MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Fractions

3.10 Linking fractions, decimals and percentages

Teacher guide | Year 6

Teaching point 1:

Some fractions are easily converted to decimals.

Teaching point 2:

These fraction–decimal equivalents can be found throughout the number system.

Teaching point 3:

Fraction–decimal equivalence can sometimes be used to simplify calculations.

Teaching point 4:

'Percent' means number of parts per hundred. A percentage can be an operator on a quantity, indicating the proportion of a quantity being considered.

Teaching point 5:

Percentages have fraction and decimal equivalents.

Teaching point 6:

If the value of a whole is known, a percentage of that number or amount can be calculated.

Overview of learning

This is the final segment in this spine. It combines work on fractions with previous work on decimals. During this segment, the children also meet percentages for the first time. They will link percentages to fractions and decimals.

The segment has an initial focus on common fraction–decimal equivalences. In *Spine 1: Number, addition and subtraction*, segments *2.23* and *2.24*, children learnt about the equivalence of $\frac{1}{10}$ and 0.1, and of $\frac{1}{100}$ and 0.01. In this segment, they will also explore decimal equivalents for $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{5}$ (and multiples of these unit fractions). Being able to divide one – and indeed any other power of ten – into two, four, five and ten equal parts, is an essential skill for reading scales in graphing and measures. As such, emphasis is placed on ensuring children are able to instantly recall these key fraction–decimal equivalents. Developing a conceptual understanding of these is essential and is addressed in this segment. However, having quick recall of these equivalences, and an ability to identify them in calculations as numbers that can easily be converted into different forms, will also be important moving forward. For example, 0.5×86 can be very quickly solved if children recognise that 0.5 is equivalent to $\frac{1}{2}$, and therefore think of the calculation as 'half of 86'. Without identifying this equivalence, children may resort to using a written multiplication algorithm to solve the calculation, which is much less efficient. Once children have learnt these equivalences, this segment moves on to calculations and comparisons that require children to be able to move confidently between numbers as fractions and numbers as decimals.

Percentages are introduced half way through the segment. Like fractions, percentages can be operators that tell us about the proportion of a number or quantity being considered. Before starting to calculate percentages of quantities, some time is spent developing a sense of percentages, so that children understand that, for example, 90% is a fairly large part of the whole and 5% is a fairly small part of the whole. Once they have an awareness of this, they can progress to linking percentages to fractions and decimals. Finally, they will learn how to use the 'benchmark' percentages of 50%, 10% and 1% to help them calculate other percentages of quantities. Some example contexts for applying percentages are suggested: statistics and measurement commonly offer suitable opportunities for practising percentages work. At secondary level, a common misconception is that percentages can't be greater than 100%. Although this is beyond the primary programme of study, percentages over 100% are briefly touched upon in this segment in order to start raising children's awareness of them. Look for further opportunities to start seeding the idea that percentages can be greater than 100%.

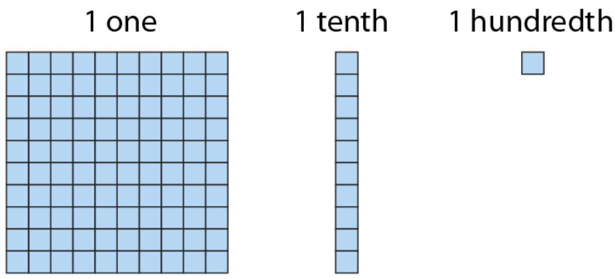
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Some fractions are easily converted to decimals.

Steps in learning

	Guidance	Representations
1:1	In their learning up to this point, children have encountered both fractions and decimals. They should know that tenths and hundredths can easily be written as decimals. Recap this with them now. Provide each pair of children with a Dienes hundred square, some tens rods and ones cubes. Let the hundred square represent one whole. Ask the children to discuss in pairs what a rod represents, justifying their answer. They should conclude that each rod represents one-tenth. Highlight the two different ways of writing one-tenth: 0.1 or $\frac{1}{10}$. Make sure that children are very clear on the connection between the '1' digit in 0.1 sitting in the tenths place, hence one-tenth, and the fractional form of writing this. It is $\frac{1}{10}$ because the whole is divided into ten equal parts, and we have one of those parts. Repeat this discussion with a small ones cube, which represents 0.01 or $\frac{1}{100}$. Here, the '1' digit is in the hundredths place, so we have one-hundredth. Children will often read the written form of $\frac{1}{10}$ and $\frac{1}{100}$ as *'one-tenth'* and *'one-hundredth'*, but they may read the numbers '0.1' and '0.01' as *'zero-point-one'* and *'zero-point-zero-one'*. Using the examples in the table opposite, encourage them to read the numbers '0.1' and '0.01' as *'one-tenth'* and *'one-hundredth'*. Model using this language	see below



Fraction notation	Decimal notation	Name
$\frac{1}{10}$	0.1	one-tenth
$\frac{1}{100}$	0.01	one-hundredth

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Missing-symbol problems:

'Fill in the missing symbols (<, > or =).'

$\frac{1}{10}$ ◯ $\frac{1}{100}$ $\qquad$ $\frac{1}{10}$ ◯ 0.01

0.1 ◯ $\frac{1}{10}$ $\qquad$ $\frac{1}{100}$ ◯ 0.1

$\frac{1}{100}$ ◯ 0.01 $\qquad$ 0.01 ◯ 0.1

in your teaching, to help them connect a value to these notations. We read '100' as '*one hundred*' rather than as '*one-zero-zero*' and applying the same convention to decimal fractions will help children associate the written notation with fractions.

Return to the Gattegno chart used in the previous spines (see the example on the previous page), and practise reading numbers from the first column. Read each number using its name (for example '*one-hundredth*' for '0.01' and '*one thousand*' for '1,000'). This should help to emphasise that they are all just one of a unit. It should also start to reinforce that 0.1 is ten times bigger than 0.01. The table and Gattegno chart can then be used alongside problems, such as those on the previous page, to compare the size of different values and complete comparison statements.

1:2

Once you have established that one can easily be split into ten parts and one hundred parts, look at how one can be split up into other equal parts. Again, give each pair of children a Dienes hundred square, and challenge them to try and spit it into equal parts by substituting it for rods and cubes. Take each fraction in turn, starting with one-half.

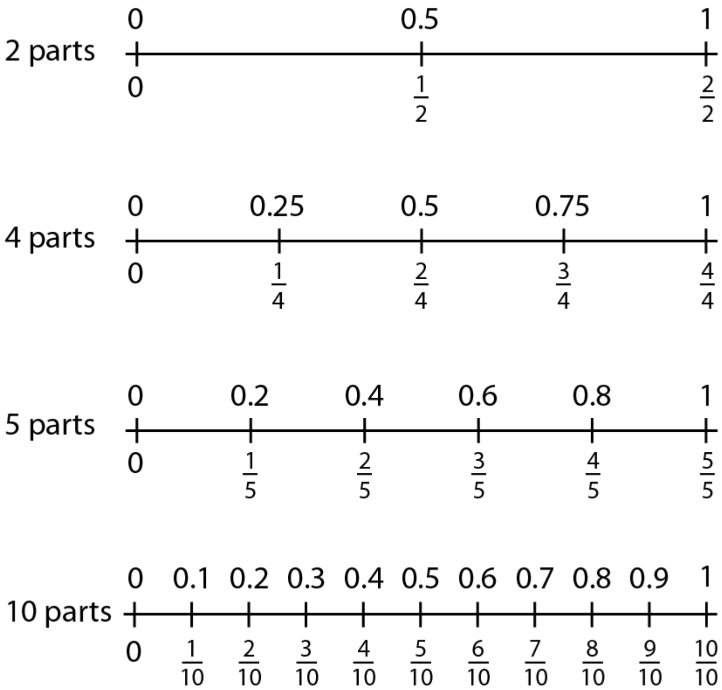
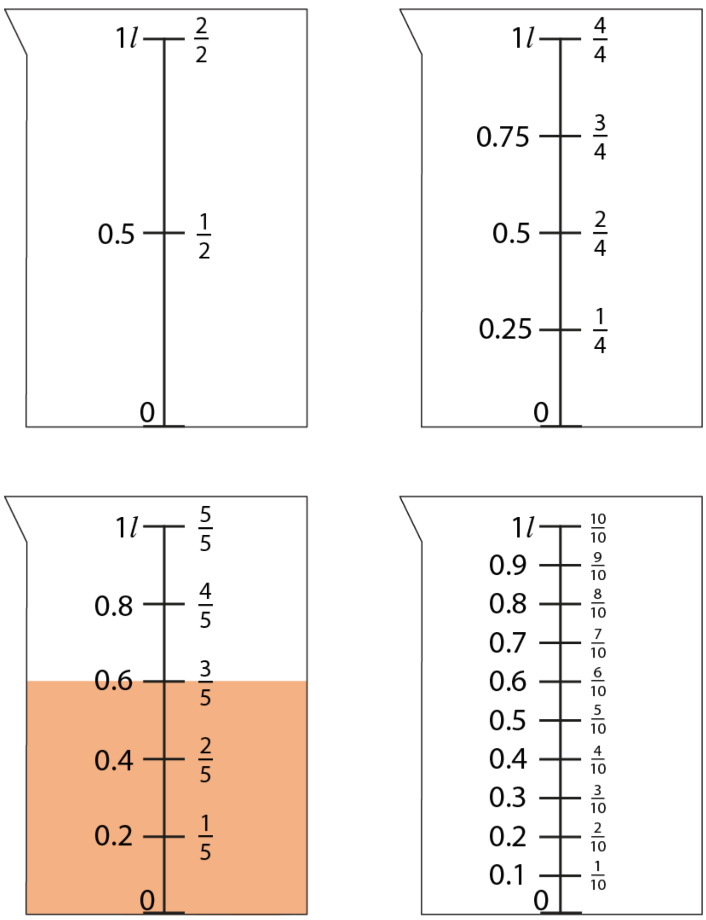
Ask the children whether one can be split into two equal parts. Give them a chance to explore with the Dienes, substituting a hundred square (representing one) for tens rods (representing tenths). They should recognise that if one is split into two equal parts, there will be five rods, or five-tenths, in each part. Record this in a table, such as the one opposite.

Then look at whether the hundred square can be divided into three equal parts. Again, give children a chance to

Fraction	Decimal
$\frac{1}{2}$	0.5
$\frac{1}{3}$	0.33333…
$\frac{1}{4}$	0.25
$\frac{1}{5}$	0.2
$\frac{1}{6}$	0.1666…
$\frac{1}{7}$	0.14285714…
$\frac{1}{8}$	0.125
$\frac{1}{9}$	0.111…
$\frac{1}{10}$	0.1

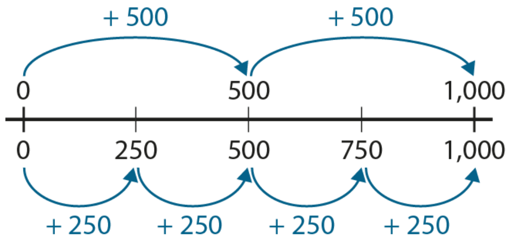
	explore substituting the hundred square (one) with rods (tenths) and cubes (representing hundredths). We can make three parts of three tenths and three hundredths, but we have one more hundredth that would need to be further split up. Explain to children that in this case, the decimal equivalent is 0.3333333… (continuing, or *recurring*, forever). Repeat this exercise with the other fractions in the table, allowing children to explore using manipulatives each time. Discuss and identify where one can be split equally, and tell the children the decimal equivalent when it can't be split equally.	
1:3	Once the table is complete, look more closely at the information shown in it. Explain that there are some of these equivalences that children are unlikely to ever use (knowing the decimal equivalents for $\frac{1}{7}$ is one such example). However, others are extremely useful and are likely to crop up again and again. Highlight $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{10}$ (two parts, four parts, five parts and ten parts, respectively). These are most common divisions used in graphs and measures. As such, they will be the focus of our learning. Because these fraction–decimal equivalents are so important and frequently used, it is necessary to learn them off by heart. $\frac{1}{8}$ is not as commonly used as the other fractions, but it is a fraction that can also be useful to know the decimal equivalent of.	see table below

Fraction	Decimal
$\frac{1}{2}$	0.5
$\frac{1}{3}$	0.33333…
$\frac{1}{4}$	0.25
$\frac{1}{5}$	0.2
$\frac{1}{6}$	0.1666…
$\frac{1}{7}$	0.14285714…
$\frac{1}{8}$	0.125
$\frac{1}{9}$	0.111…
$\frac{1}{10}$	0.1

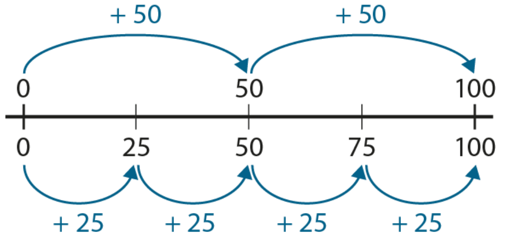
1:4	Now extend these to non-unit fractions. Look at 0–1 number lines marked and labelled with $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{10}$ unit fractions and related decimals. Discuss any patterns the children can identify and encourage them to ask any questions they might have. Children may notice that, for example, 0.5 and 0.6 occur on two different number lines. Draw attention to the '4 parts' number line, and ask *'What do you notice about these decimals?'* They should notice that 0.25 and 0.75 have two digits after the decimal point but 0.5 only has one. We could of course write 0.5 as 0.50, but as the children will know from previous learning, 0.5 and 0.50 have equal value. Take the time to count along each number line in both fractional and decimal steps.	
1:5	Present activities within a familiar measurement context to give children the chance to practise the two different ways of recording values between zero and one. Using whiteboard pens, shade one of the representations of a measuring cylinder (provided opposite), to show that it has been filled to a level of your choosing; the example opposite shows a cylinder shaded $\frac{3}{5}$ or 0.6 litres full. Ask the children to either tell you the shaded amounts or write them down. They should use both fraction and decimal notation. Select one of the four containers and cover individual labels, then ask the children to work out the hidden value and write their answers in the format of, for example, $0.25 = \frac{1}{4}$. Say a value, for example, *'zero-point-*	

	seven-five' or *'six-tenths'* and challenge the children to write it in its alternative fractional or decimal form. For example, if you say *'six-tenths'*, they should write 0.6.	
1:6	In turn, take each of the values in the examples on the following page and look at their application throughout the number system. One whole is represented in the four examples by 1,000, 100, 10 and 1. Starting with two parts (halves) and four parts (quarters), look at the bar model and associated number line for each example and discuss what children notice. Ask: • *'What patterns can you see?'* • *'Why are these repeated through the number system?'* The '50, 100' pattern is associated with two parts, and the '25, 50, 75, 100' pattern is associated with four parts. This remains the same for each place value unit within the number system because each unit is made up of ten of a smaller unit. Spend some time examining and discussing these images with the class, highlighting the links.	

