MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1

Teacher guide | Year 2

Teaching point 1:

Counting in multiples of ten can be represented by the ten times table. Adjacent multiples of ten have a difference of ten. Facts from the ten times table can be used to solve problems about groups of ten.

Teaching point 2:

Counting in multiples of five can be represented by the five times table. Adjacent multiples of five have a difference of five. Facts from the five times table can be used to solve problems about groups of five.

Teaching point 3:

Skip counting and grouping can be used to explore the relationship between the five times table and the ten times table.

Teaching point 4:

When zero is a factor, the product is zero. When one is a factor, the product is equal to the other factor (if there are only two factors).

Overview of learning

In this segment children will:

- build up the *ten* times table by combining their knowledge of skip counting in *tens* and representing equal groups with multiplication equations
- build up the *five* times table by combining their knowledge of skip counting in *fives* and representing equal groups with multiplication equations
- begin to explore the relationship between the five and ten times tables
- explore patterns in known multiplication facts (two, five and ten times tables) to generalise about the product when one of the factors is zero or one.

Teaching points 1 and *2* of this segment (ten and five times tables, respectively) follow a similar progression to segment *2.3 Times tables: groups of 2 and commutativity (part 1), Teaching point 2* (two times table). *Teaching point 4* allows children to generalise for cases where one of the factors is either zero or one, and zero and one times tables are developed. Teachers can begin to build up a class multiplication chart as each times table is covered:

×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6			15					30		
4	0	4	8			20					40		
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12			30					60		
7	0	7	14			35					70		
8	0	8	16			40					80		
9	0	9	18			45					90		
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22			55					110		
12	0	12	24			60					120		

Key:

new facts in this segment

previously learnt facts

From segment *2.7 Times tables: 2, 4 and 8, and the relationship between them*, the chart can include highlighting of previously learnt facts that are relevant to the new times table being learnt, based on children's understanding of commutativity. This will make the learning of the full set of times tables less daunting, as the further children progress through the times tables, the fewer new facts there are to be learnt. Although children already understand that the factors in a multiplication equation can be written in either order (the 'one interpretation, two equations' understanding of commutativity), they do not yet have the understanding that one equation can have two different grouping interpretations, and hence do not have the conceptual framework to link, for example, two groups of five with five groups of two. Since the idea of equal groups/unitising is used to build up the times tables, for now the new times tables in the multiplication chart (fives and tens) are not linked back to known facts in the two times table.

As with all of the times tables, regular practice will be needed in order for children to become fluent, both in reciting the times tables (for example, '*One times five is equal to five, two times five is equal to ten…*') and with isolated multiplication facts (for example, *'I know that seven times five is equal to thirty five.'*).

In *Teaching point 3*, children will make connections between the five and ten times table, by:

- 'double skip counting' (half the class skip counting in fives and the other half skip counting in tens)
- comparing a given quantity grouped into either fives or tens
- recognising which multiplication equations represent common multiples of five and ten (for example, $4 \times 5 = 20$ has the same product as $2 \times 10 = 20$).

Note that, at this stage, the language of doubling and halving (and connection to division by two), is not used to describe the relationships. For example, children are not expected to calculate 6×5 by explicitly using $(6 \times 10) \div 2$, or to express 6×10 as $2 \times 6 \times 5$. Doubling and halving will be introduced (in terms of multiplicative reasoning) in segment *2.5 Commutativity (part 2), doubling and halving*, building on an understanding of commutativity, and halving will be connected to division by two in segment *2.6 Structures: quotitive and partitive division.*

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

Counting in multiples of ten can be represented by the ten times table. Adjacent multiples of ten have a difference of ten. Facts from the ten times table can be used to solve problems about groups of ten.

Steps in learning

1:1

Guidance

Begin this teaching point by reviewing skip counting in tens, which was introduced in segment *2.1 Structures: multiplication representing equal groups*. At this point you can extend up to twelve tens.

Use familiar representations, including those from segment *2.1*; for example:

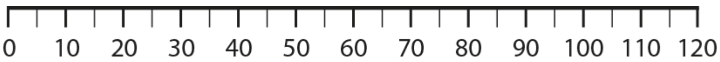
- the number line
- the Gattegno chart (have children tap the chart as they count: one tap for multiples of ten up to and including 100; two taps for multiples of ten greater than 100, for example, tap '100' then '20' on the count of 120)
- groups of ten objects (for example, tens frames with counters, boxes of ten eggs, fingers on two hands and so on; 10 p pre-money tokens)
- using skip counting to find the value of a number of 10 p coins.

Children have spent a lot of time exploring the equivalence of ten ones and one ten (unitising in tens), both in segment *2.1* and in *Spine 1: Number, Addition and Subtraction*, segment *1.8*. Remind children of this equivalence as you move from using examples where the cardinality can be seen (for example, boxes of ten eggs) to examples where the cardinality isn't visible (for example, 10 p coins; you can first count groups of ten 1 p coins, then count to the same value in 10 p coins).

As in previous segments, count in two ways:

Representations

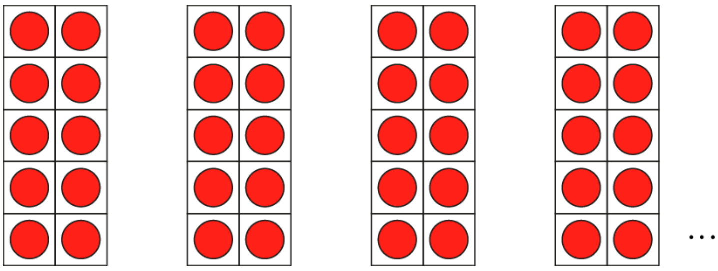
Skip counting in tens – number line:



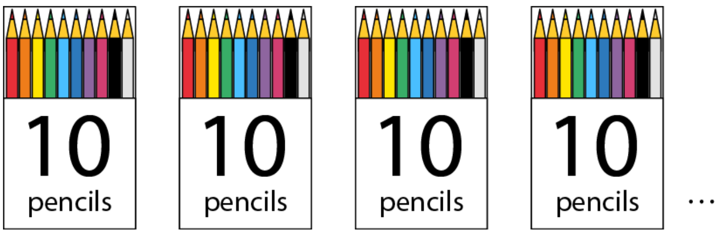
Gattegno chart:

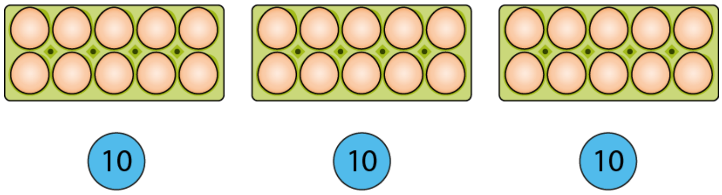
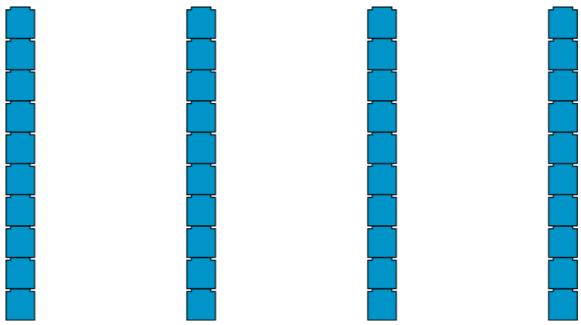
1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Skip counting with groups of ten objects – cardinality visible:



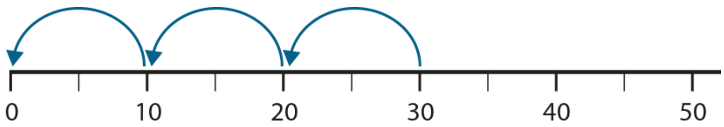
Skip counting with groups of ten objects – cardinality not clearly visible:



