Teaching for Mastery: **Variation**

Teaching for Mastery

- Access
- Pattern
- Making Connections

Representation & Structure

Coherence

Mathematical Thinking

- Chains of Reasoning
- Making Connections

Small steps are easier to take

Variation

Fluency

- Procedural
- Conceptual
- Making Connections

- Number Facts
- Table Facts
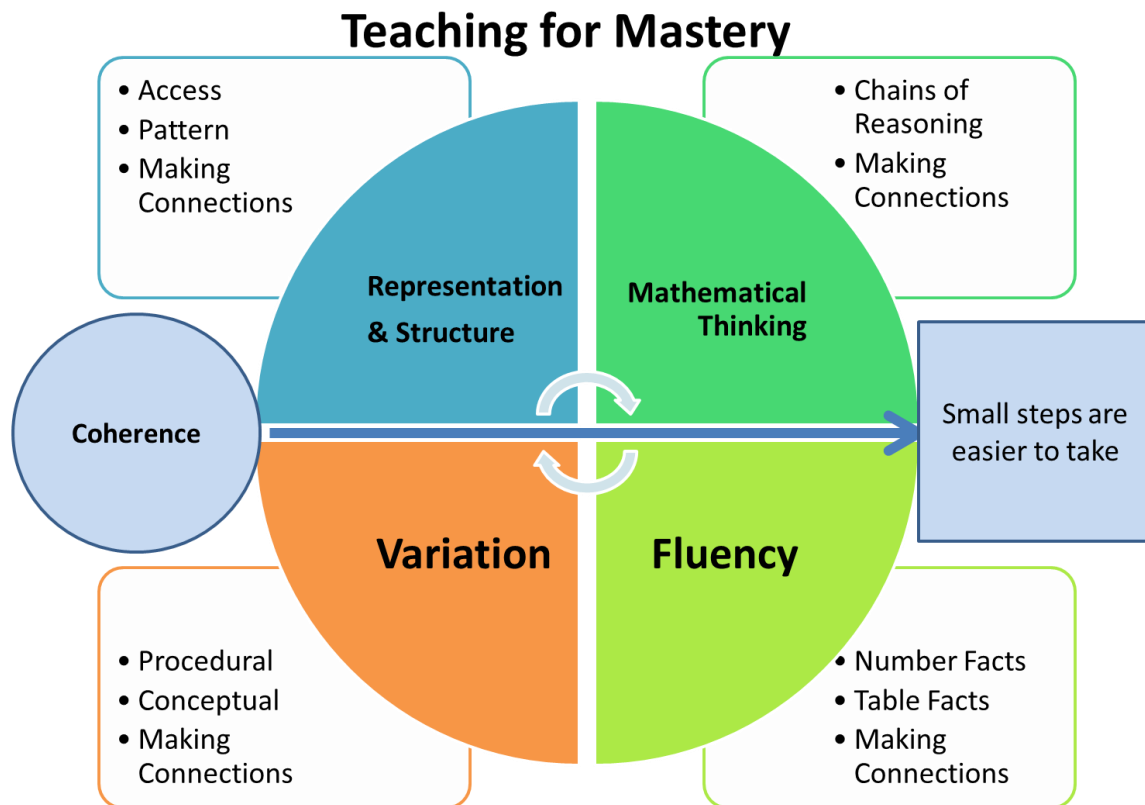- Making Connections

Key Messages

1. The central idea of teaching with variation is to **highlight the essential features of a concept or idea** through varying the non-essential features.
2. **Variation is not the same as variety** – careful attention needs to be paid to what aspects are being varied (and what is not being varied) and for what purpose.
3. When giving examples of a mathematical concept, it is useful to add variation to emphasise
   1. **What it is** (both standard and non-standard examples)
   2. **What it is not.**
4. When constructing a set of activities or questions it is important to consider what connects the examples; what mathematical structures are being highlighted? Students are encouraged to avoid mechanical practice and, instead, **to practise the thinking process (intelligent practice).**

For each exercise, find the gradient between each of the pairs of points.

Exercise A

1. (4, 3) and (8, 12)

2. (7, 4) and (-4, 8)

3. (6, -4) and (6, 7)

4. (-5, 2) and (-3, -9)

5. (-2, -1) and (-10, 1)

6. (8, -7) and (11, -1)

7. (-5, 2) and (10, 6)

8. (-6, -9) and (-6, -8)

Exercise B

1. (4, 3) and (8, 12)

2. (-2, -3) and (4, 6)

3. (5, 6) and (10, 2)

4. (-3, 4) and (8, -6)

5. (-5, 3) and (2, 3)

6. (2, 1) and (2, 9)

7. (p, q) and (r, s)

8. $(0, a)$ and $(a, 0)$

9. (0, 0) and (a, b)

Exercise C

1. (4, 3) and (8, 12)

2. (4, 3) and (7, 12)

3. (4, 3) and (6, 12)

4. (4, 3) and (5, 12)

5. (4, 3) and (4, 12)

6. (4, 3) and (3, 12)

7. (4, 3) and (2, 12)

8. (4, 3) and (1, 12)

Order the exercises in terms of which would be most effective in your teaching.

Strategy 1: procedural variation practice sets

- ✓ Strategically varies one thing at a time to draw attention to underlying structure of the concept
- ✓ Prompts students to attend to the variation by asking 'what is the same and what is different?'
- ✓ May appear easy at first glance
- ✓ Not 'variety' (Exercise A is variety)

Example

$$(y + 1)(y + 24) \qquad (y + 1)(y + 9)$$

$$(y + 2)(y + 12) \qquad (y + 2)(y + 8)$$

$$(y + 3)(y + 8) \qquad (y + 3)(y + 7)$$

$$(y + 4)(y + 6) \qquad (y + 4)(y + 6)$$

$$(y + 5)(y + 5)$$

Strategy 2: concept/non-concept

- ✓ A set of examples of what the concept is and what the concept isn't
- ✓ Helps students gain deeper understanding of concept

Example