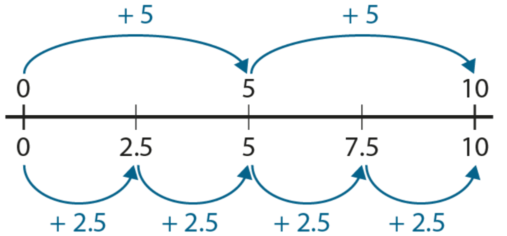
	1,000			
Halves	500		500	
Quarters	250	250	250	250



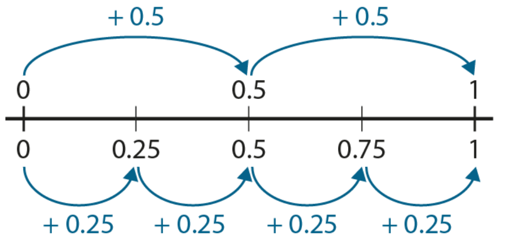
	100			
Halves	50		50	
Quarters	25	25	25	25



	10			
Halves	5		5	
Quarters	2.5	2.5	2.5	2.5



	1			
Halves	0.5		0.5	
Quarters	0.25	0.25	0.25	0.25

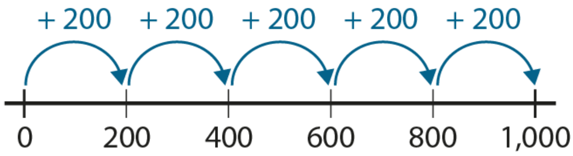


1:7 Now look at a similar model, but this time with separate examples for five parts (fifths) – noting the repetition of the '2' – and then with ten parts (tenths). See the next page.

The Gattegno chart is another powerful image to reinforce patterns through the number system. Look at the chart alongside the tenths bar models and number lines, making the link between the chart and representations. The Gattegno chart will help emphasise that 1 tenth is ten times smaller than 1. This is reiterated on the bar model, which clearly models how 10 tenths are equivalent to one.

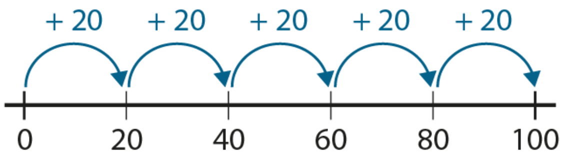
Fifths

1,000				
200	200	200	200	200



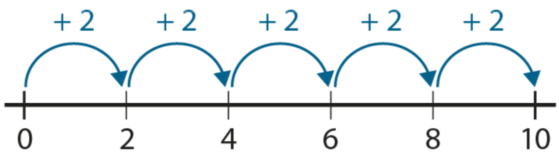
Fifths

100				
20	20	20	20	20



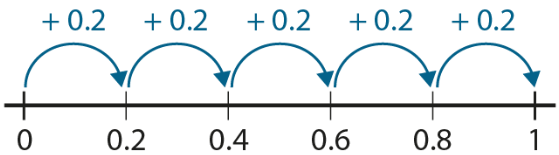
Fifths

10				
2	2	2	2	2



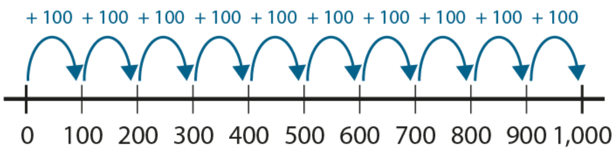
Fifths

1				
0.2	0.2	0.2	0.2	0.2



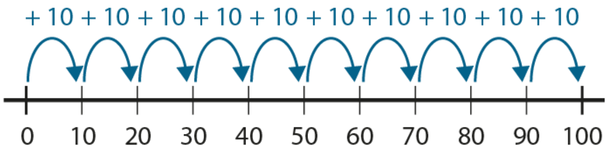
Tenths

1,000									
100	100	100	100	100	100	100	100	100	100



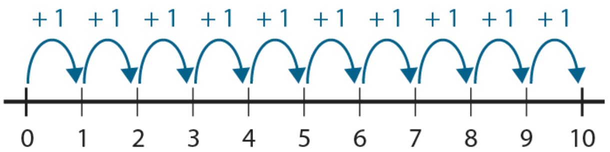
Tenths

100									
10	10	10	10	10	10	10	10	10	10



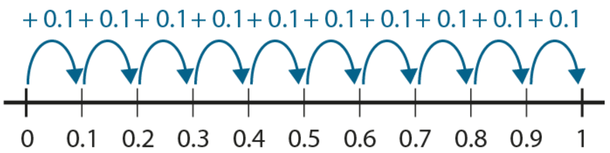
Tenths

10									
1	1	1	1	1	1	1	1	1	1



Tenths

1									
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1



1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

1:8

Complete the table opposite to generalise the patterns within the number system that you identified in the previous step. The aim is to reach the stage where children can complete these confidently using the knowledge they have accumulated, without needing to do too much working out. For now, they may need to refer back to the diagrams. Discussion points you might cover include:

- $\frac{1}{2} \times 1{,}000$, for example, can mean one-half of 1,000 (as shown in the bar

2 parts	$\frac{1}{2} \times 1{,}000$	$\frac{1}{2} \times 100$	$\frac{1}{2} \times 10$	$\frac{1}{2} \times 1$
4 parts	$\frac{1}{4} \times 1{,}000$	$\frac{1}{4} \times 100$	$\frac{1}{4} \times 10$	$\frac{1}{4} \times 1$
5 parts	$\frac{1}{5} \times 1{,}000$	$\frac{1}{5} \times 100$	$\frac{1}{5} \times 10$	$\frac{1}{5} \times 1$
10 parts	$\frac{1}{10} \times 1{,}000$	$\frac{1}{10} \times 100$	$\frac{1}{10} \times 10$	$\frac{1}{10} \times 1$

	models in the previous step) but can also be thought of as '1,000 halves'. Both give the same total. • There are links within each column: one-quarter of a number is half of one-half of a number, one-tenth of a number is half of one-fifth of a number. • There are links within each row: as the whole is made ten times smaller each time, the fractional part also becomes ten times smaller.	
1:9	Once you are confident that all children have a solid understanding of the concepts met so far, progress to comparing fractions with decimals. This will require children to be secure in comparing within fractions (segment *3.8 Common denomination: more adding and subtracting*), and in comparing within decimals (*Spine 1: Number, Addition and Subtraction*, segment *1.23*). If in doubt, check whether those skills are in place before progressing to this step. Look at a decimal–fraction comparison statement. Discuss how the two values can be compared, by converting both to either a fraction or a decimal. At this stage, you can refer to the visual prompts of the number lines in the previous steps in learning. However, do keep practising recall of the common equivalences alongside this, so that children become fluent in them. Ideally, they should reach a point where they no longer need to refer to the number lines or work out the equivalences from first principles each time.	$0.6 \bigcirc \frac{4}{5}$ • *'0.6 is equivalent to $\frac{3}{5}$.'* *'We know that $\frac{3}{5} < \frac{4}{5}$, so $0.6 < \frac{4}{5}$.'* $0.6 < \frac{4}{5}$ $0.6 = \frac{3}{5}$ $\frac{3}{5} < \frac{4}{5}$ or • *'$\frac{4}{5}$ is equivalent to 0.8.'* *'We know that $0.6 < 0.8$, so $0.6 < \frac{4}{5}$.'* $0.6 < \frac{4}{5}$ $\frac{4}{5} = 0.8$ $0.6 < 0.8$

	Use the following stem sentences: • ***'___.___ is equivalent to*** $\frac{\square}{\square}$.***'*** ***'We know that*** $\frac{\square}{\square} < \frac{\square}{\square}$***, so*** ***___.___ <*** $\frac{\square}{\square}$.***'*** or • ***'***$\frac{\square}{\square}$ ***is equivalent to ___.___.'*** ***'We know that ___.___ < ___.___,*** ***so ___.___ <*** $\frac{\square}{\square}$.***'*** Show the children how to set out written chains of reasoning, as shown in the opposite column on the previous page.	
1:10	Repeat the process from the previous step, this time using a different example, e.g. $\frac{3}{4}$ ◯ 0.5 Give children plenty of opportunities to practise these comparison statements, both with verbal reasoning and in written form, until they are confident comparing fractions and decimals.	*'Fill in the missing symbols (<, > or =).'* $\frac{1}{10}$ ◯ 0.75 0.4 ◯ $\frac{1}{4}$ 0.5 ◯ $\frac{1}{5}$ $\frac{3}{4}$ ◯ 0.75 0.8 ◯ $\frac{4}{5}$ $\frac{1}{2}$ ◯ 0.2
1:11	To complete this teaching point, summarise the fraction–decimal equivalents that children need to learn by heart (two parts, four parts, five parts and ten parts). The conceptual understanding is essential and has been addressed throughout this teaching point, but moving forward it will be important to have a quick recall of these equivalences and an ability to identify them in calculations as	

	numbers that can easily be converted into different forms. One possible activity you could try is to give children double-sided cards with a fraction on one side and equivalent decimal on the other. (A couple of examples are provided opposite, but children will need to learn all fraction–decimal equivalents from zero to one for quarters, halves, fifths and tenths.) The children can use double-sided cards to practise and self-check their level of recall. However you choose to manage this in terms of your classroom routine, it is important to reach the stage where these fraction-decimal equivalents can be instantly recalled by all children.	Front — Back $\frac{1}{4}$ — 0.25 $\frac{1}{2}$ — 0.5 $\frac{1}{5}$ — 0.2 $\frac{1}{10}$ — 0.1

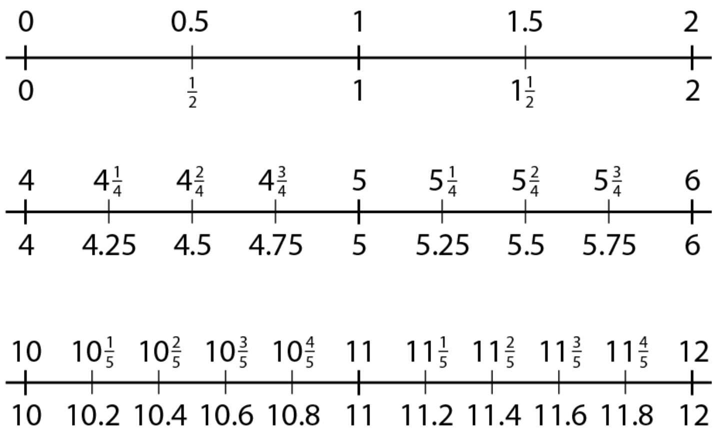
Teaching point 2:

These fraction–decimal equivalents can be found throughout the number system.

Steps in learning

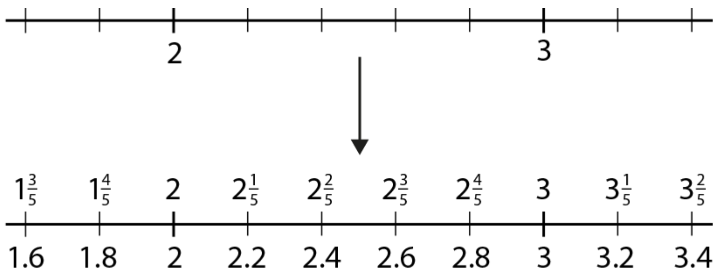
<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr><td>2:1</td><td>
Once fraction–decimal equivalents within one have been learnt by all children, extend this understanding throughout the number system. Start with some counting practice using number lines marked in two, four, five and ten parts for support. Dual-count by:

saying each number as a fraction
saying each number as a decimal.

</td><td>


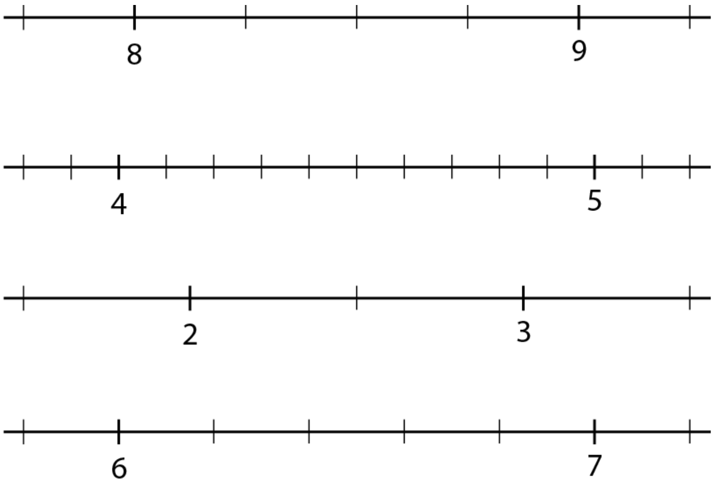
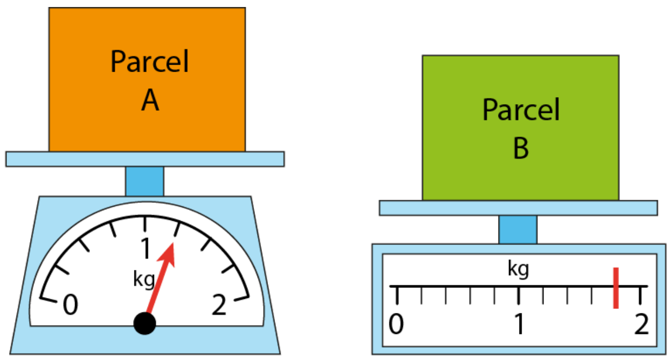
</td></tr>
<tr><td>2:2</td><td>
Now move on to identifying the missing numbers on partially-marked number lines. To support division into fractions, use the familiar stem sentences from segment 3.2 Unit fractions: identifying, representing and comparing:

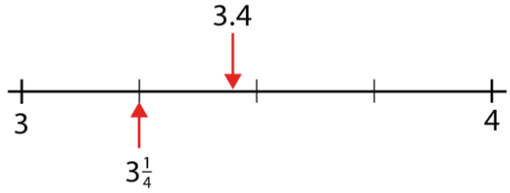
'Each whole has been divided into ___ equal parts.'
'Each part is one-___ of the whole.'