	• *'One group of ten, two groups of ten, three groups of ten…'* • *'Ten, twenty, thirty…'* You can begin to shorten the latter to: *'One ten, two tens, three tens…'* Skip counting in tens beyond 100 may be new to children, but as they are familiar with both the pattern and the representations used, they should be able to do this and discuss how they know what comes next. Practise counting both forwards and backwards in tens from different multiples of ten.	
1:2	Now connect evaluating the number of objects using skip counting (for example, three boxes of ten eggs is ten, twenty, *thirty* eggs) with representing the grouped objects with a multiplication equation (e.g., $3 \times 10 = 30$), as shown opposite. Use counters marked with the value '10' as a generalised representation. Explain to children that each counter represents a group of ten in the context (for example, one ten-value counter represents ten eggs in the example opposite). Ask children to describe what each number in the equation represents: • *'What does the "3" represent?'* *'The "3" represents the number of groups of eggs.'* • *'What does the "10" represent?'* • *'The "10" represents the number of eggs in a box/group.'* • *'What does the "30" represent?'* • *'The "30" represents how many eggs there are altogether.'* Remember, when describing a multiplication fact such as $3 \times 10 = 30$ use the language '*three times ten is equal to thirty.*' Avoid saying '*times by*' or '*multiplied by*'. For more on this, see	Example 1: *'How many eggs are there? Count in groups of ten.'*  • *'Ten, twenty, thirty. There are thirty eggs.'* • *'There are three groups of ten; there are thirty altogether.'* $3 \times 10 = 30$ • *'Three is a factor.'* • *'Ten is a factor.'* • *'The product of three and ten is thirty.'* • *'Thirty is the product of three and ten.'* Example 2: *'Show me four groups of ten.'* 

<table>
<tr>
<td></td>
<td>segment 2.2 Structures: multiplication representing equal groups, Overview of learning.

Also continue to use the language of factors and products to describe the multiplication equation:
'___ is a factor.'
'___ is a factor.'
'The product of ___ and ___ is ___.'
'___ is the product of ___ and ___.'
Work through several examples in this way, varying the representations used.</td>
<td>'How many is this altogether? Count in groups of ten.'
'Ten, twenty, thirty, forty. There are forty cubes.'
'There are four groups of ten; there are forty altogether.'
$4 \times 10 = 40$
'Four is a factor.'
'Ten is a factor.'
'The product of four and ten is forty.'
'Forty is the product of four and ten.'</td>
</tr>
<tr>
<td>1:3</td>
<td>Now explore 0 × 10. Use skip counting backwards in groups of ten, supported by the number line, starting with, say, three groups of ten. Write the multiplication equation to represent each multiple of ten.

You can use a contextual representation alongside the number line, such as the boxes of eggs shown opposite; remove one box at a time, until there are zero groups of ten eggs and, therefore, zero eggs.</td>
<td>

'Three tens, two tens, one ten, zero tens.'
'Thirty, twenty, ten, zero.'
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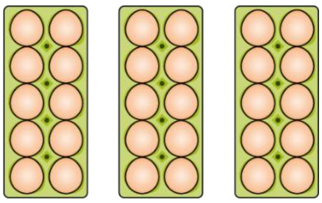
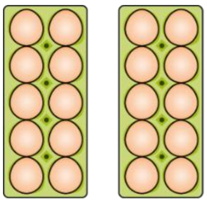
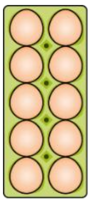
<tr><td>
$3 \times 10 = 30$</td><td>'Three tens…'</td><td>'Thirty…'</td></tr>
<tr><td>
$2 \times 10 = 20$</td><td>'two tens…'</td><td>'twenty…'</td></tr>
<tr><td>
$1 \times 10 = 10$</td><td>'one ten…'</td><td>'ten…'</td></tr>
<tr><td>$0 \times 10 = 0$</td><td>'zero tens.'</td><td>'zero.'</td></tr>
</table></td>
</tr>
</table>

1:4

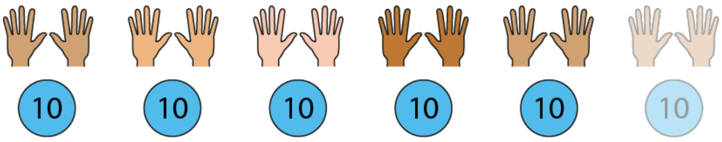
Now, using a familiar context alongside the ten-value counters, work systematically to build up the ten times table (to twelve tens) as you count in multiples of ten, beginning with zero tens. Use a ratio chart to record the number of groups and the product. As you complete the ratio chart, also write the multiplication equations. Children learnt in segment *2.3 Times tables: Groups of 2 and commutativity (part 1)*, that the factors in a multiplication equation can be written in any order, so write pairs of equations, describing them using the language introduced in segment *2.3, Teaching point 3*, for example:

- $3 \times 10 = 30$
  - *'Three groups of ten is equal to thirty.'*
  - *'Three times ten is equal to thirty.'*
- $10 \times 3 = 30$
  - *'Ten, three times is equal to thirty.'*
  - *'Ten times three is equal to thirty.'*

At each stage:

- encourage children to describe what each equation represents, for example:
  - *'There are three groups of ten fingers.'*
  - *'There are thirty fingers altogether.'*
  - *'The product of three and ten is thirty.'*
- then add another ten fingers, and work with children to complete the next column of the table, using their knowledge of what comes next in the skip-counting sequence.

Building up the ten times table:



$1 \times 10 = 10$	$10 \times 1 = 10$
$2 \times 10 = 20$	$10 \times 2 = 20$
$3 \times 10 = 30$	$10 \times 3 = 30$
$4 \times 10 = 40$	$10 \times 4 = 40$
$5 \times 10 = 50$	$10 \times 5 = 50$
$\mathbf{6 \times 10 = 60}$	$\mathbf{10 \times 6 = 60}$

Number of children	Number of fingers
0	0
1	10
2	20
3	30
4	40
5	50
6	**60**

1:5

Once the ratio chart and full set of equations are complete, ask children questions, encouraging them to use the table/equations for support, for example:

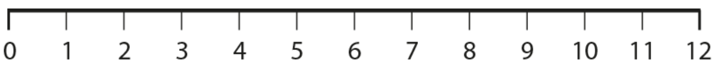
- *'If there are six children, how many fingers are there altogether?'*
- *'How many children are needed to show fifty fingers?'*
- *'When the product is seventy, what are the factors?'*
- *'Why are five times ten and ten times five both equal to fifty?'*

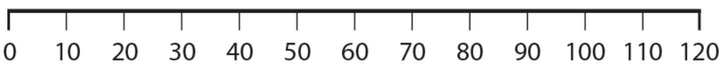
Complete ratio chart and ten times table:

Number of children	Number of fingers
0	0
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100
11	110
12	120

$0 \times 10 = 0$	$10 \times 0 = 0$
$1 \times 10 = 10$	$10 \times 1 = 10$
$2 \times 10 = 20$	$10 \times 2 = 20$
$3 \times 10 = 30$	$10 \times 3 = 30$
$4 \times 10 = 40$	$10 \times 4 = 40$
$5 \times 10 = 50$	$10 \times 5 = 50$
$6 \times 10 = 60$	$10 \times 6 = 60$
$7 \times 10 = 70$	$10 \times 7 = 70$
$8 \times 10 = 80$	$10 \times 8 = 80$
$9 \times 10 = 90$	$10 \times 9 = 90$
$10 \times 10 = 100$	$10 \times 10 = 100$
$11 \times 10 = 110$	$10 \times 11 = 110$
$12 \times 10 = 120$	$10 \times 12 = 120$

1:6 Now practise chanting the ten times table, with the written times table for support, using a variety of representations, including:





	• stacked number lines (as shown opposite) • the Gattegno chart • concrete representations • pictorial representations. Use the following language: • *'One group of ten is equal to ten.'* *'Two groups of ten is equal to twenty…'* • *'One times ten is equal to ten.'* *'Two times ten is equal to twenty…'* then shortening to *'One ten is ten, two tens are twenty…'* and • *'Ten, one time is equal to ten...'* *'Ten, two times is equal to twenty...'* • *'Ten times one is equal to ten...'* *'Ten times two is equal to twenty...'* Regular practice should be undertaken, including outside the main maths lesson, until children are fluent.	

