being either prime or composite.

Working with Cuisenaire® rods

http://www.cuisenaire.co.uk/index.php/home/videos/cuisenaire-rods-in-the-classroom/video/0

A number of videos by The Cuisenaire® Company, showing students working with Cuisenaire® rods.

NCETM interview with Caroline Ainsworth

https://youtu.be/euG4xBBFhBI

Pete Griffin interviews Caroline Ainsworth about the three important stages in the use of Cuisenaire® rods.

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