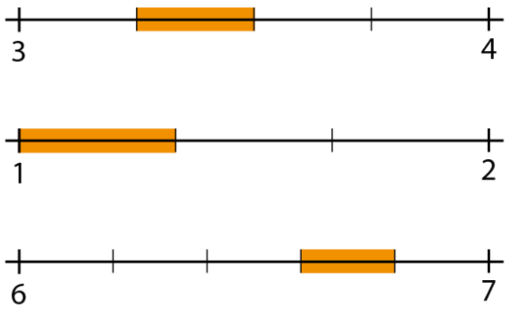
Once the fraction labels have been identified, equivalent decimal labels can then be added.
Make sure children count the number of parts the line has been divided into (here: five), and not the number of marks or notches between integers (here: four).
</td><td>


'Each whole has been divided into five equal parts.'
'Each part is one-fifth of the whole.'

</td></tr>
</table>

2:3	Give children practice marking decimal and fraction intervals on partially-marked number lines. As they practise, ensure you link, for example, four parts to the 25, 50, 75, 100 pattern and five parts to the 2, 4, 6, 8, 10 pattern, so that the children don't need to rely so heavily on the stem sentence. The stem sentence is a great scaffold, but developing fluency in labelling two-, four-, five- and ten-part scales without it will be extremely helpful when it comes to applying these skills to graphing and measurement contexts.	
2:4	Once you feel the children have the skills necessary to convert between common fraction–decimal equivalences across the number system, provide them with practice examples.	• *'Convert these decimal measurements to fractions.'* 1.2 km 5.75 m 25.5 kg • *'Convert these fractions to their decimal equivalents.'* $1\frac{1}{4}$ l $10\frac{1}{2}$ cm $4\frac{4}{5}$ m
2:5	To challenge children's understanding, provide problems set in a real-life context that require the conversion between common fractions and decimals. Are they able to identify and convert the information successfully?	• *'My brother weighs 7.3 kg. I weigh $10\frac{1}{2}$ kg. How much more than my brother do I weigh?'* • *'Year 6 set off on a $2\frac{3}{4}$ km woodland walk. By lunch, they had walked 1.5 km. How much further do they need to walk?'* • *'I need $5\frac{1}{2}$ litres of cola for a science experiment. One bottle contains 1.5 litres of cola. How many bottles do I need?'* • *'Here are two parcels:'*  • *'What is the total combined weight of the parcels?'*

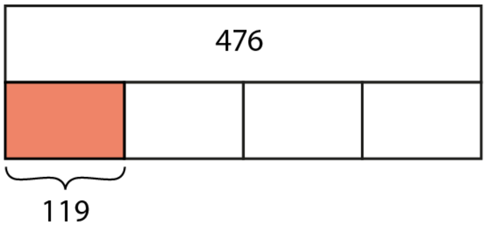
2:6	Next, practise comparing numbers where one is presented as a decimal and the other as a fraction. Show an inequality statement between two numbers, e.g. $3\frac{1}{4} < 3.4$, and ask children to convince you that this statement is true. They are likely to offer various justifications as to why the statement is true. Some of their potential methods are shown opposite. Make sure that all children have a strategy or method that they can successfully apply to compare fractions and decimals from these two-, four-, five- and ten-part groups. Often, conversion to a decimal will make comparison easier. As with fractions, there may sometimes need to be an additional step of finding a common denominator (as shown opposite). As children's fraction and decimal sense improves, you may find that they don't need to explicitly convert them in order to compare them. They may instead be able to solve them by, for example, thinking about their relative positions in the number system.	Method 1 – converting to decimals: $3\frac{1}{4} < 3.4$ $3\frac{1}{4} = 3.25$ $3.25 < 3.4$ Method 2 – converting to fractions with a common denominator: $3\frac{1}{4} < 3.4$ $3.4 = 3\frac{4}{10} = 3\frac{16}{40}$ $3\frac{1}{4} = 3\frac{10}{40}$ $3\frac{10}{40} < 3\frac{16}{40}$ Method 3 – positioning on a number line: 
2:7	Provide further varied practice opportunities for comparing fractions and decimals. Encourage children to clearly show their reasoning to justify their answers. You may find it useful to have a class example on the board as a scaffold for the children, so they can follow the steps, if needed. To promote depth of understanding, use a dòng nǎo jīn problem like the one shown on the following page. A question such as this should encourage children to use their existing	Missing-symbol problems: *'Fill in the missing symbols (<, > or =). Justify your answers.'* $1.5 \bigcirc 1\frac{1}{5}$ $10\frac{1}{4} \bigcirc 10.4$ $2\frac{1}{10} \bigcirc 2.1$ $6\frac{1}{2} \bigcirc 0.62$

	understanding of the positional value of numbers on a number line. They may need to think outside of the two-, four-, five- and ten-part divisions.	Ordering: *'Put the sets of numbers in order from smallest to greatest.'* 1.4 $4\frac{1}{4}$ 4.1 4.4 $1\frac{4}{4}$ $3\frac{1}{5}$ 3.5 $3\frac{5}{5}$ $1\frac{3}{5}$ 1.3 Dòng nǎo jīn: *'Write some fractions and matching decimals that you would find within the coloured sections on these number lines. Try to include at least one of the common fraction–decimal equivalents you have been learning about.'* 

Teaching point 3:

Fraction–decimal equivalence can sometimes be used to simplify calculations.

Steps in learning

	Guidance	**Representations**
3:1	By now, children should be confident with recognising decimals and fractions with common equivalence when they see them. Now advance to using these equivalents to simplify calculations. Opposite is a sequence of four calculations. The first calculation in the sequence is one that all children should all be able to attempt. The calculations progress through to the final calculation, which some children may be unsure of how to approach. Begin with the first calculation: *476 ÷ 4*. Show the bar model opposite, and discuss how it relates to the calculation. You can prompt the discussion with questions such as: • *'What is the whole?'* • *'How many equal parts is 476 being divided into?'* • *'How many parts do we have?'* All children should be able to solve this quickly through short division (some may be able to use mental methods, but short division is likely to be the quickest and most accurate for many children). Reveal the other calculations sequentially, discussing each as they appear and relating each one back to the previous calculation. Use the bar model as a reference point.	$476 \div 4$ $\frac{1}{4}$ of 476 $\frac{1}{4} \times 476$ 0.25×476 

<table>
<tr>
<td>**3:2**</td>
<td>When looking at the final calculation, *0.25 × 476*, emphasise that for this we have to rely on our prior knowledge that $0.25 = \frac{1}{4}$ in order to realise that we can solve this by dividing by four. If children have a solid understanding of the concepts they have learnt so far, then 0.25 should be instantly recognisable as a decimal that can easily be substituted for a fraction. Without this recognition, children may select an inefficient long multiplication method in order to solve it.

Compare the two calculation methods shown opposite and focus on how much more efficient the second method is.</td>
<td>Method 1:
$$\begin{array}{r} \cancel{1}\,\cancel{1} \\ \cancel{3}\;\;\cancel{3} \\ 4\;\;7\;6 \\ \times\quad 0\,.\,2\;5 \\ \hline 2\;3\,.\,8\;0 \\ 9\;5\,.\,2\;0 \\ \hline 1\;1\;9\,.\,0\;0 \\ \hline 1 \end{array}$$
Method 2:
$$\begin{array}{r} 1\;\;1\;\;9 \\ 4\,\overline{)\,4\;\;7\;\;{}^{3}6} \end{array}$$</td>
</tr>
<tr>
<td>**3:3**</td>
<td>Look at another calculation that can be simplified by substitution; this time with the decimal after the multiplication sign: *65 × 0.5.* Ask children if they have any suggestions as to how this calculation could be simplified. Discuss with the class and conclude that it can be rewritten as $65 \times \frac{1}{2}$. This can now be solved by halving 65.

Recognising this requires children to have a solid understanding of commutativity, in order to identify that $65 \times \frac{1}{2}$ can be considered as '$\frac{1}{2}$ of 65'. This should not be new to children; it has been covered extensively in previous segments.</td>
<td></td>
</tr>
<tr>
<td>**3:4**</td>
<td>Sometimes, there are several possible efficient approaches to dealing with calculations such as these. Present them with an example that lends itself to a variety of approaches, such as *0.2 × 75.*

Discuss the range of strategies that could be used to solve this. Three</td>
<td>Method 1:
$0.2 \times 75 = \frac{1}{5} \times 75$
$$\begin{array}{r} 1\;\;5 \\ 5\,\overline{)\,7\;\;{}^{2}5} \end{array}$$</td>
</tr>
</table>

	possible strategies are shown on the previous page and opposite, but children may offer other strategies as well. Prompt them to explain their method clearly and give value to all their ideas, but if a suggested strategy seems particularly inefficient, steer the child towards a more efficient strategy.	Method 2: $\frac{1}{10}$ of $75 = 7.5$ $\frac{2}{10}$ of $75 = 15$ Method 3: $2 \times 75 = 150$ $0.2 \times 75 = 15$ ÷ 10
3:5	To consolidate children's skills in identifying and working with efficient methods, provide mixed calculation practice, similar to the examples presented opposite.	0.25×6 93×0.5 0.75×396 105×0.4 $5{,}250 \times 0.2$ 0.8×455
3:6	Provide some examples that will enable children to practise their skills within a real-life context. (Note that the structure of these word problems is repeated addition, e.g. 120 lots of 0.25 l, rather than scaling, e.g. $\frac{1}{4}$ of 120. However, once written as *120* $\times$ *0.25*, it is probably most efficiently solved by finding $\frac{1}{4}$ of 120 (*120* $\div$ *4*), rather than multiplying 0.25 by 120. To support this critical step, encourage children to write a multiplication expression for each story, e.g. *120* $\times$ *0.25* or *0.25* $\times$ *120*.) Offer further practice through dòng nǎo jīn problems, such as those provided on the next page. Challenging children to find different ways to solve a calculation provides an opportunity for them to evaluate different methods, and it requires depth of understanding. Two possible methods for solving the first calculation are provided.	• *'Mo makes 120 glasses of juice for the school fair. He needs 0.25 litres of juice for each glass. How much juice does Mo need altogether?'* • *'Jake needs 0.2 metres of ribbon to make one head dress. He is making 90 head dresses for the school play. How much ribbon does Jake need?'* • *'One bag of modelling clay weighs 0.75 kg. How much do 24 bags of modelling clay weigh?'* • *'Izzy swims 0.5 km five mornings a week, before school. How far does she swim in four weeks?'*

Dòng nǎo jīn:

'Find two different ways to calculate each of the following.'

- 685×0.2
- 0.75×484
- 110×0.9

Possible methods to solve *685×0.2:*

Method 1:

$$685 \times 0.2 = 137$$

$$685 \times \tfrac{1}{5}$$

$$\begin{array}{r} 1\ \ 3\ \ 7 \\ 5\overline{)\,6\ {}^{1}8\ {}^{3}5} \end{array}$$

Method 2:

$$685 \times 0.2 = \tfrac{2}{10} \text{ of } 685 = 137$$

$$\tfrac{1}{10} \text{ of } 685 = 68.5$$

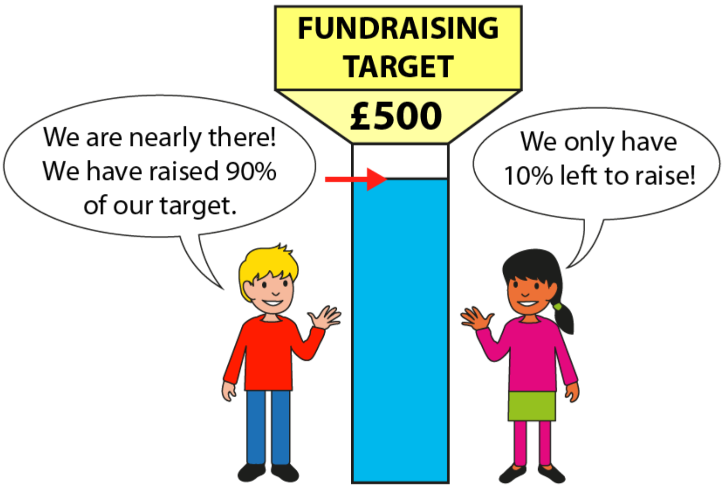
$$\begin{array}{r} 6\ 8\ .\ 5 \\ \times \qquad 2 \\ \hline 1\ 3\ 7\ .\ 0 \\ \hline {}_{1\ \ 1}\quad \end{array}$$

Teaching point 4:

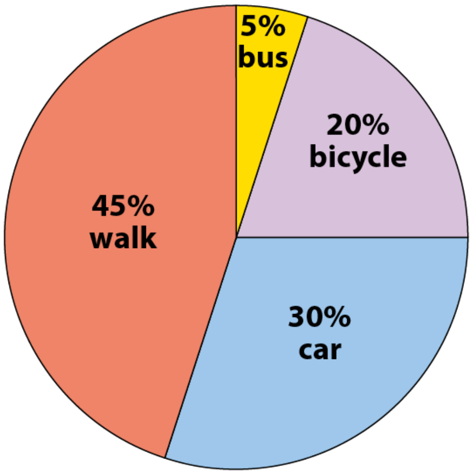
'Percent' means number of parts per hundred. A percentage can be an operator on a quantity, indicating the proportion of a quantity being considered.