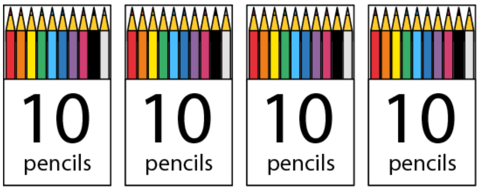
1:7	Provide practice, including:
	• completing/writing multiplication equations for contextual examples
	• missing-number sequences and problems
	• true/false style questions
	• word problems, for example:
	  ◦ *'What is the product of "8" and "10"?'*
	  ◦ *'I have eleven 10 p coins. How much is this altogether?'*
	  ◦ *'If there are five boxes of ten eggs, how many eggs are there altogether?'*
	  ◦ Dòng nǎo jīn: *'If there are ten eggs in each box, and fifty eggs altogether, how many boxes of eggs are there?'*
	At this stage, children can recite the ten times table up to the number they need to find the answers or use the multiplication chart for reference.	

Plenty of practice will be needed over an extended period until children are fluent in the isolated multiplication facts (for example, just knowing that seven tens is seventy, rather than having to recite the times table up to seven tens).

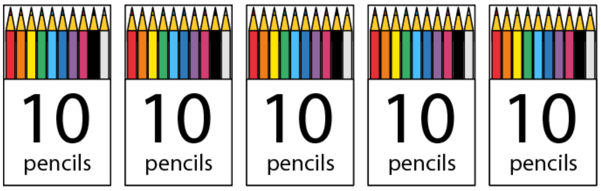
Children should write a multiplication equation for each problem, rather than simply writing the product.

Completing multiplication equations:

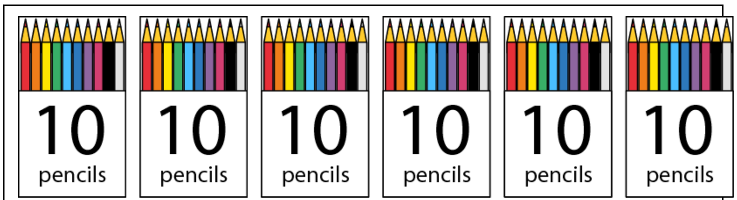
- *'For each picture, complete the equations to show how many pencils there are altogether.'*



$4 \times 10 = \square$ $\quad$ $10 \times 4 = \square$

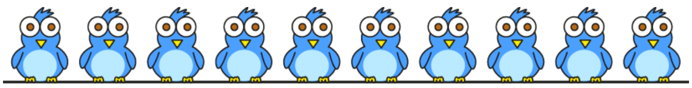


$\square \times 10 = \square$ $\quad$ $10 \times \square = \square$



$\square \times \square = \square$ $\quad$ $\square \times \square = \square$

- *'There are ten birds on each wire. Altogether there are 30 birds. How many wires are there?'*



$\square \times 10 = 30$

Missing-number sequences/problems:

'Fill in the missing numbers.'

0	10	20	30	40								

120	110	100										

1		☐	0		☐
3		☐	2		☐
5	$\times 10 =$	☐	4	$\times 10 =$	☐
7		☐	6		☐
9		☐	8		☐

$10 \times 1 =$ ☐ $10 \times 3 =$ ☐

$10 \times 5 =$ ☐ $10 \times 7 =$ ☐

$10 \times 9 =$ ☐

$10 \times 0 =$ ☐ $10 \times 2 =$ ☐

$10 \times 4 =$ ☐ $10 \times 6 =$ ☐

$10 \times 8 =$ ☐

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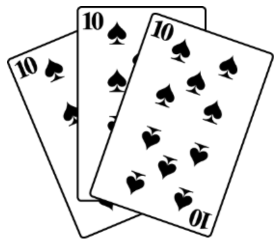
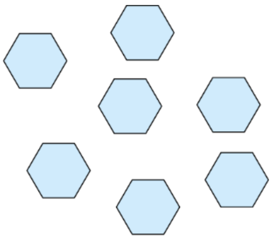
True/false style problems:
'Tick the examples that represent facts in the ten times table.'
<table>
<tr><td></td><td>10 × 9 = 90</td></tr>
<tr><td>4 + 10 = 40</td><td></td></tr>
<tr><td></td><td>4 × 10 = 40</td></tr>
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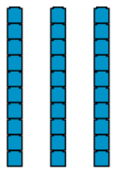
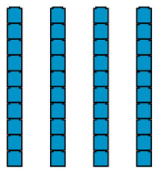
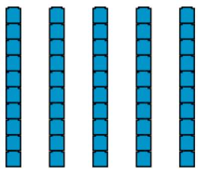
<td>1:8</td>
<td>
Now explore patterns in the ten times table. Show the multiplication chart and/or ratio chart and ask children what patterns they can see. Draw out the following:

The products all end with a zero.
Working down the list, the product increases by ten each time.

Then focus in on the fact that adjacent multiples of ten have a difference of ten, and that this knowledge can be used to find the next or previous multiple of ten from a given multiple. Use the language of groups, alongside Dienes ten rods, to describe the difference between each multiple, as shown opposite.

Present a ratio chart with some missing numbers, and work with children to use the adjacent multiples rule to fill in the missing numbers. You can use a number line with a
</td>
<td>
Using the language of groups:
<table>
<tr><th colspan="3">10 less ← multiple of 10 → 10 more</th></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>30</td><td>40</td><td>50</td></tr>
</table>

'Forty has one more group of ten than thirty.'
'Thirty has one fewer group of ten than forty.'

'Fifty has one more group of ten than forty.'
'Forty has one fewer group of ten than fifty.'

</td>
</tr>
</table>

rabbit/frog/kangaroo beginning at the known fact, then jumping ten to get to the missing fact.

Also use arrays, adding or removing (revealing or hiding) a row to move from the known multiple to the next/previous multiple. You can use a portion of a 12 × 12 chart as shown on the next page.

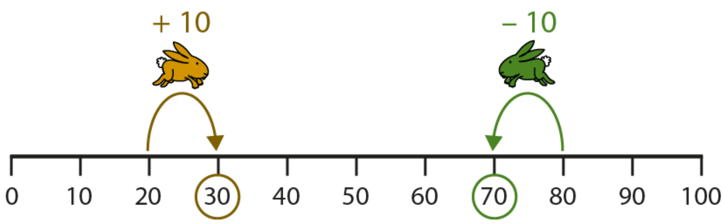
Throughout, write the corresponding equation to represent the operations, for example:

$3 \times 10 = 2 \times 10 + 10$

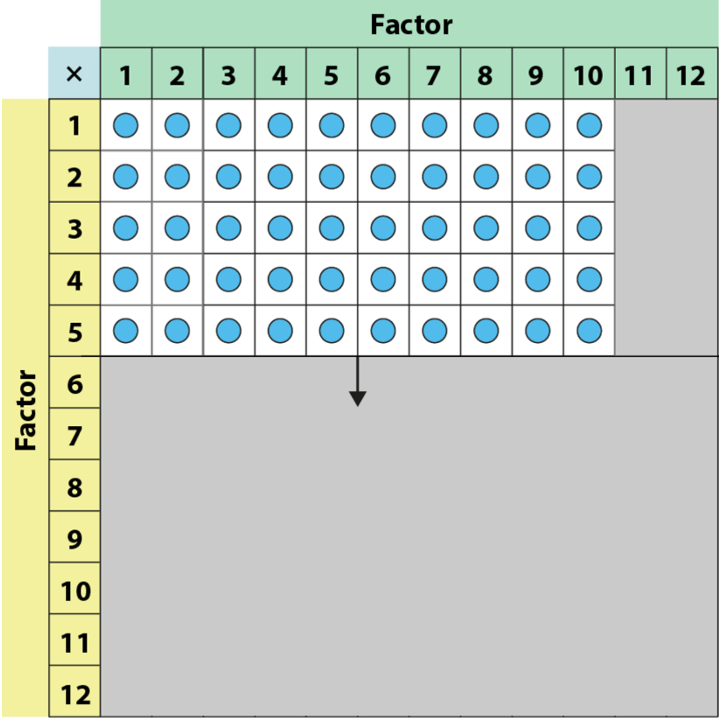
$7 \times 10 = 8 \times 10 - 10$

Finding adjacent multiples – ratio chart and number line:

	$\times$ **10**	
0	0	
1	10	
2	20	↓ + 10 $3 \times 10 = 2 \times 10 + 10$
3		
4	40	
5	50	
6	60	
7		↑ − 10 $7 \times 10 = 8 \times 10 - 10$
8	80	
9	90	
10	100	
11	110	
12	120	



Finding adjacent multiples – array chart:



$6 \times 10 = 5 \times 10 + 10$

1:9 Provide children with varied practice, applying what they have learnt throughout this teaching point, including:

- ratio charts with missing numbers (ask children to explain how they know what the missing number is using their knowledge of adjacent multiples of ten having a difference of ten)
- sequences of missing-number problems
- true/false style questions
- real-life problems, including measures contexts, for example:
  - *'If one toy car costs £10, how much do five cost?'*
  - *'If six children are each holding up all of their fingers, how many fingers are there altogether?'*
  - *'How many 10 p coins will I need to buy a cake that costs 80 p?'*

Missing-number problems:

- *'Fill in the missing numbers.'*

	× 10
0	
	10
2	20
	30
4	
5	
	60
7	
	80
	90
10	100
11	110
12	

$10 \times 10 = 100$

$9 \times 10 = \square$

$8 \times 10 = \square$

$7 \times \square = 70$

$6 \times \square = 60$

$5 \times \square = 50$

$\square \times 10 = 40$

$\square \times 10 = 30$

$\square \times 10 = 10$

- *'Maja has eight buckets. Each bucket contains ten litres of water.'*
  - *'How much water does Maja have altogether?'*
  - *'Maja uses ten litres of water to wash her Dad's car. How many buckets of water does she have now?'*
- *'Becca needs 90 p to buy a pen. She has eight 10 p coins. How much more money does she need to buy the pen?'*

$12 \times 10 = 120$ so $11 \times 10 = \square$

$10 \times 8 = 80$ so $10 \times 9 = \square$

$10 \times 6 = 60$ so $7 \times 10 = \square$

- *'Fill in the missing symbols (<, > or =).'*

$12 \times 10 \bigcirc 11 \times 10$

$12 \times 10 \bigcirc 11 \times 10 + 10$

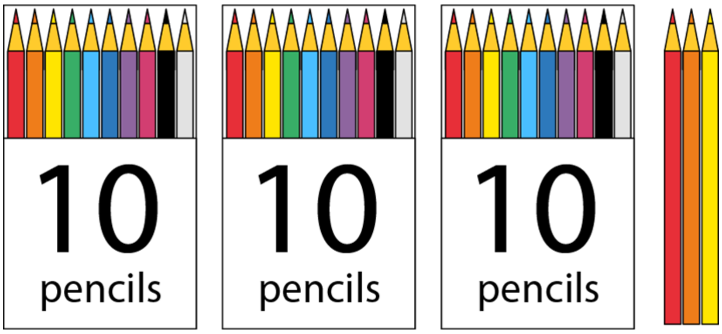
$12 \times 10 \bigcirc 12 \times 10 + 10$

$12 \times 10 \bigcirc 13 \times 10 - 10$

True or false question:

'These sets of numbers are products in the ten times table.'

	These are products in the ten times table. True (✓) or false (×)?
10, 50, 70, 90	
20, 10, 5, 0	
1, 10, 0, 111	
130, 140, 150, 160	

		Dòng nǎo jīn:  *'Which expression(s) match this picture? Explain.'* 4×10 4×3 $10 \times 3 + 3$ $3 \times 10 + 3$

Teaching point 2:

Counting in multiples of five can be represented by the five times table. Adjacent multiples of five have a difference of five. Facts from the five times table can be used to solve problems about groups of five.

Steps in learning

2:1

Guidance

This teaching point follows the same progression as *Teaching point 1*, but for groups of five/the five times table. Use similar representations and contextual examples as you did for the ten times table, to support children with making connections and seeing patterns. This will pave the way for a more detailed look at the relationship between the five times table and ten times table in *Teaching point 3*, and in segment *2.5 Commutativity (part 2), doubling and halving*.

Since this teaching point mirrors *Teaching point 1*, the steps here just include a brief summary and any key differences, with a selection of representations. Refer back to *Teaching point 1* for more detail.

Begin by reviewing skip-counting in fives (introduced in segment *2.1 Counting, unitising and coins*), and extend up to twelve fives, supported by:

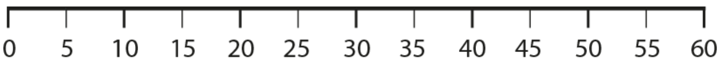
- ordinal representations (number line, Gattegno chart)
- groups of five objects where the cardinality is clear (using similar representations as in *Teaching point 1*); you can also use a 'tally' arrangement, to link to and support work on statistics
- examples where the cardinality isn't clearly visible (for example, 5 p coins).

Representations

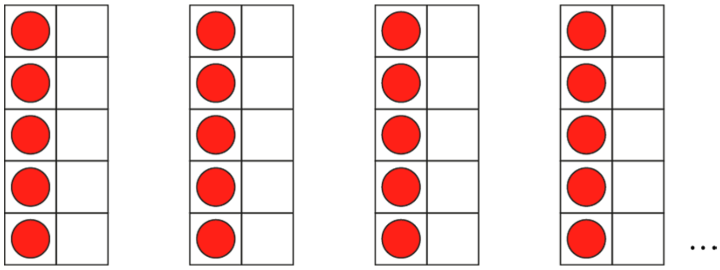
Counting in three ways:

- *'One group of five, two groups of five, three groups of five…'*
- *'One five, two fives, three fives…'*
- *'Five, ten, fifteen…'*

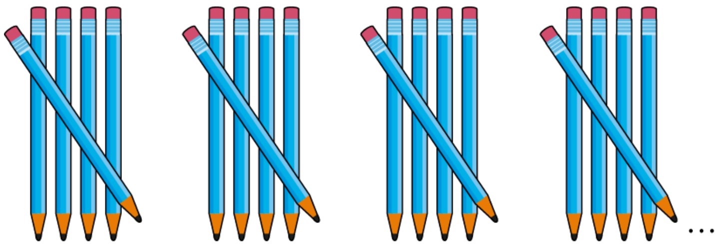
Skip counting in fives – number line:



Skip counting with groups of five objects – cardinality visible:

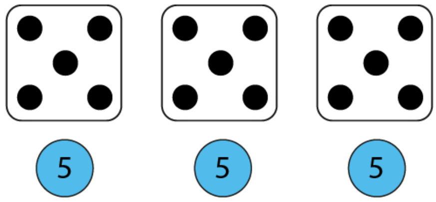


Skip counting groups of five – tally arrangement:



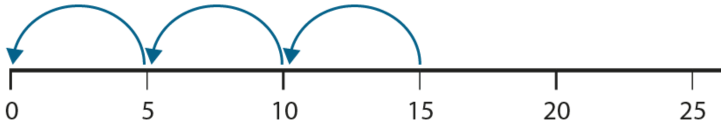
Skip counting in fives – five-pence coins:

	When using the Gattegno chart the tapping pattern will differ from that used in step *1:1*:
	• one tap each for the first two multiples (five and ten)
	• then
	  • two taps for each *odd* two-digit multiple of five (15, 25…)
	  and
	  • one tap for each *even* two-digit multiple of five (20, 30…).	

