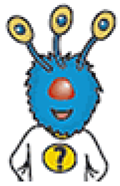
Primary
& Early Years
Magazine 79



National Centre
for Excellence in the
Teaching of Mathematics

Welcome to another issue of our Primary Magazine. This magazine has been serving primary teachers for 78 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are accessible through the Primary Magazine Archive.

Contents

In each issue we have a selection of interesting and useful articles. New National Curriculum in Focus is dedicated to unpicking the new curriculum, which most schools have been following for a year now. In this edition we look in some depth at the recently-published NCETM materials to help teachers assess their pupils' mastery of the curriculum.

This month, Where's the Maths in That? looks at the concept of conjecture - a thinking and discussion skill not confined to maths classrooms. In particular we consider how the imaginary figure of Captain Conjecture, who appears in the new assessment materials, can help children flex their conjecturing skills in maths lessons.

Finally, Maths in the Staff Room provides simple plans for CPD meetings in your school to be led by a member of staff. These are short meetings that can be used exactly as indicated or adapted to meet the CPD needs of the school. This month we offer an idea for a meeting centred on developing reasoning skills in the maths classroom.

But first, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

News

NCETM assessment materials downloads hits 68 000
As this issue went to press, the number of downloads of our assessment materials, published at the end of July, hit 68 000. The materials, written by an expert group including primary teachers working through the Maths Hubs programme, and produced in collaboration with Oxford University Press (OUP), aim to help teachers assess the degree to which pupils have acquired mastery of mathematical concepts. They consist of specially constructed questions, tasks and activities aligned with key topics of the National Curriculum.

Maths textbook trials in primary schools continue
The project exploring how using high quality textbooks can help teachers and pupils in KS1 maths lessons is being broadened this year. The number of schools participating in the trial, as part of the Maths Hubs programme, is now well over 100. If you're interested in learning more from a school in your area, contact your local Maths Hub.

Regular round-up of Twitter-based maths education news
If your appetite for maths education news extends beyond what we provide in this magazine, and in our monthly national newsletter, you might like to subscribe to Maths education, a neatly packaged selection of Twitter-based primary and secondary news, from maths lecturer Steve Bishop (@SteveBishop100). It's free, by the way.

The psychology of learning maths
You can't fail to have heard of the concept of mindset and how most people are now convinced that a 'growth mindset' can help learning. This was the topic of a recent episode in BBC Radio Four's series Mind Changers, which starts in a primary mathematics lesson in an English school. Well worth a listen!

New National Curriculum in Focus

New National Curriculum in Focus *is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series* *here*.

National Curriculum Assessment Materials

In July, the NCETM produced a suite of materials to support the assessment of the National Curriculum for mathematics. The materials have been designed to support teachers with assessment of mathematics with a particular focus on mastery of the curriculum. The introduction is worth reading and provides a good introduction to the materials and the concept of mastery. There is a downloadable booklet for each year from Y1 to Y6. Each booklet is structured in an identical way and provides examples of questions and activities that address key areas of learning in each year group.

The materials have been written with the intention of supporting teachers to make sense of what mastery means in the context of assessing mathematics and also what it means to evidence greater depth of understanding. Within a mastery curriculum, pupils who grasp concepts quickly are not accelerated into new content, but instead have the opportunity to develop greater depth.

What's included?

Here is a typical page, in this case from the Y3 booklet addressing the strand Addition and Subtraction:

Addition and Subtraction

Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- add and subtract numbers mentally, including:
  - a 3-digit number and ones
  - a 3-digit number and tens
  - a 3-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction

The Big Ideas

Relating numbers to 5 and 10 helps develop knowledge of the number bonds within 20. For example, given 8 + 7, thinking of 7 as 2 + 5, and adding the 2 and 8 to make 10, then the 5 to 15. This should then be applied when calculating with larger numbers.

Subtraction bonds can be thought of in terms of addition: for example, in answering 15 – 8, thinking what needs to be added to 8 to make 15. Counting on for subtraction is a useful strategy that can also be applied to larger numbers.

Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?', 'What happens if ...?', and checking that pupils can use the procedures or skills to solve a variety of problems.

Mastery	Mastery with Greater Depth
What do you notice? Is there a relationship between the calculations? 500 + 400 =	523 + 400 =	523 + 28 = 400 + 500 =	423 + 500 =	423 + 28 = 300 + 600 =	323 + 600 =	323 + 28 = 200 + 700 =	223 + 700 =	223 + 28 = 100 + 800 =	123 + 800 =	123 + 48 =	For positive integers are the following statements always, sometimes or never true? ■ The sum of 2 odd numbers is even. ■ The sum of 3 odd numbers is even. ■ Adding 5 to a number ending in 6 will sum to a number ending in 1. ■ Adding 8 to a number ending in 2 will always sum to a multiple of 10. Explain why in each case.