Steps in learning

	Guidance	Representations
4:1	Fractions provide one way to describe the proportion of a number or amount being considered. For example, if I run $\frac{1}{3}$ of a race, this is further than if I run $\frac{1}{10}$ of a race. This teaching point looks at a different way to describe the proportion of a number or amount: percentages. Start by discussing the etymology of the word 'percent'. It means '*for/out of* every hundred'. A full amount, or quantity, is referred to as 100%. For example, if 100% of the children in a class have completed their homework, it means that everyone in the class has completed it. If 90% of a class have completed their homework, then most of the class have completed it. If 50% have completed their homework, then half of the class have completed it. If only 10% have completed their homework, then only a small proportion of the class have completed it. Look at a range of examples, similar to those shown below, and discuss what the information given about each percentage tells us. Focus discussions on whether a large part of the whole or a small part of the whole is being considered. Ask: *'Can you think of any examples of where you have used or seen percentages before?'*	



Year 6 transport to school



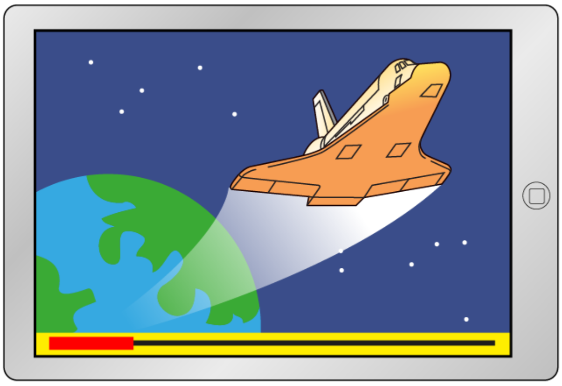
4:2

One of the things the children will need to understand about percentages is that it tells us about the *proportion of* a number or amount being considered. Later in this segment, children will learn that 10% of a large quantity may be a bigger number or amount than 90% of a small quantity. For example, 10% of 500 is 50, but 90% of 60 is only 48. At this stage, don't discuss particular percentages of quantities with the children, but start to develop the sense that percentages tell us about a *proportion of* a number or amount, independent of the size of the number or amount. That is, the percentage tells us whether the 'part' is a large or a small part of the whole.

Look at example 1, opposite. Ask the children to estimate the percentage of the film that has been watched. Don't make marks to try to work it out exactly, but encourage the children to estimate. If we know that at the start Nevaeh had watched 0% of the film, and at the end she has watched 100% of it, then we can see that she has watched around 15–20% of the film so far. She is reasonably near the start of the film and still has plenty left to watch.

Example 1:

- *'Nevaeh is watching a film on a tablet. Estimate the percentage of the film she has watched so far.'*
- *'Can you tell how long it will take to watch the rest of the film?'*



Example 2:

'This icon shows how much power Evie has left in her phone battery. Approximately what percentage of the power remains?'

<table>
<tr>
<td></td>
<td>

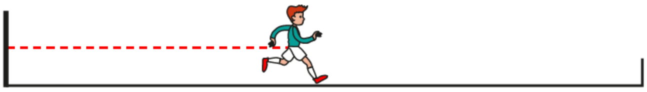
Discuss with the children whether we have enough information to work out how much longer the film will last. We *can* say that Nevaeh has around 80–85% of the film still to watch, but we *can't* say how long this will take as we don't know the duration of the film.

The first three examples on the previous page and opposite can easily be related to a 0%–100% number line. As such, you may wish to write 0% and 100% on the diagrams to support children with their estimations. The fourth example is a little more challenging as it is not linear. However, children should be able to see that a bit less than half of the city suffered earthquake damage and base their estimations on this.

</td>
<td>

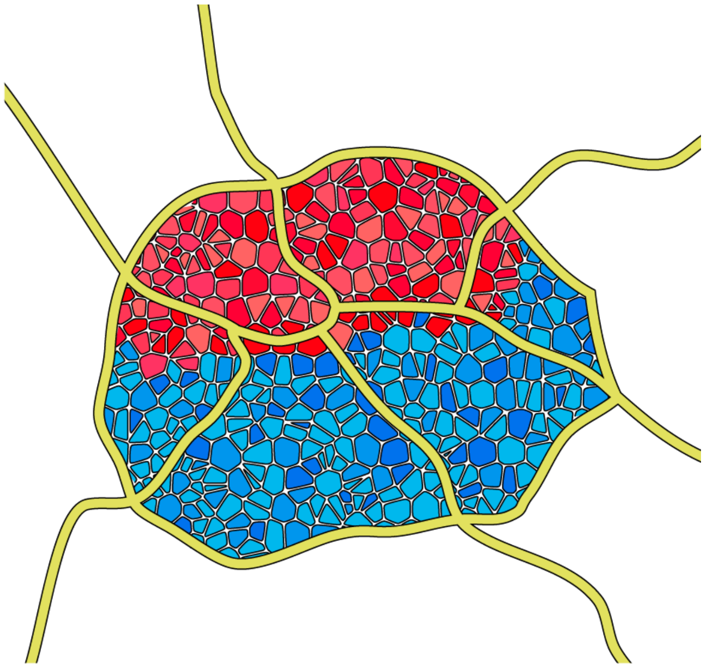
Example 3:

'The diagram shows the percentage of a race that Finn has run so far. Roughly what percentage of the race has he already run?'



Example 4:

'This plan shows a city that has been damaged by an earthquake. The red sections show the damaged areas. Estimate the percentage of the city that has been damaged by the earthquake.'



</td>
</tr>
<tr>
<td>

4:3

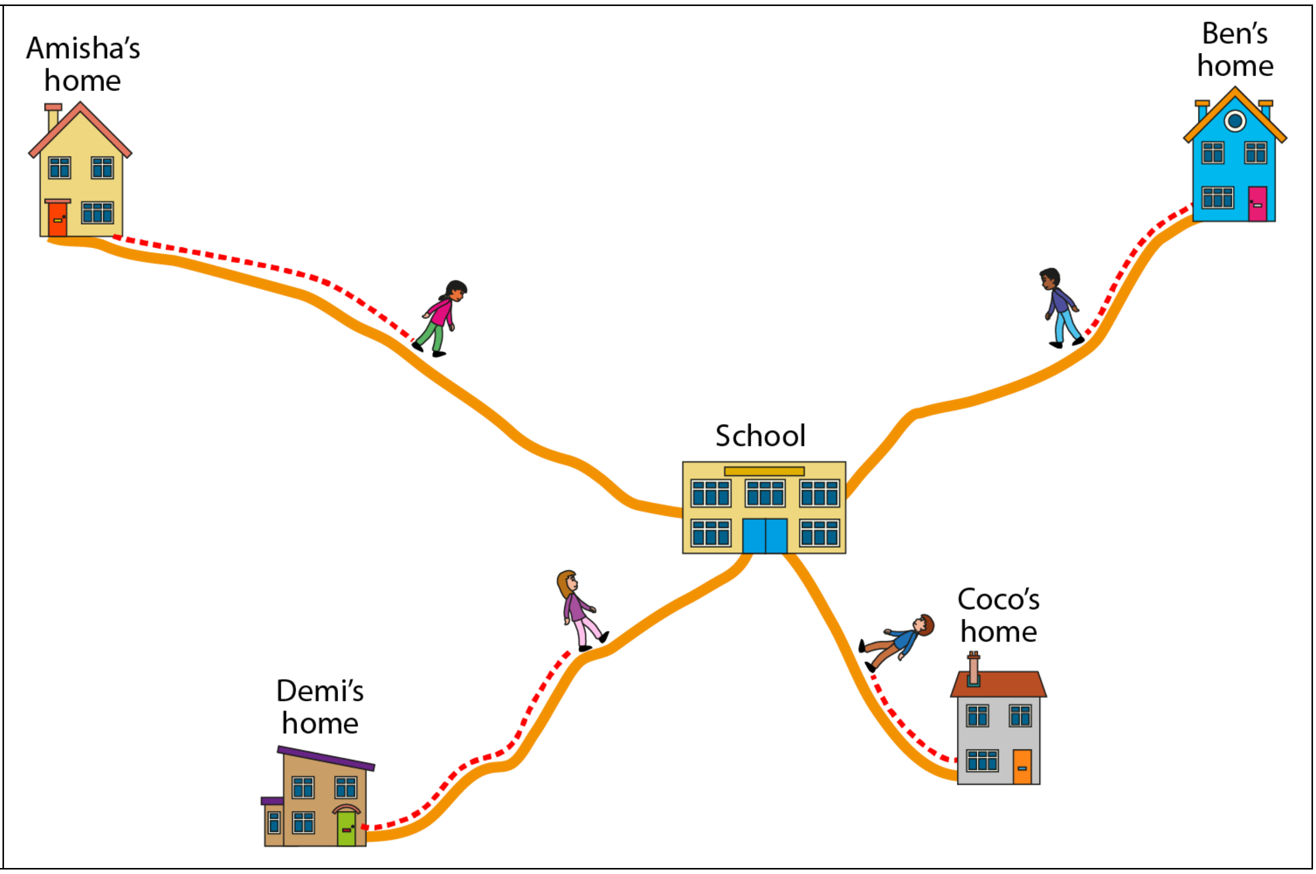
</td>
<td colspan="2">

Now look at the diagram on the next page, and the positions of four children on their journeys to school. Ask the children to estimate the percentage of the journey that each child has completed so far. You may also like to estimate the percentage of the journey that they still have to complete, noting that these two numbers must add up to 100%.

Some potential questions or discussion points around this diagram include:

- *'Which two children have walked about the same distance so far?'*
(Amisha and Demi have, although Demi has completed a greater percentage of her journey.)
- *'Which two children have walked about the same percentage of their journey?'*
(Both Amisha and Coco have completed about 50% of their journey.)
- *'Who has walked the furthest?'*
(Amisha and Demi, who have walked about the same distance.)
- *'Who has completed the greatest percentage of their walk?'* (Demi)
- *'Who is less than 50% of the way through their walk to school?'* (Ben)
- *'Who has walked the shortest distance so far?'* (Coco)

</td>
</tr>
</table>

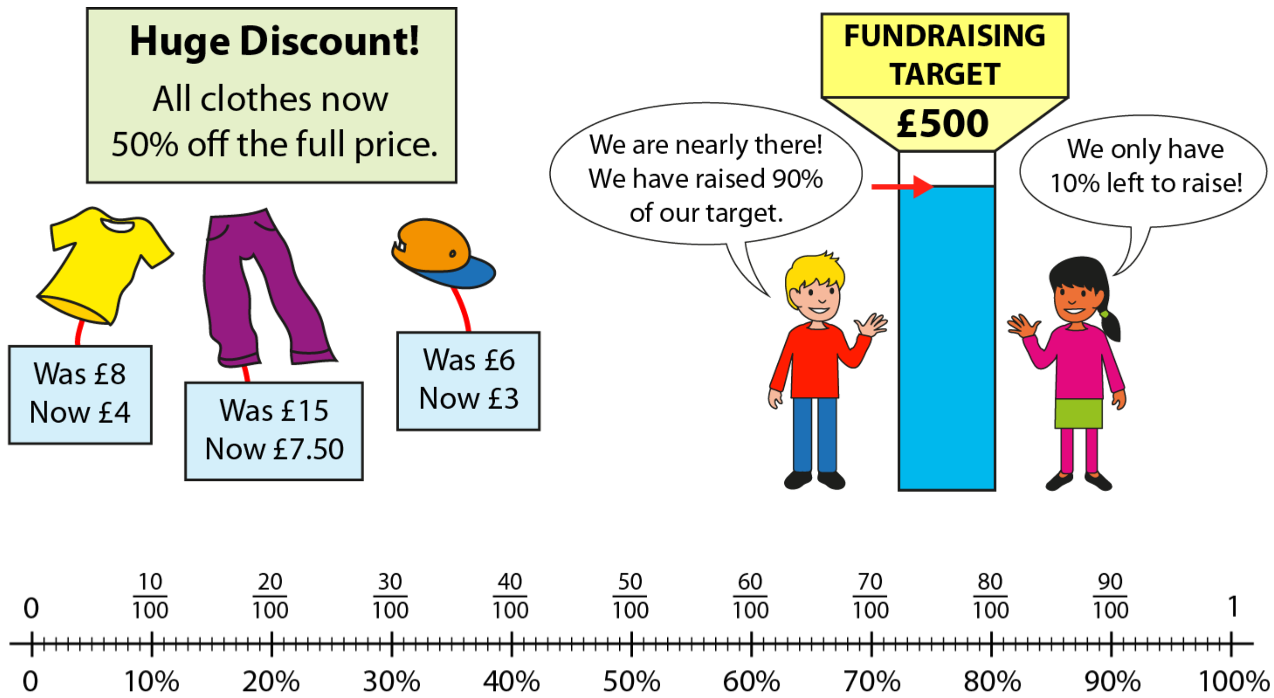

Amisha's
home
Ben's
home
School
Coco's
home
Demi's
home

Teaching point 5:

Percentages have fraction and decimal equivalents.