2:2	Now connect evaluating the number of objects using skip counting (for example, three dice showing five dots is five, ten, *fifteen* dots) with representing the grouped objects with a multiplication equation (e.g., $3 \times 5 = 15$), as shown opposite. Begin to use counters marked with the value '5' as a generalised representation to support unitising. When working with hands (*Example 2* opposite), some children may notice the connection between six groups of five and three groups of ten ($6 \times 5 = 30 = 3 \times 10$). If so, ask them to explain how many groups of five they see and how many groups of ten. This relationship is explored further in *Teaching point 3*.	Example 1: *'How many dots are there? Count in groups of five.'*  • *'Five, ten, fifteen. There are fifteen dots.'* • *'There are three groups of five; there are fifteen altogether.'* $3 \times 5 = 15$ • *'Three is a factor.'* • *'Five is a factor.'* • *'The product of three and five is fifteen.'* • *'Fifteen is the product of three and five.'* Example 2: *'Show me six groups of five.'*  *'How many is this altogether? Count in groups of five.'* • *'Five, ten, fifteen…thirty. There are thirty fingers.'* • *'There are six groups of five; there are thirty altogether.'* $6 \times 5 = 30$

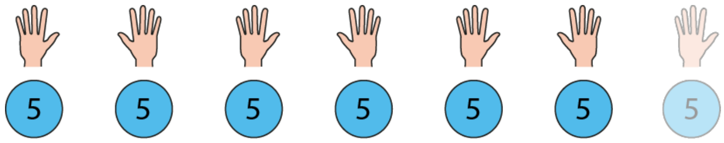
- *'Six is a factor.'*
- *'Five is a factor.'*
- *'The product of six and five is thirty.'*
- *'Thirty is the product of six and five.'*

2:3 Briefly explore 0×5, using skip counting backwards in groups of five, supported by the number line.



2:4 Now work systematically to build up the five times table as you count in multiples of five from zero to sixty. Record using a ratio chart and multiplication equations. The representations and language opposite exemplify adding the seventh group of five.

Building up the five times table:



$0 \times 5 = 0$	$5 \times 0 = 0$
$1 \times 5 = 5$	$5 \times 1 = 5$
$2 \times 5 = 10$	$5 \times 2 = 10$
$3 \times 5 = 15$	$5 \times 3 = 15$
$4 \times 5 = 20$	$5 \times 4 = 20$
$5 \times 5 = 25$	$5 \times 5 = 25$
$6 \times 5 = 30$	$5 \times 6 = 30$
$\mathbf{7 \times 5 = 35}$	$\mathbf{5 \times 7 = 35}$

- *'Seven groups of five is equal to thirty-five.'*
- *'Seven times five is equal to thirty-five.'*
- *'Five, seven times is equal to thirty-five.'*
- *'Five times seven is equal to thirty-five.'*

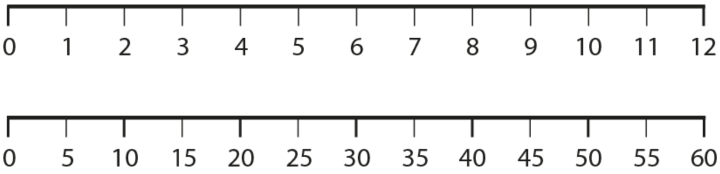
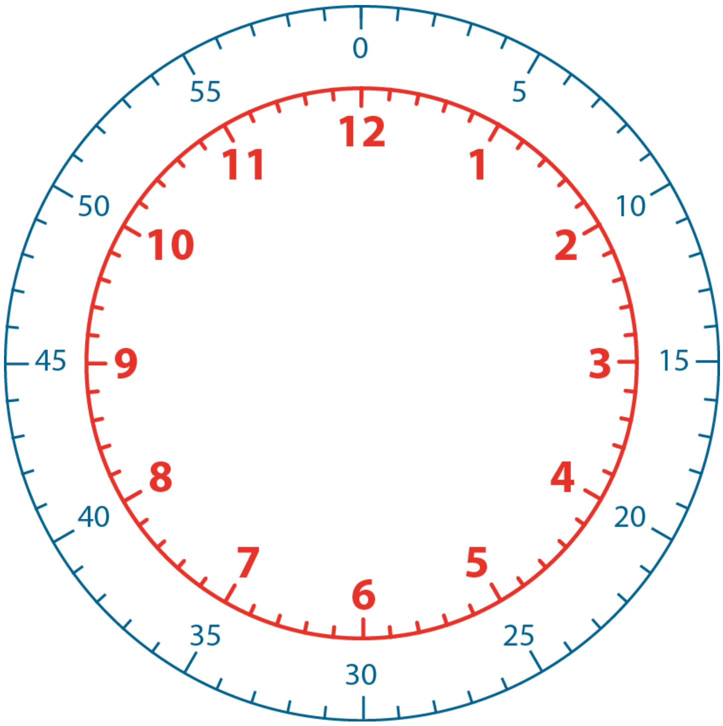
- *'There are seven groups of five fingers.'*
- *'There are thirty-five fingers altogether.'*
- *'The product of seven and five is thirty-five.'*

Number of hands	0	1	2	3	4	5	6	**7**
Number of fingers	0	5	10	15	20	25	30	**35**

2:5	Ask children questions, using the ratio chart/equations for support, for example: • *'If there are nine hands, how many fingers are there altogether?'* • *'How many hands are needed to show twenty-five fingers?'* • *'When the product is fifty, what are the factors?'* • *'Why are five times ten and ten times five both equal to fifty?'*	Complete ratio chart and five times table:

Number of hands	Number of fingers
0	0
1	5
2	10
3	15
4	20
5	25
6	30
7	35
8	40
9	45
10	50
11	55
12	60

$0 \times 5 = 0$	$5 \times 0 = 0$
$1 \times 5 = 5$	$5 \times 1 = 5$
$2 \times 5 = 10$	$5 \times 2 = 10$
$3 \times 5 = 15$	$5 \times 3 = 15$
$4 \times 5 = 20$	$5 \times 4 = 20$
$5 \times 5 = 25$	$5 \times 5 = 25$
$6 \times 5 = 30$	$5 \times 6 = 30$
$7 \times 5 = 35$	$5 \times 7 = 35$
$8 \times 5 = 40$	$5 \times 8 = 40$
$9 \times 5 = 45$	$5 \times 9 = 45$
$10 \times 5 = 50$	$5 \times 10 = 50$
$11 \times 5 = 55$	$5 \times 11 = 55$
$12 \times 5 = 60$	$5 \times 12 = 60$

2:6	Now practise chanting the five times table, with the written times table for support, using a variety of representations, including: • stacked number lines (as shown opposite) • the Gattegno chart • concrete representations • pictorial representations. You could use the context of a clock and show how a double number line around it (as shown opposite) helps us to tell the time; practise telling the time, counting in fives to identify minutes past/to the hour for a variety of different times.	Stacked number lines:  • *'One group of five is equal to five.'* *'Two groups of five is equal to ten…'* • *'One times five is equal to five.'* *'Two times five is equal to ten…'* and • *'Five, one time is equal to five...'* *'Five, two times is equal to ten...'* • *'Five times one is equal to five...'* *'Five times two is equal to ten...'* Clock with double number line:  *'If the minute hand is pointing at "3", how many minutes past the hour is it? Count in fives.'*

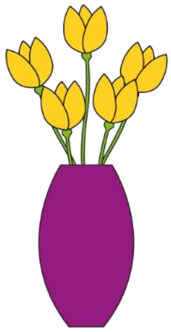
2:7

Provide practice, including:

- completing/writing multiplication equations for contextual examples
- missing-number sequences and problems (note that separating the odd and even multiples begins to draw children's attention to patterns, and the connection with the ten times tables; this will be explored in more detail in *Teaching point 3*).
- true/false style questions
- word problems, for example:
  - *'What is the product of "11" and "5"?'*
  - *'I have seven 5 p coins. How much is this altogether?'*
  - *'I rolled two dice, and they both showed five. What did I roll altogether?'*
  - *'There are four bunches of five balloons, and two extra balloons. How many balloons are there altogether?'*
  - Dòng nǎo jīn: *'I have invited sixty people to a party. If five people can sit at each table, how many tables do I need?'*

Completing multiplication equations:

'In each vase there are five flowers.'



$1 \times 5 = 5$ $\qquad$ $5 \times 1 = 5$

- *'How many flowers are there in four vases?'*

$4 \times 5 = \square$ $\qquad$ $5 \times 4 = \square$

- *'How many flowers are there in eight vases?'*

$\square \times 5 = \square$ $\qquad$ $5 \times \square = \square$

- *'How many flowers are there in twelve vases?'*

$\square \times \square = \square$ $\qquad$ $\square \times \square = \square$

- *'How many vases are needed to hold 30 flowers?'*

$\square \times 5 = 30$ $\qquad$ $5 \times \square = 30$

Missing-number sequences/problems:

'Fill in the missing numbers.'