Each strand begins with relevant statements from the **National Curriculum**. The booklets do not attempt to address all the statements, but cover what are considered to be key areas that should be understood and mastered at the end of each year. A section on big ideas is included, which highlight aspects in learning the concepts identified in the programmes of study, and teachers should find these helpful to ensure they are included in teaching. The mastery check statement is helpful as a reminder that just being able to get the correct answer is not necessarily evidence of sufficient understanding; teachers will need to probe more deeply. However many of the questions in the materials are designed to identify deeper understanding.

The core content of the booklets is examples of questions and activities that pupils who have mastered that particular key area, will be able to answer quickly and easily. There are two sets of examples: **Mastery** and **Mastery with Greater Depth**. Below is a summary, taken from the introduction of what each of the two sets of examples is attempting to reveal about a pupil's understanding:

A pupil really understands a mathematical concept, idea or technique if he or she can:

- *describe it in his or her own words;*
- *represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach);*
- *explain it to someone else;*
- *make up his or her own examples (and non-examples) of it;*
- *see connections between it and other facts or ideas;*
- *recognise it in new situations and contexts;*
- *make use of it in various ways, including in new situations.*

Developing mastery with greater depth is characterised by pupils' ability to:

- *solve problems of greater complexity (i.e. where the approach is not immediately obvious), demonstrating creativity and imagination;*
- *independently explore and investigate mathematical contexts and structures, communicate results clearly and systematically explain and generalise the mathematics*

[p7 of each booklet].

Explain your reasoning

Children are asked to explain their reasoning at the end of many of the questions.

Use in school…

Select some of these questions, perhaps one for each year group, and discuss as a staff what explanations you consider children might give that evidence their understanding.

Intelligent Practice

Many of the examples reflect the notion of intelligent practice, as in the first column of the Y3 example above, where children are not just expected to work through each example carrying out the same procedure, but are also expected to notice connections between the calculations, and use them to calculate subsequent questions. Finding easy ways to calculate and using *what I know* are key aspects of mastering mathematics.

Use in school…

Look at some of the examples where connections between calculations are evident. Teachers can create their own examples, trial them with pupils and share with each other. Initially children may not see the connections and their attention will need to be drawn to them. However over time they should start to recognise them, providing opportunities for mathematical reasoning.

Reducing the burden of assessment

Use in school…

Consider whether these materials are sufficient for measuring and evidencing pupils' progress. Some schools are using only these materials and don't see the need for tracking etc, and assigning a number to pupils' progress. The removal of levels was intended to reduce the burden of assessment. If children are able to answer questions in the first column then a teacher can be fairly confident that the child has mastered the curriculum in line with national expectations; if they can also answer questions in the second column, they are evidencing greater progress. Only if children are not evidencing mastery of a concept might there be the need for more detailed analysis. Evidencing progress using questions has been shown to produce more reliable information than a tick-box list of criteria. The questions are matched to the National Curriculum and exemplify the required standards in line with national expectations.

Where's the Maths in That? – Maths across the curriculum
Conjecturing and Convincing

This month, instead of looking at where maths appears in other curriculum subjects, we look at a concept - conjecture - that has a place in every lesson on the timetable (English, history, art, for example), but which we feel can play an especially powerful part in mathematics learning. You can find previous ***Where's the Maths in That?*** *features here.*

The character *Captain Conjecture* appears throughout the new assessment materials. He voices a conjecture as in the example below, and children are invited to discuss whether or not they agree, seeking to convince others that their explanation is correct:

Captain Conjecture says, 'I can double any number, but I can only halve some numbers.'

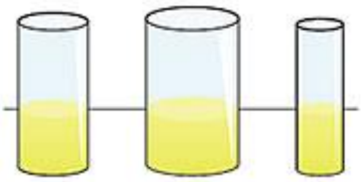
Do you agree?

Explain your reasoning

The process of conjecturing and convincing are important in learning mathematics and are promoted in the middle aim of the National Curriculum to ***reason mathematically*** *by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language* (National Curriculum p3). Sometimes a conjecture may need a qualification, such as in the one above where the conjecture is correct when we are working with whole numbers only, or objects that cannot be dissected. However the statement is incorrect where we allow for fractions or decimal numbers to be involved, since any number can be halved, although this will result in a decimal or fraction for odd numbers. This question appears in the Year 1 materials, where either explanation is acceptable as long as it is accompanied by a valid argument. It is the argument that is important; it demonstrates the child's ability to give an explanation based on mathematical reasoning. Teachers often comment that children find these types of questions the most difficult In SATs tests. However they provide good opportunities for demonstration of mastery. Not only does the child have to be familiar with the mathematics involved, but they also have to reason about it and communicate their understanding.

The process of conjecturing and convincing is promoted in the book *Thinking Mathematically* (Mason et al 2010). It is identified as one of a set of *natural powers* that young children arrive at school with, that can be very useful when learning mathematics. Others that are included are, specialising and generalising; imagining and expressing; and sorting and classifying. These are all things that children do quite naturally in an attempt to make sense of the world around them. They do however need to be nurtured and developed within mathematics lessons if they are to be effectively applied to learning mathematics.