Steps in learning

	Guidance	**Representations**
5:1	As we know from the previous teaching point, percentage means ‘*for/out of* every hundred’. As such, percentages can easily be written as fractions with a denominator of 100. For example, in the two pictures we looked in step *4:1*, we could also say that the clothes are now $\frac{50}{100}$ of the full price, and we could say the school has raised $\frac{90}{100}$ of their fundraising target. As a class, explore the number line below, which shows the equivalent percentages and fractions with a denominator of 100. Look at the two scales and ask the children: • *‘What is the same?’* • *‘What is different?’* Once children are comfortable with these equivalences, discuss some percentages which fall in between the labelled intervals. Pose questions, such as: • *‘Where would 31% be? How is this written as a fraction?’* • *‘What about* $\frac{68}{100}$ *? How is this written as a percentage?’*	

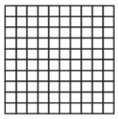
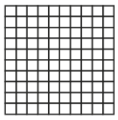
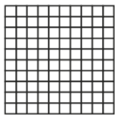
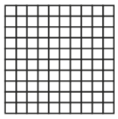


5:2

Show the children the table opposite, and discuss the following representations of each percentage with them:

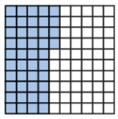
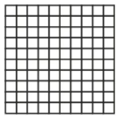
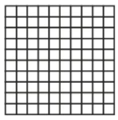
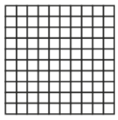
- written percentage
- percentage written as a fraction with a denominator of 100
- percentage out of 100, shaded on a hundred square
- estimated position of given percentage on a 0%–100% number line.

Children may notice that some of the fractions can be simplified. Confirm that this is the case, but at this stage we are going to stick with working in hundredths. Work together, using the information that is given to complete each row of the table.

Percentage	Fraction	Hundred square	Number line
80%	$\frac{\square}{100}$		0% 100%
45%	$\frac{\square}{100}$		0% 100%
31%	$\frac{\square}{100}$		0% 100%
9%	$\frac{\square}{100}$		0% 100%

5:3

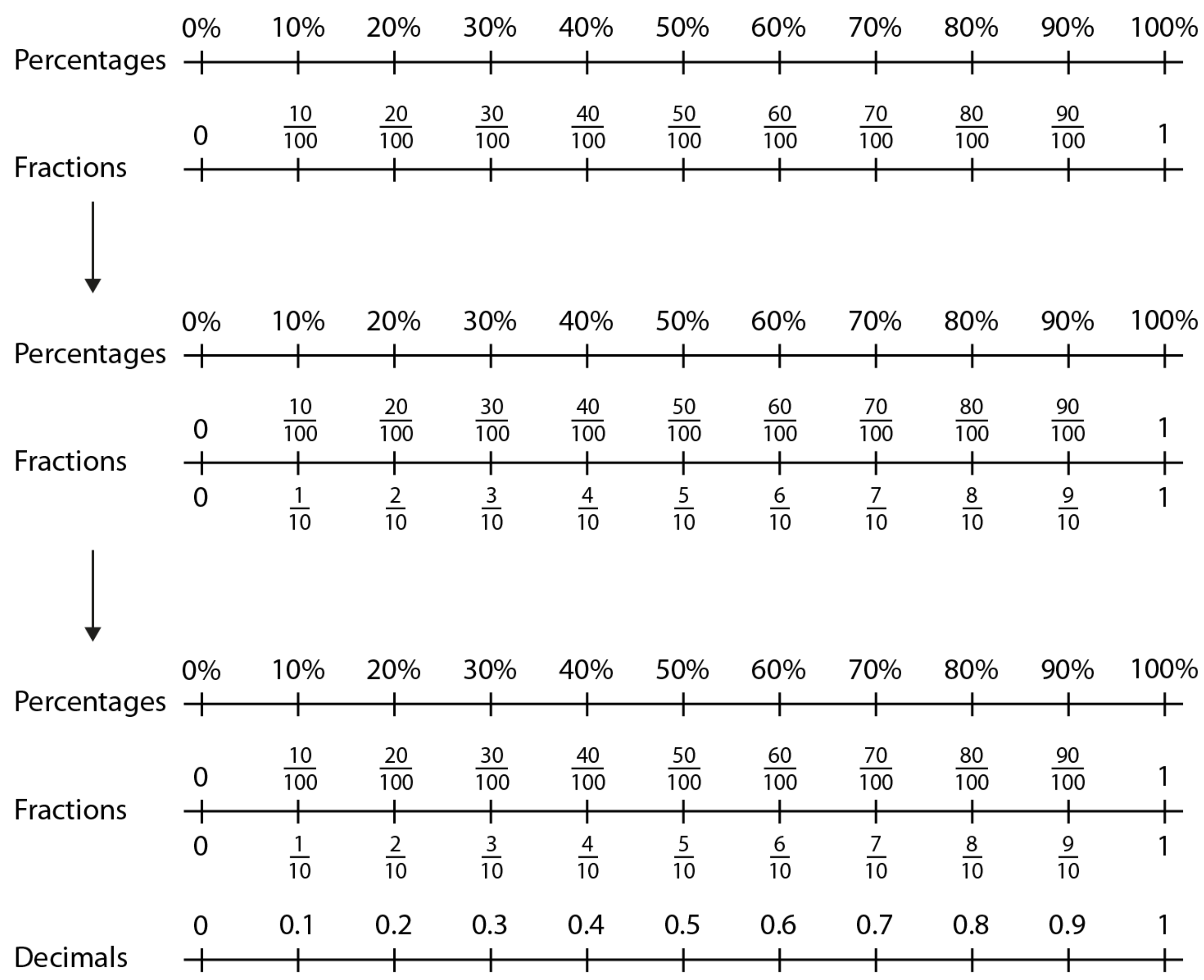
Once children are demonstrating confidence in working with all the various forms in the table, provide a similar table but with some columns completed and others left empty. Challenge them to work from any of the given representations to complete the missing information in the other columns.

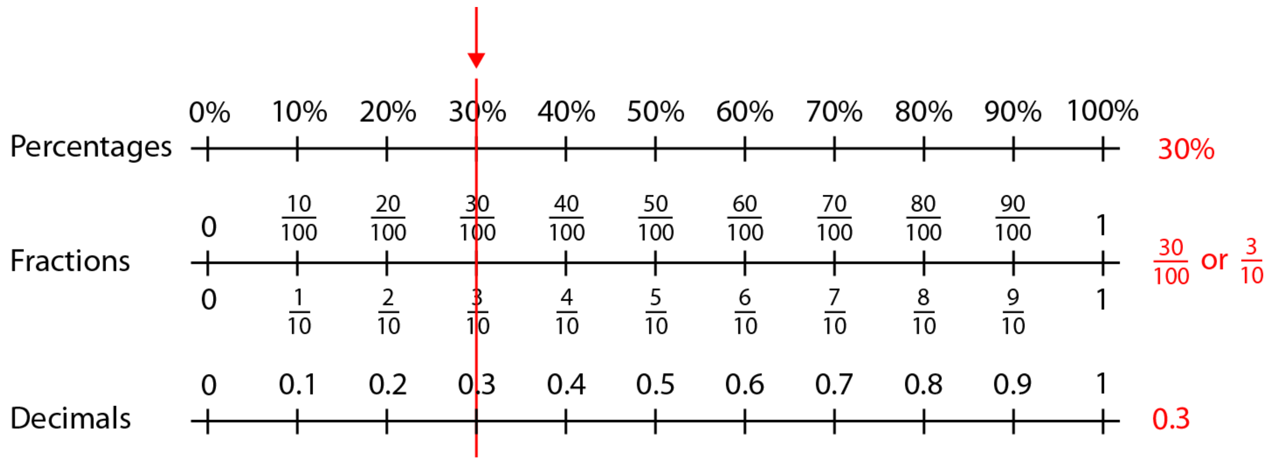
Percentage	Fraction	Hundred square	Number line
	$\frac{\square}{100}$		0% 100%
	$\frac{10}{100}$		0% 100%
	$\frac{\square}{100}$		45% 0% 100%
6%	$\frac{\square}{100}$		0% 100%

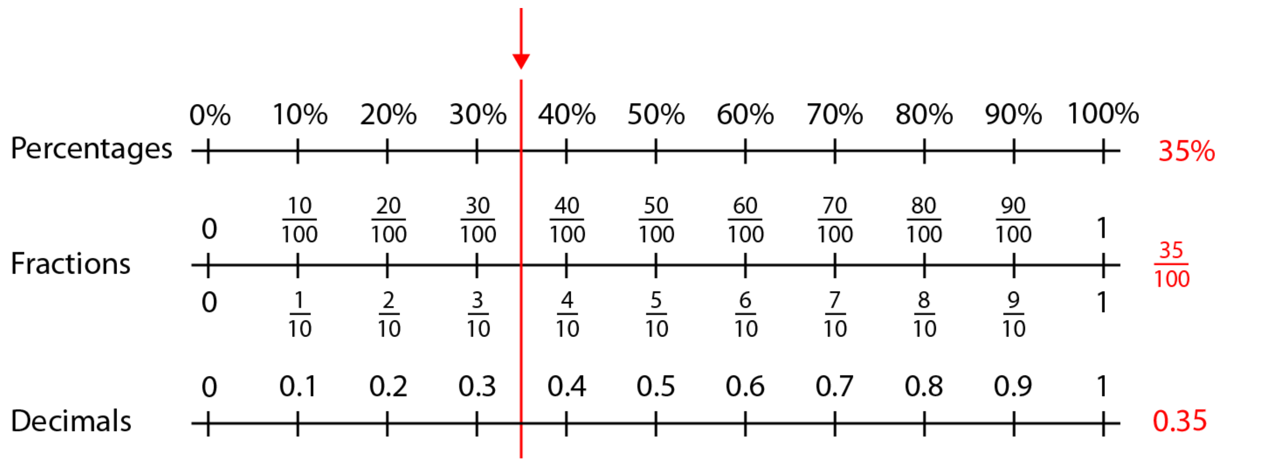
5:4

This is a good stage to introduce decimals. Through work in *Spine 1: Number, Addition and Subtraction*, segment *1.24*, children should be confident recognising equivalences between decimals and fractions with a denominator of 100, but briefly review this now.

Start by looking at the percentage and fraction number lines from step *5:1*. Then, introduce fractions with a denominator of ten to the fractions number line. Examine the equivalences shown here, drawing on the equivalent fractions work that children have already done throughout this spine. When they are confident with this and can clearly see, for example, why 60% is equivalent to 0.6, add a third number line for decimals, and link this back to the fractions and percentages.

	
5:5	After discussion of the various equivalences shown on the number lines, add a vertical line through all the number lines and discuss how to read equivalent values. For example, start with a vertical line at a multiple of 10%. (In the example provided on the next page, 30% is shown, and it will be easier to read the values by using a multiple of 10%, as all of these points are marked and labelled.) Once all children are secure in their ability to read these equivalent values, move your line along the number line so that it rests between marked/labelled intervals (here: 35%). The percentages and fractions with a denominator of 100 should be fairly easy to give. You will need to discuss why we can no longer give this value as a fraction with a denominator of 10, and also how to write it as a decimal. Move the line to various other positions until you are convinced that children can accurately verbalise the percentage, fraction and decimal equivalents, both for multiples of 10% and for other percentages.





5:6 Return to the table from step *5:2* and insert another column for the decimal equivalents. Discuss how each of the percentages and fractions shown so far can also be written as a decimal. Some points that may arise from your discussion include:

- 80% can be written as 0.80 (make sure children do not read this as *'zero-point-eighty'*) but can also be written as 0.8; this is equivalent to 8 tenths, or 80 hundredths.
- 9% is 0.09 as a decimal. Look out for children who write 9% as 0.9. You can minimise this misconception by making sure children get used to reading out 0.09 as *9 hundredths* and 0.9 as *9 tenths* (see step *1:1*).
- Writing 45% and 31% as decimals is less likely to result in an error.

Percentage	Fraction	Hundred square	Number line	Decimal
80%	$\frac{\square}{100}$		0% 100%	
45%	$\frac{\square}{100}$		0% 100%	
31%	$\frac{\square}{100}$		0% 100%	
9%	$\frac{\square}{100}$		0% 100%	

	Repeat for the table from step *5:2* to provide children with further opportunities to practise.	
5:7	Now that children are confident converting percentages to both decimals and to fractions with a denominator of 100, extend this to fractions with other denominators. Show children the equation opposite and ask them to discuss how to find the missing values with their partner. If they have a secure understanding, they should be reasoning along the lines of: • *'We know from the previous steps that $80\% = \frac{80}{100}$.'* • *'We know from work in previous segments that $\frac{80}{100} = \frac{4}{5}$.'* (both the numerator and denominator can be divided by 20 to preserve the value of the fraction). Repeat with a different example. After solving both these examples, focus on the fractions and double-check that they make sense. For instance, with 45%, we know that this is just less than $\frac{1}{2}$, and we can also see that $\frac{9}{20}$ is just less than $\frac{1}{2}$, so that sounds like it should be correct.	$80\% = \frac{\square}{100} = \frac{\square}{5}$ $45\% = \frac{\square}{100} = \frac{\square}{20}$
5:8	Next, introduce an example where the interim step is removed, e.g., $12\% = \frac{\square}{25}$ At first glance, this appears much more challenging, as 12% can't easily be thought of in terms of 'twenty-fifths'. Remind children of the interim step of converting to hundredths first. Once they have rewritten 12% as $\frac{12}{100}$, this	

can then be simplified to $\frac{3}{25}$ by dividing both the numerator and denominator by four.

Reach the generalisation that: ***'In order to convert a percentage to a fraction, first convert it to a fraction with a denominator of 100.'***

$$12\% = \frac{\square}{25}$$

$$12\% = \frac{\square}{100} = \frac{\square}{25}$$

$$12\% = \frac{12}{100} \underset{\div 4}{\overset{\div 4}{=}} \frac{3}{25}$$

5:9

Now focus on percentages that are multiples of ten. Return to the number line from step *5:1*, (example 1, opposite), and examine how the fractions in hundredths can also be written as tenths. Count up from zero to one in tenths, and then 0%–100% in steps of 10%. Next, practise moving between different equivalents, for example asking:

- *'What is 90% as a fraction?'*
- *'What is $\frac{4}{10}$ as a percentage?'*

Review the table in example 2, and link these percentages to fractions, decimals and the hundred square representation.