0	5	10	15	20								

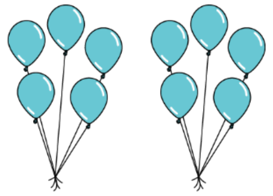
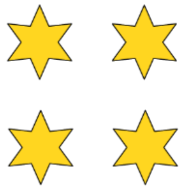
60	55	50										

1		
3		
5	× 5 =	
7		
9		

0		
2		
4	× 5 =	
6		
8		

True/false style problems:

'Tick the examples that represent facts in the five times table.'

 ☐	$5 \times 9 = 45$ ☐
$9 + 5 = 45$ ☐	 ☐
 ☐	$9 \times 5 = 45$ ☐

2:8	Now ask children what patterns they can see in the five times table, prompting for the following:
	• The products end with either a zero or a five (and these alternate).
	• There are patterns in the tens digits and the ones digits:
	  ◦ ones digit: *0, 5, 0, 5, 0, 5, 0, 5...*
	  ◦ tens digit: *0, 0, 1, 1, 2, 2, 3, 3...*
	• The product of five and an odd number ends with five.	

- The product of five and an even number ends with zero.
- Working down the list, the product increases by five each time.

Focus in on the fact that adjacent multiples of five have a difference of five, and that this knowledge can be used to find the next or previous multiple of five from a given multiple. Use the same representations as in step *1:8* and, throughout, write the corresponding equations, for example:

$3 \times 5 = 2 \times 5 + 5$

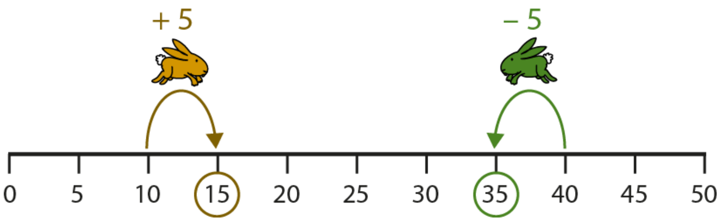
$7 \times 5 = 8 \times 5 - 5$

Finding adjacent multiples – ratio chart and number line:

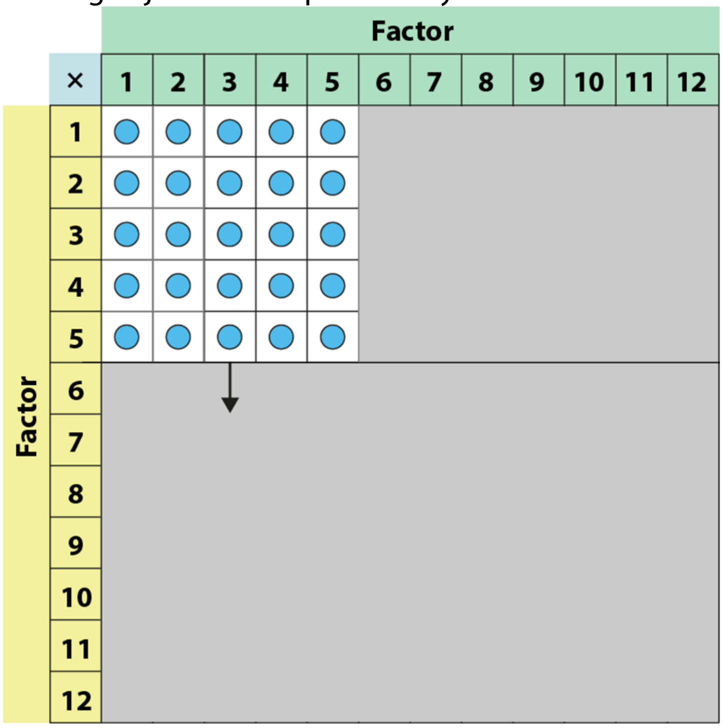
	× 5
0	0
1	5
2	10
3	
4	20
5	25
6	30
7	
8	40
9	45
10	50
11	55
12	60

↓ + 5 $3 \times 5 = 2 \times 5 + 5$

↑ − 5 $7 \times 5 = 8 \times 5 - 5$



Finding adjacent multiples – array chart:



$6 \times 5 = 5 \times 5 + 5$

2:9

Provide similar practice to that in step *1:9.*

Example word problems:

- *'Five children can fit around each table at a party. There are four full tables and five children standing. How many children are there altogether?'*
- Dòng nǎo jīn:

  'Lily added one to all the products in the five times table to make a list of numbers.'
  - *'What patterns are there in Lily's list of numbers?'*
  - *'What is the difference between two adjacent numbers in Lily's list?'*

Missing-number/symbol problems:

- *'Fill in the missing numbers.'*

	× 5
0	
	5
2	10
	15
4	
5	
	30
7	
	40
	45
10	50
11	55
12	

$10 \times 5 = 50$

$9 \times 5 = \square$

$8 \times 5 = \square$

$7 \times \square = 35$

$6 \times \square = 30$

$5 \times \square = 25$

$\square \times 5 = 20$

$\square \times 5 = 15$

$\square \times 5 = 10$

$5 \times 10 = 50$ so $5 \times 9 = \square$

$14 \times 5 = 70$ so $13 \times 5 = \square$

$17 \times 5 = 85$ so $18 \times 5 = \square$

$5 \times 4 = 5 \times 3 + \square$

$5 \times 4 - 5 = \square \times \square$

- *'Fill in the missing symbols (<, > or =).'*

  $14 \times 5 \bigcirc 15 \times 5$

  $14 \times 5 \bigcirc 15 \times 5 - 5$

  $14 \times 5 \bigcirc 14 \times 5 + 5$

  $14 \times 5 \bigcirc 13 \times 5 + 5$

True or false question:

'These sets of numbers are products in the five times table.'

	These are products in the ten times table. True (✓) or false (×)?
25, 60, 55, 10, 5	
30, 20, 102, 10, 80, 0, 40	
55, 65, 15, 35, 59	
80, 85, 58, 90, 95, 59	

Teaching point 3:

Skip counting and grouping can be used to explore the relationship between the five times table and the ten times table.

Steps in learning

3:1

In this teaching point, children begin to make connections between the five times table and the ten times table. At this stage, exploration will be in terms of 'double skip counting' (step *3.1*), comparing a given quantity grouped either into fives or tens (steps *3.2–3.3*), and recognising multiplication equations that represent common multiples (step *3.4*). In segment *2.5 Commutativity (part 2), doubling and halving* the relationship will be explored further.

Begin with skip counting. First practise counting forwards from zero in multiples of five, then in multiples of ten. Use representations such as:

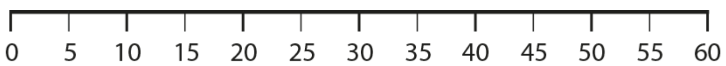
- a number line with both multiples of five (and therefore, multiples of ten) labelled
- the Gattegno chart
- a hundred square with multiples of five (and therefore, multiples of ten) highlighted.

Then split the class in half, with one half counting in multiples of five and the other half in multiples of ten, up to 60. The group counting in fives should count on every 'beat', while the group counting in tens will count on every other 'beat', such that both groups say the multiples of ten at the same time.

Then ask children what they notice, prompting for the following:

- All of the numbers said by the 'tens' group' are also said by the 'fives group'.
- Not all of the numbers said by the 'fives group' are also said by the 'tens group'.
- For every number said by the 'tens group', the 'fives group' says two numbers.
- After discussion, double count again, recording the pattern in a table as shown below.