Add another column to your table, as in example 3, with a vertical bar divided into ten equal parts. Ask the children to think carefully about how 10% of this bar could be shaded. They should discuss their ideas with a partner. As the whole is now ten parts, one of these parts is 10%. Once the new bars are all correctly shaded, focus on the resulting image.

10% of the hundred square is ten squares, but 10% of the bar is one square. Remind children that percentages tell us about the size of the

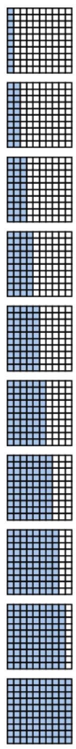
Example 1:

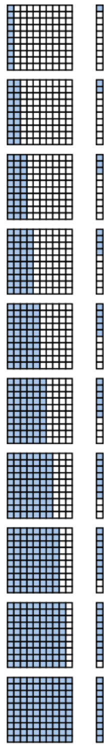
0	$\frac{10}{100}$	$\frac{20}{100}$	$\frac{30}{100}$	$\frac{40}{100}$	$\frac{50}{100}$	$\frac{60}{100}$	$\frac{70}{100}$	$\frac{80}{100}$	$\frac{90}{100}$	1
0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	1
0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%

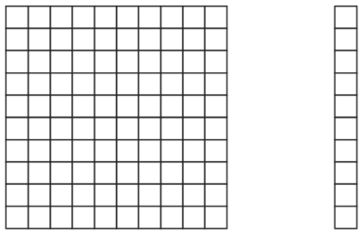
Example 2:

10%	$\frac{1}{10}$	0.1
20%	$\frac{2}{10}$	0.2
30%	$\frac{3}{10}$	0.3
40%	$\frac{4}{10}$	0.4
50%	$\frac{5}{10}$	0.5
60%	$\frac{6}{10}$	0.6
70%	$\frac{7}{10}$	0.7
80%	$\frac{8}{10}$	0.8
90%	$\frac{9}{10}$	0.9
100%	$\frac{10}{10}$	1

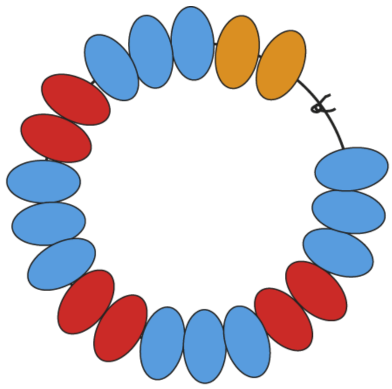


<table>
<tr>
<td></td>
<td>part in relation to the whole. If the size of the whole is reduced, then the size of the shaded part is also reduced.</td>
<td>Example 3:
<table>
<tr><td>10%</td><td>$\frac{1}{10}$</td><td>0.1</td></tr>
<tr><td>20%</td><td>$\frac{2}{10}$</td><td>0.2</td></tr>
<tr><td>30%</td><td>$\frac{3}{10}$</td><td>0.3</td></tr>
<tr><td>40%</td><td>$\frac{4}{10}$</td><td>0.4</td></tr>
<tr><td>50%</td><td>$\frac{5}{10}$</td><td>0.5</td></tr>
<tr><td>60%</td><td>$\frac{6}{10}$</td><td>0.6</td></tr>
<tr><td>70%</td><td>$\frac{7}{10}$</td><td>0.7</td></tr>
<tr><td>80%</td><td>$\frac{8}{10}$</td><td>0.8</td></tr>
<tr><td>90%</td><td>$\frac{9}{10}$</td><td>0.9</td></tr>
<tr><td>100%</td><td>$\frac{10}{10}$</td><td>1</td></tr>
</table>

</td>
</tr>
<tr>
<td>5:10</td>
<td>Consolidate learning by providing children with varied practice in moving between fractions, decimals and percentages, including working with equivalent fractions.</td>
<td>

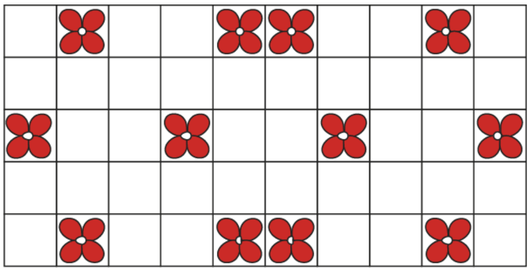
'Shade 70% of each of these shapes.'



'What percentage of the beads in this bracelet are red?'


</td>
</tr>
</table>

- *'Below is a pattern of tiles on a bathroom wall. Clare says, "12% of the tiles have flowers on them." Explain why Clare's reasoning is incorrect.'*



- *'Complete the missing information in this table.'*

Fraction	Decimal	Percentage
		43%
	0.27	
$\frac{1}{5}$		
		75%
	0.09	
$\frac{30}{100}$		
	0.61	
		80%

- *'Convert these percentages to fractions in their simplest form.'*

  80%   53%   50%   45%   68%

<table>
<tr><td></td><td></td><td>

- *'Match these percentages to their equivalent fractions.'*

50%	$\frac{3}{5}$
75%	$\frac{680}{1,000}$
60%	$\frac{3}{4}$
70%	$\frac{7}{14}$
68%	$\frac{7}{10}$

- *'Circle all the fractions that are equivalent to 40%.'*

$\frac{2}{5}$ $\frac{40}{10}$ $\frac{8}{20}$ $\frac{14}{35}$ $\frac{1}{40}$ $\frac{3}{5}$

</td></tr>
<tr><td>**5:11**</td><td>

At the end of *Teaching point 1*, children memorised some key fraction-decimal equivalents. Return to these now, and extend them to include percentages. When presented with any value from the table opposite, the children should be able to complete the other two columns.

This practice can be extended by including the other multiples of 10%, as per the focus of step *5:9*.

As mentioned in *Teaching point 1*, the conceptual understanding of these equivalents is essential, but having quick recall of them will be important going forward as children begin to calculate percentages of quantities. Display these equivalents clearly on the classroom wall, and practise recall of them as part of your classroom routine until all the children are fluent.

</td><td>

Fraction	Decimal	Percentage
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%

</td></tr>
</table>

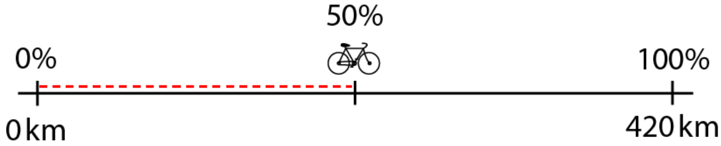
Teaching point 6:

If the value of a whole is known, a percentage of that number or amount can be calculated.

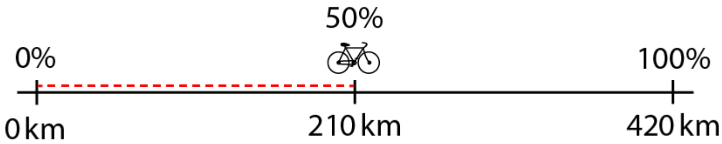
Steps in learning

<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr><td>6:1</td><td>This teaching point considers calculating percentages of quantities. Introduce this through a scenario, such as: 'Zara is doing a 420 km charity bike ride. So far, she has completed 50% of the route. How far has she cycled?'

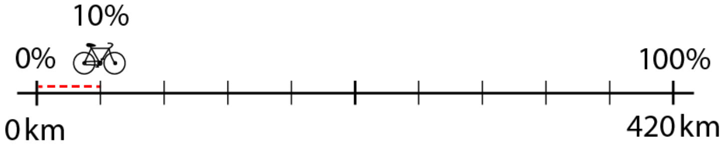
Work towards the following generalisation: 'To find 50% of a number, halve it.'</td><td>'Zara is doing a 420 km charity bike ride. So far, she has completed 50% of the route. How far has she cycled?'



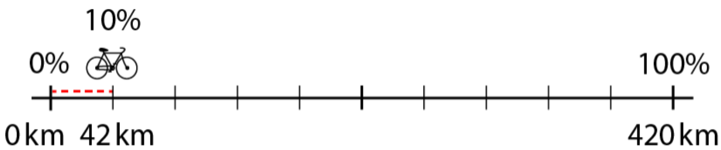
• '100% of 420 km is 420 km.'
• '50% of 420 km is $\frac{1}{2}$ of 420 km.'
• 'Zara has cycled 210 km.'

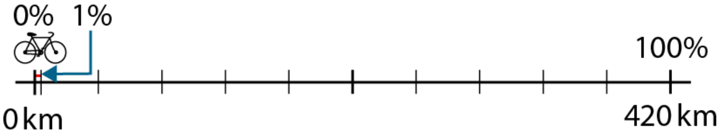
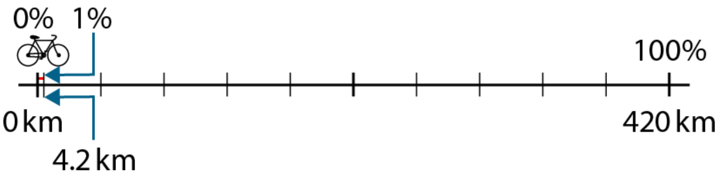
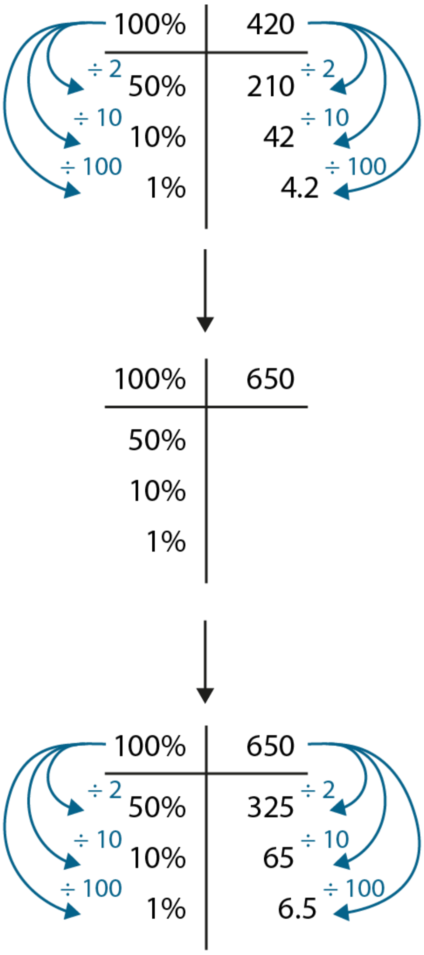
</td></tr>
<tr><td>6:2</td><td>Extend the scenario: 'Rishi has completed 10% of the same bike ride. How far has he cycled?'

Work towards the following generalisation: 'To find 10% of a number, divide it by ten.'</td><td>'Rishi has completed 10% of the same bike ride. How far has he cycled?'



• '100% of 420 km is 420 km.'
• '10% of 420 km is $\frac{1}{10}$ of 420 km.'
• 'Rishi has cycled 42 km.'

</td></tr>
</table>

6:3	Extend the scenario further: *'James has completed 1% of the same bike ride. How far has he cycled?'* Work towards the following generalisation: ***'To find 1% of a number, divide it by one hundred.'*** This is a good point at which to start introducing the idea that percentages can be greater than 100%. For example, if Zara finished the bike ride and then decided to turn around and cycle back to the start, we would say that she had completed 200% of the original bike ride.	*'James has completed 1% of the same bike ride. How far has he cycled?'*  • *'100% of 420 km is 420 km.'* • *'1% of 420 km is* $\frac{1}{100}$ *of 420 km.'* • *'James has cycled 4.2 km.'* 
6:4	Summarise the examples from steps *6:1–6:3* in written form. Be careful not to write *'10% = 42'*. It is true that 10% *of 420* = 42, but it is not mathematically accurate to say 10% = 42. Recording the percentages in a vertical ratio table, as shown opposite, is a clear and useful way to summarise different percentages of the whole. Repeat finding these key percentages of the whole with another number, such as 650. Initially focus on finding these 'benchmark' percentages (50%, 10% and 1%), as they, in turn, can be used to find other percentages of a whole. Again, this might be a good place to briefly discuss the idea that percentages can be more than 100%. For example, 200% of 650 would be 1,300 and 300% of 650 would be 1,950. This type of calculation is not included in the general practice here, but mentioning it prepares children for encountering percentages greater than 100% at secondary school.	

6:5

At this point, after working through various examples as a class, provide individual practice opportunities with a variety of problem types, such as those provided opposite. To consolidate learning, present dòng nǎo jīn problems, similar to those provided.

Ratio tables:

'Complete the following tables.'

100%	**30**
50%	
10%	
1%	

100%	**230**
50%	
10%	
1%	

100%	**84**
50%	
10%	
1%	

Missing-number problems:

'Fill in the missing numbers.'

$\square$ % of 54 = 5.4

14 = $\square$ % of 1,400

$\square$ % of 170 = 85

Word problems:

- *'We set the target of raising £200 for our class charity. So far, we have raised 10% of the money. How much have we raised?'*
- *'There are 45,000 tickets available for a rugby match. 50% of the tickets have been sold. How many tickets have been sold?'*
- *'A zoo has 300 butterflies in its butterfly house. 1% of them are the rare banded peacock butterflies. How many banded peacock butterflies does the zoo have?'*

Dòng nǎo jīn:

- *'Find different ways to complete the following statements. What do you notice about the relationships between each pair of numbers?'*

  10% of $\square$ = 50% of $\square$

  1% of $\square$ = 10% of $\square$

- *'Class 6 are doing a sponsored silence for the whole of the six-hour school day. So far, they are 10% of the way through. How long is left?'*

6:6

Now that children are able to find 1%, 10% and 50% of a number, progress to finding some other useful benchmark percentages.