'Double skip counting' – number line:



Gattegno chart:

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

'Double skip counting' – hundred square:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Comparing counting in multiples of five and ten:

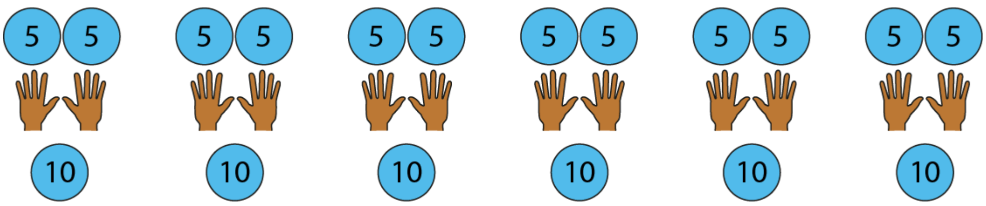
Number	**0**	1	2	3	4	**5**	6	7	8	9	**10**	11	12	13	14	**15**	16	17	18	19	**20**	21	22	23	24	**25**
Counting in 5s	✓					✓					✓					✓					✓					✓
Counting in 10s	✓										✓										✓					

3:2 Now discuss the relationship between the number of fives and the number of tens in terms of *groups* of five or *groups* of ten, using hands (pictorial representation). Show one pair of hands, and ask:

- *'How many groups of five are there?'*
- *'How many groups of ten are there?'*

Use counters to represent one group of ten and two groups of five. Then add another pair of hands, ask the questions again and add the next set of counters. Continue until there are about six pairs of hands (as shown below).

Work towards the generalisation: ***'For every one group of ten, there are two groups of five.'***

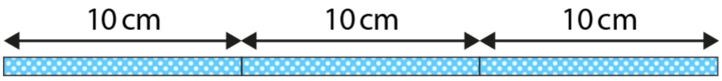


3:3

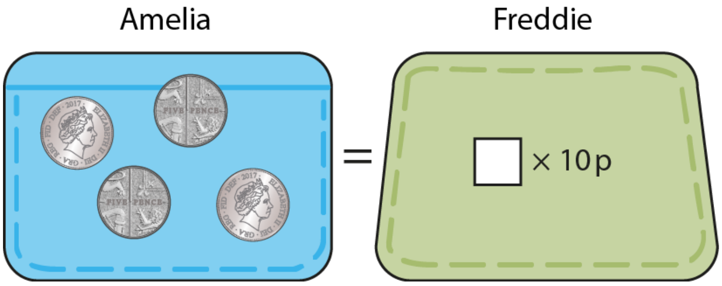
Before moving on to comparing multiplication equations, provide children with practice comparing groups of five and groups of ten, as shown opposite.

Also give children practice working with manipulatives. For example, provide each child with 30 counters and ask them to arrange the counters first into groups of five, then into groups of ten; encourage children to combine pairs of groups of five to make groups of ten, rather than starting again. Similarly, start with, say, 40 counters and ask children to arrange the counters first into groups of ten then into groups of five; encourage children to split up their existing groups into fives, rather than starting again.

- *'There are three 10 cm lengths of ribbon? How many 5 cm lengths can be made?'*



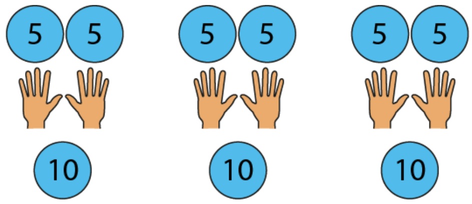
- *'Amelia has four five-pence coins. Freddie has the same amount of money, in ten-pence coins. How many ten-pence coins does Freddie have?'*



3:4

Now look at a given number of pairs of hands. Ask children to write a multiplication equation to describe the groups of five and another to describe the groups of ten. Draw attention to the fact that one equation is in the five times table and one is in the ten times table, but both have the same product.

Repeat for a few examples, before showing both times-table charts side by side so that children can compare the products and look for patterns. Highlight or circle equations with the product equal to a multiple of ten, working toward the generalisation: ***'Products in the ten times table are also in the five times table.'***



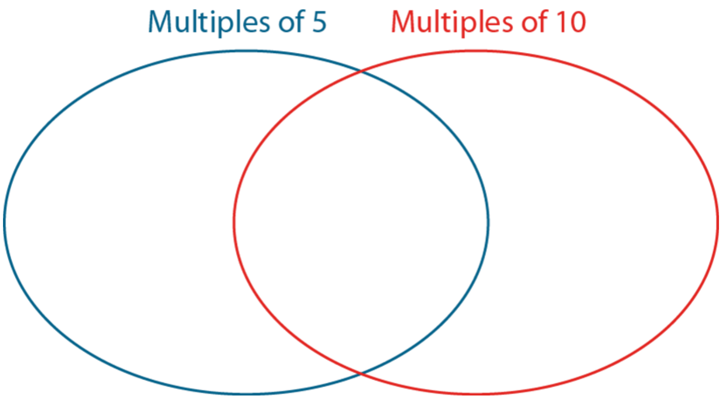
$6 \times 5 = 30$

$3 \times 10 = 30$

Comparing the five and ten times tables:

$0 \times 5 = 0$	$0 \times 10 = 0$
$1 \times 5 = 5$	$1 \times 10 = 10$
$2 \times 5 = 10$	$2 \times 10 = 20$
$3 \times 5 = 15$	$3 \times 10 = 30$
$4 \times 5 = 20$	$4 \times 10 = 40$
$5 \times 5 = 25$	$5 \times 10 = 50$
$6 \times 5 = 30$	$6 \times 10 = 60$
$7 \times 5 = 35$	$7 \times 10 = 70$
$8 \times 5 = 40$	$8 \times 10 = 80$
$9 \times 5 = 45$	$9 \times 10 = 90$
$10 \times 5 = 50$	$10 \times 10 = 100$
$11 \times 5 = 55$	$11 \times 10 = 110$
$12 \times 5 = 60$	$12 \times 10 = 120$

<table>
<tr>
<td>3:5</td>
<td>As a class, sort some numbers into a Venn diagram as shown opposite. Once the numbers are sorted, ask questions to draw children's attention to the patterns and connections:

'Which section does not have any numbers in it?'<br>'Why?'
'What do you notice about the numbers in the section where the two sets overlap?'
'What do you notice about the numbers that don't go inside the circles?'
</td>
<td>'Place these numbers in the diagram. In the overlapping section, you should place the numbers that are both multiples of 5 and 10. Numbers that are neither multiples of 5 nor 10 should go outside the circles.'
<table><tr><td>0</td><td>5</td><td>10</td><td>27</td><td>35</td><td>31</td><td>40</td><td>15</td><td>100</td></tr></table>

</td>
</tr>
<tr>
<td>3:6</td>
<td>Children have already noticed the following patterns in the five times table (step 2:8):

Even multiples of five have a ones digit of zero.
Odd multiples of five have a ones digit of five.

Use intelligent practice to draw further attention to the link between even multiples of five and multiples of ten. Work towards the generalisation: 'Even multiples of five are also multiples of ten.'</td>
<td>
<table>
<tr><td>0</td><td></td><td>0</td><td></td><td>0</td></tr>
<tr><td>2</td><td></td><td>10</td><td></td><td>1</td></tr>
<tr><td>4</td><td></td><td>☐</td><td></td><td>2</td></tr>
<tr><td>6</td><td>$\times 5 =$</td><td>☐</td><td>$= 10 \times$</td><td>3</td></tr>
<tr><td>8</td><td></td><td>☐</td><td></td><td>4</td></tr>
<tr><td>10</td><td></td><td>☐</td><td></td><td>5</td></tr>
<tr><td>12</td><td></td><td>☐</td><td></td><td>6</td></tr>
</table></td>
</tr>
<tr>
<td>3:7</td>
<td>Complete this teaching point by providing children with practice, including:

missing-number problems
contextual problems, for example:
'Charlotte has eight sheets of stickers; each sheet has ten stickers.'
'Eloise has four sheets of stickers; each sheet has five stickers.'
'True or false: Charlotte and Eloise have the same number of stickers.'
</td>
<td>'Fill in the missing numbers.'
$2 \times 5 = 1 \times 10$
$4 \times 5 = \square \times 10$
$6 \times 5 = \square \times 10$
$8 \times 5 = \square \times 10$
$10 \times 5 = \square \times 10$</td>
</tr>
</table>

	To promote and assess depth of understanding, present children with the following dòng nǎo jīn problem: ‘*Rishi says that all multiples of five are multiples of ten. Is he correct? Explain.*’	$1 \times 10 = 2 \times 5$ $3 \times 10 = \square \times 5$ $4 \times 10 = \square \times 5$ $5 \times 10 = \square \times 5$

Teaching point 4:

When zero is a factor, the product is zero. When one is a factor, the product is equal to the other factor (if there are only two factors).

Steps in learning

4:1

In the context of the two, five and ten times tables, children have seen that when one is a factor (one group of two, five or ten), then the product is equal to the group size. They have also seen, through backward skip counting in twos, fives or tens, that zero 'groups' is equal to zero. Note that the term 'groups' becomes abstract in the case when one of the factors is zero; children also often have difficulty seeing that, for example, five individual objects can also be seen as five 'groups' of one (this will be explored in segment *2.5 Commutativity (part 2), doubling and halving*). As such, at this stage, children will work with non-contextualised multiplication equations, using pattern spotting to generalise about cases when zero or one is a factor, irrespective of factor order or whether we identify the zero/one as the number of 'groups' or the size of the 'groups'.

Begin by presenting the first few multiplication equations of each familiar times table, for comparison. Highlight the equations with zero as a factor. Encouraging children to use the language of 'factor' and 'product' ask:

- *'What's the same?'*
  (one of the factors is zero; the other factor is not zero; the product is zero)
- *'What's different?'*
  (the other (non-zero) factor varies)

Then, based on the highlighted equations, ask questions of the form:

- *'What is the product of ___ and zero?'*
- *'What is the product of zero and ___?'*

Encourage children to answer with the stem sentences:

- ***'The product of __ and zero is zero.'***
- ***'The product of zero and ___ is zero.'***

As the pattern emerges, use the following generalised statement: ***'When zero is a factor, the product is zero.'***

Comparing multiplications equations when zero is a factor:

Two times table		Five times table		Ten times table	
$0 \times 2 = 0$	**$2 \times 0 = 0$**	**$0 \times 5 = 0$**	**$5 \times 0 = 0$**	**$0 \times 10 = 0$**	**$0 \times 10 = 0$**
$1 \times 2 = 2$	$2 \times 1 = 2$	$1 \times 5 = 5$	$5 \times 1 = 5$	$1 \times 10 = 10$	$1 \times 10 = 10$
$2 \times 2 = 4$	$2 \times 2 = 4$	$2 \times 5 = 10$	$5 \times 2 = 10$	$2 \times 10 = 20$	$2 \times 10 = 20$

<table>
<tr>
<td>4:2</td>
<td>Now, as a class, complete a ‘zero times table’, written both with zero as the first factor, and as the second factor.</td>
<td>
<table>
<tr><td>$0 \times 0 = 0$
$1 \times 0 = 0$
$2 \times 0 = 0$
$3 \times 0 = 0$
$4 \times 0 = 0$
$5 \times 0 = 0$
$6 \times 0 = 0$
$7 \times 0 = 0$
$8 \times 0 = 0$
$9 \times 0 = 0$
$10 \times 0 = 0$
$11 \times 0 = 0$
$12 \times 0 = 0$</td>
<td>$0 \times 0 = 0$
$0 \times 1 = 0$
$0 \times 2 = 0$
$0 \times 3 = 0$
$0 \times 4 = 0$
$0 \times 5 = 0$
$0 \times 6 = 0$
$0 \times 7 = 0$
$0 \times 8 = 0$
$0 \times 9 = 0$
$0 \times 10 = 0$
$0 \times 11 = 0$
$0 \times 12 = 0$</td></tr>
</table>
</td>
</tr>
<tr>
<td>4:3</td>
<td>Provide children with some practice for cases when zero is a factor, including missing-number/symbol problems, extending beyond 12 as the other factor.</td>
<td>Missing-number/symbol problems:
• ‘Fill in the missing numbers.’
$0 \times 0 = \square$ $10 \times 0 = \square$
$2 \times 0 = \square$ $12 \times 0 = \square$
$4 \times 0 = \square$ $14 \times 0 = \square$
$6 \times \square = 0$ $16 \times \square = 0$
$8 \times \square = 0$ $18 \times \square = 0$

$0 \times 10 = \square$ $\square \times 1 = 0$
$0 \times 30 = \square$ $\square \times 10 = 0$
$0 \times 50 = \square$ $\square \times 100 = 0$</td>
</tr>
</table>

- *'Fill in the missing symbols (<, > or =).'*

0×5 ◯ 0×6

0×6 ◯ 0×6

0×6 ◯ $0 \times 5 + 0$

0×7 ◯ 0×6

4:4

Now, follow the same process as in steps *4:1* to *4:3* for cases where one is factor. Compare equations in the two, five and ten times table, asking:

- *'What's the same?'*
  (one of the factors is one; the other factor is *not* one)
- *'What's different?'*
  (the other factor varies; the product varies)
- *'What do you notice about the other factor and the product?'*
  (the product is equal to the other factor)

Then, using the equations for support, ask questions of the form:

- *'What is the product of ___ and one?'*
- *'What is the product of one and ___?'*

Encourage children to answer with the stem sentences:

- ***'The product of ___ and one is ___.'***
- ***'The product of one and ___ is ___.'***

As the pattern emerges, use the following generalised statement: ***'When one is a factor, the product is equal to the other factor.'***

Note that this is, of course, only true when there are two factors (for example, $2 \times 1 = 2$ and $3 \times 1 = 3$, but $2 \times 3 \times 1 = 6$). At this stage, however, we are working in the context of the times tables, and children only have experience of multiplication equations with two factors; as such the generalised sentence has been kept simple, with language that implies only two factors.

Comparing multiplications equations when one is a factor:

Two times table		**Five times table**		**Ten times table**	
$0 \times 2 = 0$	$2 \times 0 = 0$	$0 \times 5 = 0$	$5 \times 0 = 0$	$0 \times 10 = 0$	$0 \times 10 = 0$
$\mathbf{1 \times 2 = 2}$	$\mathbf{2 \times 1 = 2}$	$\mathbf{1 \times 5 = 5}$	$\mathbf{5 \times 1 = 5}$	$\mathbf{1 \times 10 = 10}$	$\mathbf{1 \times 10 = 10}$
$2 \times 2 = 4$	$2 \times 2 = 4$	$2 \times 5 = 10$	$5 \times 2 = 10$	$2 \times 10 = 20$	$2 \times 10 = 20$

4:5	Now, as a class, complete a 'one times table'.	$0 \times 1 = 0$ $1 \times 1 = 1$ $2 \times 1 = 2$ $3 \times 1 = 3$ $4 \times 1 = 4$ $5 \times 1 = 5$ $6 \times 1 = 6$ $7 \times 1 = 7$ $8 \times 1 = 8$ $9 \times 1 = 9$ $10 \times 1 = 10$ $11 \times 1 = 11$ $12 \times 1 = 12$ $1 \times 0 = 0$ $1 \times 1 = 1$ $1 \times 2 = 2$ $1 \times 3 = 3$ $1 \times 4 = 4$ $1 \times 5 = 5$ $1 \times 6 = 6$ $1 \times 7 = 7$ $1 \times 8 = 8$ $1 \times 9 = 9$ $1 \times 10 = 10$ $1 \times 11 = 11$ $1 \times 12 = 12$
4:6	Provide children with some practice for cases when one is a factor, including missing-number/symbol problems.	Missing-number/symbol problems: • *'Fill in the missing numbers.'* $0 \times 1 = \square$ $2 \times 1 = \square$ $4 \times 1 = \square$ $6 \times \square = 6$ $8 \times \square = 8$ $10 \times 1 = \square$ $12 \times 1 = \square$ $14 \times 1 = \square$ $16 \times \square = 16$ $18 \times \square = 18$ $1 \times 10 = \square$ $1 \times 30 = \square$ $1 \times 50 = \square$ $\square \times 1 = 1$ $\square \times 10 = 10$ $\square \times 100 = 100$

- *'Fill in the missing symbols (<, > or =).'*

1×5 ◯ 6×1

1×6 ◯ 6×1

1×6 ◯ $1 \times 5 + 1$

1×7 ◯ $1 \times 6 + 6$

Dòng nǎo jīn:

'Fill in the missing numbers.'

	factor	**factor**	**product**
factor			0
factor			5
product	0	10	