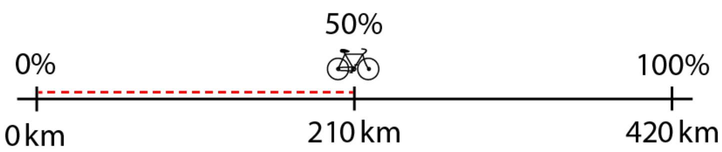
Return to the story of Zara from step *6:1*, 50% of the way through her bike ride. Ask the class to consider how to work out what distance Zara had cycled when she was 25% of the way through her bike ride. Possible responses may include the following methods:

- Method 1:
  25% is half of 50%. So, 210 km can be halved to work out that 25% of 420 km is 105 km.
- Method 2:
  Use the equivalence of 25% to $\frac{1}{4}$, in order to work out that 420 km can be divided by four to give 105 km. So, 25% of 420 km is 105 km.

Complete the ratio table opposite to show that 25% of 420 km is 105 km. Discuss both of the methods and the connections between them. Also discuss any alternative strategies offered by the children. If they are writing methods on the board, make sure that they do not write *'25% = 105 (km)'*. Instead, they must write *'25% of 420 (km) = 105 (km)'*.

Once 25% of the bike ride has been calculated, ask the children to work out how far Zara had cycled after completing 75% of the bike ride. The children may offer various strategies, including the following methods:

- Method 1:
  Add 50% of 420 (210) and 25% of 420 (105) to get 75% of 420. 75% of 420 km is 315 km.
- Method 2:
  Multiply 25% of 420 (105) by 3 to get 75% of 420. $105 \times 3 = 315$ (km).



100%	420 km
50%	210 km
25%	
75%	

100%	420 km
50%	210 km
25%	105 km
75%	

100%	420 km
50%	210 km
25%	105 km
75%	315 km

Finding 75% of 420:

Method 1

$$\begin{aligned} 75\% \text{ of } 420 &= 210 + 105 \\ &= 315 \end{aligned}$$

Method 2

$$\begin{aligned} 75\% \text{ of } 420 &= 105 \times 3 \\ &= 315 \end{aligned}$$

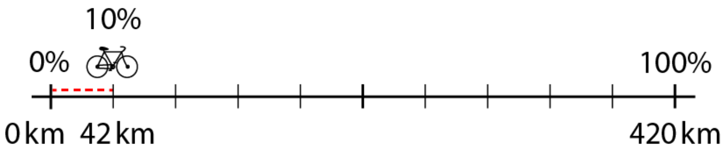
Method 3

$$\begin{aligned} 75\% \text{ of } 420 &= 420 - 105 \\ &= 315 \end{aligned}$$

<table>
<tr>
<td></td>
<td>
Method 3:
75% of 420 can be calculated by subtracting 25% of 420 (105) from 100% of 420.
Complete the ratio table for 75% of 420. Discuss all three methods and the connections between them.
If none of the children suggest method 3, show the workings for it on the board. Ask the children to try and identify the method that has been used. Sometimes, percentages of a quantity can be worked out by adding together other known percentages of that quantity. Methods 1 and 2 are ‘adding together’ methods. Sometimes though, it might be most efficient to use a ‘subtracting from’ method (like method 3) to calculate percentages of quantities.
</td>
<td></td>
</tr>
<tr>
<td>6:7</td>
<td colspan="2">
Now look back at the example of Rishi in step 6:2. Rishi had completed 10% of the bike ride. As in the previous step, use this information to derive further percentages of 420.
Start by looking at 5% of 420 km in the ratio table on the next page. This can be calculated by halving 10% of 420. It can also be found by multiplying 1% of 420 by 5, but in this instance, that method would involve a decimal calculation.
Move on to deriving 20% of 420 km by doubling 10% of 420. Then look at all of the other multiples of 10%, and discuss how 10% of 420 can be used to calculate all of these. Of course, this is not always the most efficient method (it is easier to find 50% of 420 km by halving 420), but it is a method that is very useful for the multiples of 10% other than 50%.
Highlight some alternative ways to calculate percentages of 420 km, for example:

60% of 420 km can be calculated by adding 50% of 420 (210) and 10% of 420 (42). This is the ‘adding- together’ method.
90% of 420 km can also be calculated by subtracting 10% of 420 (42) from 100% of 420. This is the ‘subtracting-from’ method.

The children will probably think of lots of other methods, which you should also discuss, encouraging them to consider the relative ease and efficiency of the various methods proposed.
Once the ratio table is complete, you may choose to note that this offers an alternative method to calculate 25% of 420 km to those in step 6.6; we can also add 5% of 420 and 20% of 420.
</td>
</tr>
</table>



100%	420 km
10%	42 km
5%	21 km
20%	84 km
30%	
40%	
50%	
60%	
70%	
80%	
90%	

→

100%	420 km
10%	42 km
5%	21 km
20%	84 km
30%	3×42 km
40%	4×42 km
50%	5×42 km
60%	6×42 km
70%	7×42 km
80%	8×42 km
90%	9×42 km

→

100%	420 km
10%	42 km
5%	21 km
20%	84 km
30%	126 km
40%	168 km
50%	210 km
60%	252 km
70%	294 km
80%	336 km
90%	378 km

6:8

Before going any further, provide practice in calculating the percentages of a variety of numbers, using the benchmark percentages as a starting point. Initially, scaffold the steps by providing the ratio tables for support, then challenge children to do this part independently.

'Find the following percentages:'

- 40% of 15

100%	**15**
10%	
40%	

- 25% of 680

100%	**680**
50%	
25%	

- 5% of 1,400

100%	**1,400**
10%	
5%	

- 75% of 240

100%	**240**
50%	
25%	
75%	

- 25% of 280
- 30% of 55
- 5% of 80
- 90% of 1,500
- 75% of 900

6:9	It is only a small step further to combine these percentages so that children can calculate any percentage of any number. Choose an example, such as *'Find 45% of 64'*, and ask children to discuss different approaches they could use to solve it. Prompt them to think about both 'adding-together' methods and 'subtracting-from' methods. Using the ratio tables is a good way to organise their work and show the 'building block' percentages as they go. Two potential methods are shown opposite, but children will probably offer others as well. Discuss all their ideas and, as before, prompt them to consider the relative ease and efficiency of their suggestions. Repeat with other questions, such as those provided opposite, discussing the various methods for each one. Ask: *'When might a "subtracting-from" method be more efficient than an "adding-together" method?'*	*'Find 45% of 64.'* **100%** | **64** 50% | 32 10% | 6.4 5% | 3.2 Method 1: 45% of $64 = 32 - 3.2$ $= 28.8$ Method 2: 45% of $64 = 4 \times 6.4 + 3.2$ $= 25.6 + 3.2$ $= 28.8$ • *'Find 69% of 800.'* **100%** | **800** 50% | 400 10% | 80 20% | 160 1% | 8 Method 1: 69% of $800 = 400 + 160 - 8$ $= 560 - 8$ $= 552$ Method 2: 69% of $800 = 7 \times 80 - 8$ $= 560 - 8$ $= 552$ Method 3: 69% of $800 = 69 \times 8$ $= 552$

<table>
<tr><td></td><td></td><td>

- *'Now calculate these.'*

  92% of 720

  31% of 90

</td></tr>
<tr><td>6:10</td><td>By now, children should be able to find a percentage of any number. Provide individual practice opportunities with a variety of problem types. Include dòng nǎo jīn problems, such as the examples on the next page, to consolidate and further deepen children's understanding.</td><td>

Calculations:

'Calculate the following:'

48% of 380   75% of 3 m

12% of 5   34% of £10

19% of 40,000   61% of 5 kg

True/false style problems:

'Are the following statements true or false? Why? Challenge yourself to do this <u>without calculating the exact percentages</u>.'

- 25% of 379 is a bit less than 100.
- 90% of 520 is around 400.
- 45% of 210 is more than 105.

Word problems:

- *'Each class in a school is given a box of 120 pens.'*
  - *'Year 3 have used $\frac{1}{4}$ of the pens.'*
  - *'Year 4 have used 35 pens.'*
  - *'Year 5 have used $\frac{7}{20}$ of the pens.'*
  - *'Year 6 have used 30% of the pens.'*

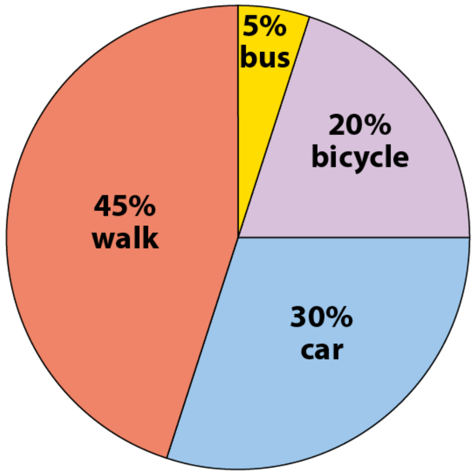
  'Which class has used the most pens so far?'

- *'The full price of a computer is £299. There is 10% discount in a sale. How much does the computer cost now?'*

</td></tr>
</table>

- *'There are 120 children in Year 6. Calculate how many children use each mode of transport to get to school.'*

Year 6 transport to school



Dòng nǎo jīn:

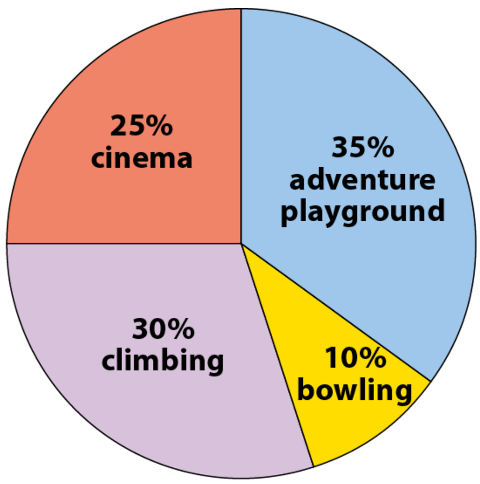
'Fill in the missing numbers.'

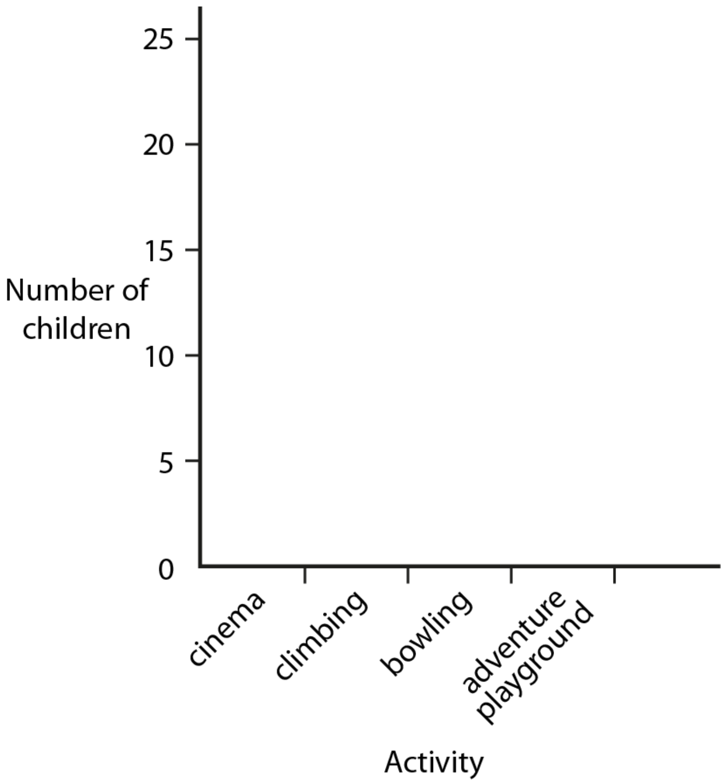
$9 = \boxed{}$ % of 60

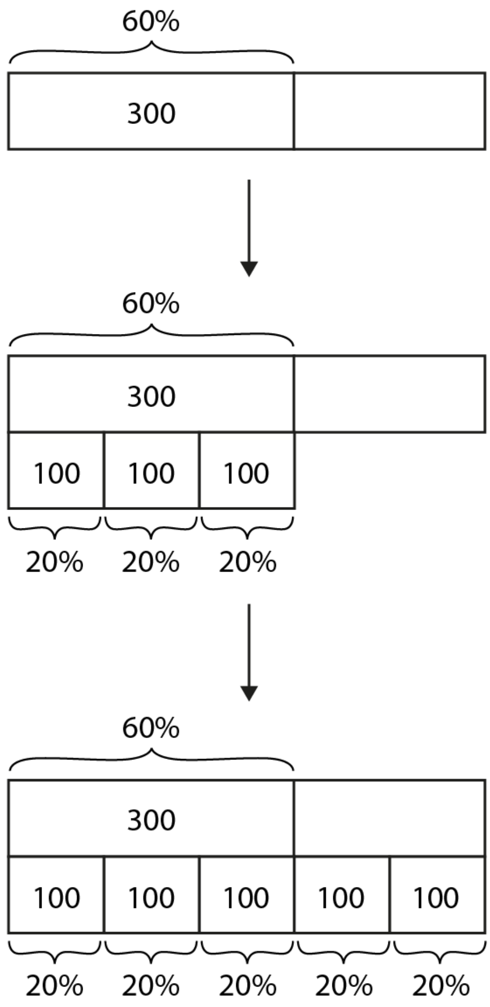
36% of $\boxed{}$ = 9% of $\boxed{}$

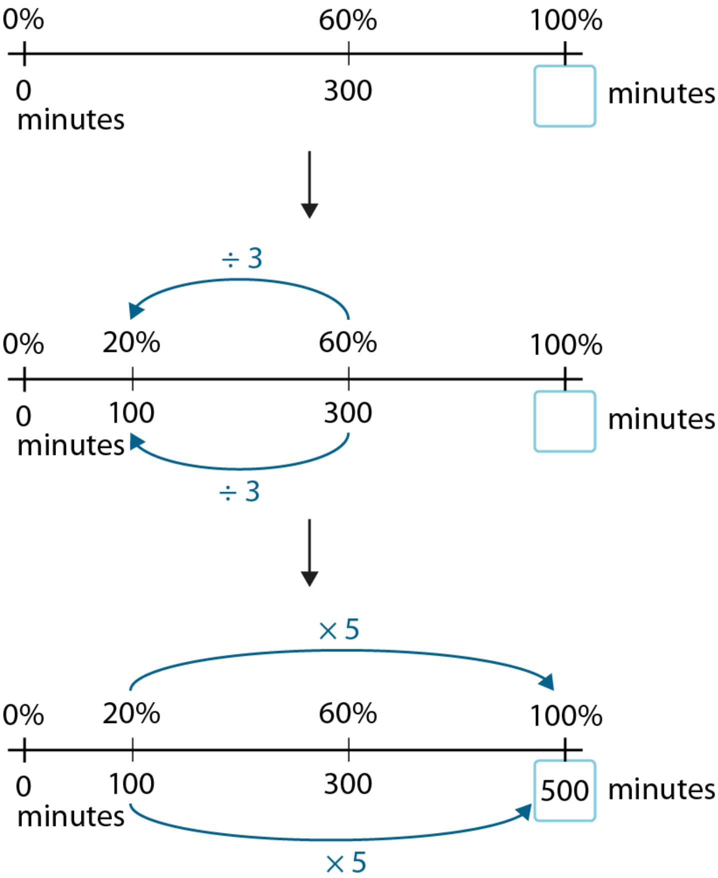
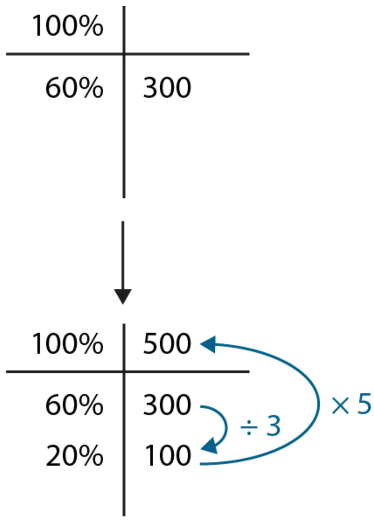
$\boxed{}$ % of 160 = $\boxed{}$ % of 120

- *'This pie chart shows what 60 Year 6 children voted to do for their end-of-year treat.'*



		• *'Construct a bar chart to show the number of children who voted for each activity.'* 
6:11	Now look at examples where a percentage part is known and children have to calculate the whole amount from the part. Present the following scenario: *'I have made 300 minutes of calls so far this month. This is 60% of my monthly "free-call" allowance. How many minutes of free calls do I get each month?'* Discuss this scenario, posing the types of questions that children will need to consider independently in order to make sense of situations like this and solve them. For example: • *'Have I used all of my "free-call" minutes yet?'* • *'Have I used more than half of my minutes?'* • *'Is my total monthly "free-call" minutes more or less than 300?'*	*'I have made 300 minutes of calls so far this month. This is 60% of my monthly "free-call" allowance. How many minutes of free calls do I get each month?'*

	Once it has been established that 300 minutes is part of the monthly allowance, and the whole is currently unknown, think about how this could be represented with a model. As mentioned previously, when working with models, children need to get to a stage where they can use them independently to show the underlying structure of a situation. Avoid teaching 'set rules' for how to draw a model, and make sure that the children are using them to support sense-making, and can describe what their models show. When using multiple models, you will need to balance the benefits that comparison and discussion of different representations bring in terms of developing conceptual understanding, with the risk that using too many models at one time can cause confusion and hinder learning. The bar model and double number line are shown here as two *possible* ways to help children make sense of situations where we are given a part and need to find a whole. However, you will need to think carefully about whether to use both, making connections between them, or just to focus on one. Work through your chosen model with the children. In both cases, you need to find a factor of 60% that is also a factor of 100%, in order to calculate the whole. 20% of the whole is the highest factor, although 10% of the whole could also be used. It is *extremely important* to use language such as *'60% of the whole'* and *'20% of my total minutes'* rather than just talking about 60% and 20%, so that you are always emphasising what the whole is that you are referring to.	Bar model: 

	Once children have a firm understanding of what is happening in structures like this using the bar model or double number line, summarise the workings in a ratio table.	Double number line:  Ratio table: 

6:12

Now look at a series of examples where both the size of the part and the percentage that the part forms of the whole are varied. Again, the three models used in step *6:11* can be applied. They are shown opposite for the first example in the sequence.

- *'I am thinking of a number. 10% of my number is 15. What is the number I am thinking of?'*
- *'I am thinking of a number. 20% of my number is 15. What is the number I am thinking of?'*
- *'I am thinking of a number. 25% of my number is 15. What is the number I am thinking of?'*
- *'I am thinking of a number. 25% of my number is 30. What is the number I am thinking of?'*
- *'I am thinking of a number. 75% of my number is 90. What is the number I am thinking of?'*

For the first four questions, we can work directly from the part to the whole. For the final example, you will need to work backwards to a common factor (in this case 25% of the whole, which is 30), as per step *6:11*.

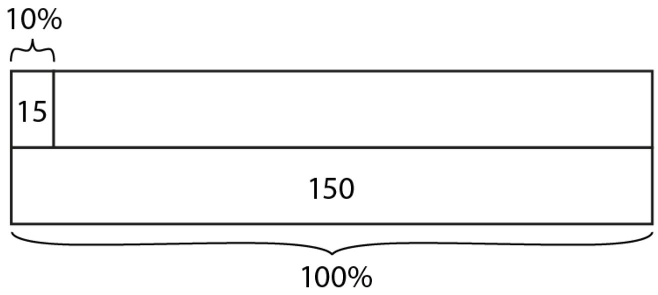
As you work through the series of examples, discuss similarities and differences compared to the previous example and predict how the value of the whole will change. Ask:

- *'What is the same?'*
- *'What is different?'*

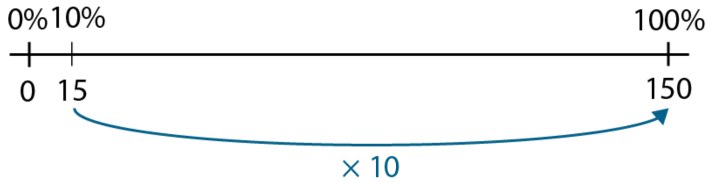
In particular, when moving from the first to the second example, some children may predict the whole will increase when they see the percentage increase from 10% to 20%. However, as the same amount (15) now forms a greater percentage of the whole, in fact the whole will be smaller than in the first example.

'I am thinking of a number. 10% of my number is 15. What is the number I am thinking of?'

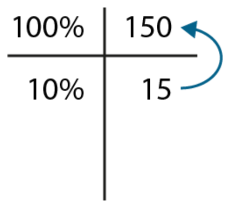
Bar model:



Double number line:



Ratio table:



6:13	Explore some more similar examples as a class, and ask children to draw their preferred model each time, until they develop confidence with it. Look at the examples opposite. In example 1, the children can work directly from the part to the whole. In examples 2 and 3, they need to work 'backwards' to a benchmark percentage, and then use this to work out the full amount (100%). Note that for example 3, drawing each individual percentage out on a bar model is both laborious and unnecessary. This is a good example of the need for children to 'sense-make' with models and adapt them to individual situations – once the children have calculated that 1% of the walk is 3.5 km, they can use this to work out that the whole walk is 350 km by multiplying by 100, without needing to draw it in a model. As before, ratio tables can be used to summarise the situations.	Example 1: *'I am thinking of a number. 25% of my number is 80. What is the number I am thinking of?'* Example 2: *'I am thinking of a number. 30% of my number is 75. What is the number I am thinking of?'* Example 3: *'Molly is doing a long-distance walk. So far, she has walked 14 km. This is just 4% of the total distance. How far is Molly aiming to walk?'*
6:14	Move on to a slightly more complex scenario, where the children are given the percentage reduction and asked to find an original amount, for example: *'A coat has 10% off in a sale. The price is now £45. What was the original price?'* Here, a common (incorrect) assumption is that if an amount has been reduced by 10%, we can calculate 10% of the reduced amount and add it back on to find the original. As before, discuss this scenario, posing the types of questions that children will need to consider independently in order to make sense of situations like this and solve them. For example: • *'Is the coat more or less expensive now than when it was full price?'*	*'A coat has 10% off in a sale. The price is now £45. What was the original price?'* Incorrect reasoning: 10% of £45 = £4.5 ✗ = £45 + £4.5 = £49.50

- *'Is a 10% reduction a big reduction or a small reduction?'*
- *'If it is reduced by 10%, what percentage of the full price does it now cost?'*

We want the children to ask themselves *'What do I know?'* and *'What do I want to know?'* and then find a model that links these considerations.

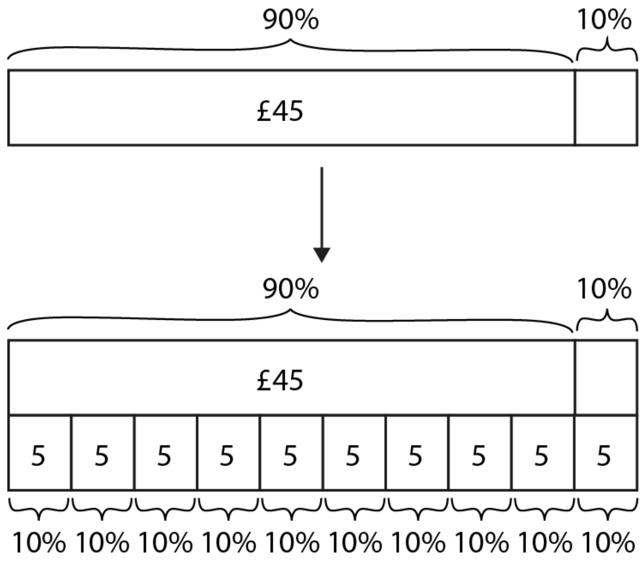
Representing this situation on a model, be it a bar model or a double number line, helps to reveal the structure. The key understanding that is needed to calculate the whole, is identifying that because the cost has been reduced by 10% of the full price, £45 is equivalent to 90% of the full price.

As previously, a ratio table can also be a helpful model to use to record the steps in finding the full price. Because its layout is not proportional (in the way that the bar model and double number line are proportional), it doesn't expose as visually why 90% of the total is £45, but once children have understood the concept, a ratio table can be useful.

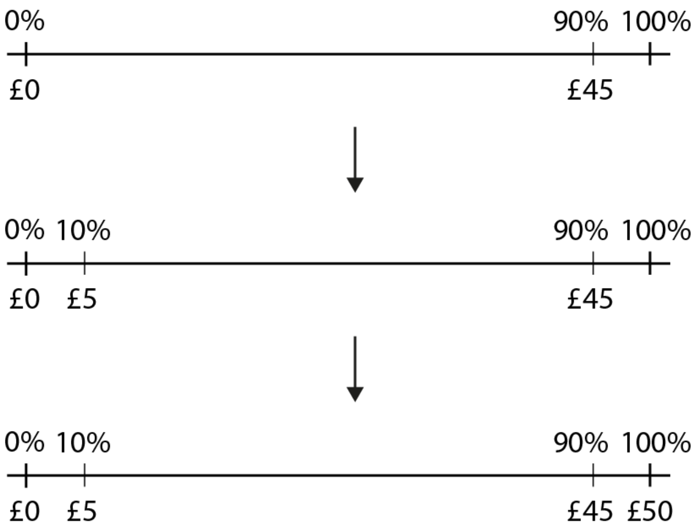
Present some examples with a similar structure. As you work on these questions with the class, ask the children to identify what a common mistake might be for each. This will raise their awareness of the common pitfalls to try and avoid themselves.

Correct reasoning:

Bar model



Double number line



Ratio table

100%	
90%	45
10%	

→

100%	50
90%	45
10%	5

Notes

- 90% of original price = £45
- 10% of original price = £5
- 100% of original price = £50

<table>
<tr><td></td><td></td><td>Further practice:
'I have a length of rope. I cut off 20% of the rope. The remaining piece is 10 m long. How long was the rope to start with?'
'A farmer sells 25% of her sheep. She has 600 sheep left. How many sheep did she have to start with?'</td></tr>
<tr><td>6:15</td><td>Conclude this segment by presenting examples in real-life contexts, where the size of the part and the percentage it forms of the whole are known, and children use this to then calculate the whole.
Children should be strongly encouraged to continue to use diagrams and models to support them in solving these. Without them, it is quite likely they may make some of the common mistakes discussed in the previous step.</td><td>Real-life contexts:
'30% of the seats at a cricket match are taken. So far, there are 750 people present. How many people will be there when all of the seats are filled?'
'So far, Adam has read 180 pages, or 60%, of his book. How many pages are in Adam's book in total?'
'A pair of trainers is reduced by 25% in a sale. They now cost £36. What did they cost at full price?'
'90% of the runners in a race have crossed the finish line. There are still 12 runners on the course. How many runners took part altogether?'
Ratio tables:
'Complete these ratio tables.'
<table><tr><th>100%</th><th></th></tr><tr><td>40%</td><td>28</td></tr><tr><td>10%</td><td></td></tr></table><table><tr><th>100%</th><th></th></tr><tr><td>15%</td><td>60</td></tr><tr><td>5%</td><td></td></tr></table>Find my number:
'I am thinking of a number. 30% of my number is 105. What is my number?'
'I am thinking of a number. 7% of my number is 49. What is my number?'</td></tr>
</table>