Another example from the Y1 assessment materials is:



Captain Conjecture says, 'All of the glasses contain the same quantity of lemonade.'

Do you agree?

Explain your reasoning.

In this example the purpose of the question is to assess whether a child is able to reason about the capacity of a container. Conservation of liquid is an important concept that children need to grasp. Some children may ignore the width of the container and only focus on the level of the liquid, hence claiming that the three containers contain the same quantity. A child however that has mastered the concept will not only take into account the level of the liquid but also the width of the containers. They will disagree with Captain Conjecture and be able to explain why they disagree.

A further example is taken from Y5:

Captain Conjecture says, 'Using the digits 0 to 9 we can write any number, no matter how large or small.'

This might seem an obvious generalisation; however it may be something that many children have not explicitly thought about before. Although presented in Year 5, it is a question that might be valuable as a discussion point in any year group, where children can think about the numbers they are currently familiar with. In Y5 children should make reference to place value in their explanations: *These digits can take on different values, depending on where we place them, so the digit one might have a value of one thousand as in the number 1249 or one tenth as in the number 45.1, and so by placing them in different positions we can construct different numbers, in fact any number. Our place value system is a base ten system and so only uses the digits nought to nine. Once ten is reached, we move into the next column or position to express the value of the number, and ten ones becomes one ten, and ten tens becomes one hundred etc, so we only need the digits nought to nine to express any number.*

Most of the conjectures posed by Captain Conjecture lead to a mathematical generalisation. Generalisation lies at the heart of mathematics and is mentioned in the middle aim of the curriculum, as referenced above. Generalisation involves making a statement that is true for all examples. In making a generalisation we cut down on the amount of mathematics to learn, and we deepen our conceptual understanding. Generalisation is a key process in the development of mastery of mathematics. If we understand an idea well enough to generalise it for the range of examples it applies to, then we have probably mastered the concept.

Captain Conjecture appears many times across the assessment materials and is a valuable tool in providing opportunities for children to reason and generalise, and for the teacher to assess their understanding. You can of course make up your own Captain Conjecture statements, as can your pupils. This will support the journey to mastering mathematics.

Reference

MASON, J., BURTON, L., & STACEY, K. (2010), *Thinking mathematically*, 2nd Edition Pearson

Maths in the Staff Room – Short Professional Development Meetings

Maths in the Staff Room *provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school. You can find previous features in this series here*

Reasoning in the Classroom

Meeting Aim

- Understand how reasoning can be a part of every lesson

Timing

- 1.5 hours

Resources

- Problem solving with EYFS, Key Stage 1 and Key Stage 2 children - Logic problems and puzzles *(NB: this links to a document for the Primary National Strategy, which was formally discontinued in 2011. However, the resources have the potential to complement teaching in line with the new 2014 mathematics curriculum)*
- Developing opportunities and ensuring progression in the development of reasoning skills

Introduction

- Share the aim of the professional development meeting

1. Share the 'reasoning' aim of the new National Curriculum:

> The National Curriculum for mathematics aims to ensure that all pupils:
>
> *reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language*

Research by Nunes (2009) identified the ability to reason mathematically as the most important factor in a pupil's success in mathematics. It is therefore crucial that opportunities to develop mathematical reasoning skills are integrated fully into the curriculum. Such skills support deep and sustainable learning and enable pupils to make connections in mathematics.

Ask the teachers to jot down how they are developing reasoning in their lessons.

Developing teachers' subject knowledge

2. Ask teachers to consider which is the odd one out of the following:

Share rules that might be introduced to develop the activity in the class. These could include asking one person to identify the odd one out and for others to give a reason for that person's choice or demanding that the other two elements from the odd one out must have something in common.

Try some other examples such as:

5	9	10
1	2	3
0.5	5	½
0.2	½	0.5

Decide what aspects of the reasoning aim from the National Curriculum are being covered with this type of activity.

3. Look at Developing opportunities and ensuring progression in the development of reasoning skills.

Select an appropriate area of the national curriculum for the needs of your school.

Look at the questions provided in the document, select a few examples and discuss what might be expected as a response from the children.

For example:

Always, sometimes, never?

Is it always, sometimes or never true that an even number that is divisible by 3 is also divisible by 6?

Is it always, sometimes or never true that the sum of four even numbers is divisible by 4?

What responses might we expect from children?

4. Look at Problem solving with EYFS, Key Stage 1 and Key Stage 2 children - Logic problems and puzzles.

Each group of teachers look at an age-appropriate lesson.

Again, decide which elements of the National Curriculum aim are being met by the lesson.

These three approaches have given three ways of developing reasoning in the classroom. The first is a short activity designed to encourage children to question each other's thinking. The second shows how reasoning is part of any lesson and the third is a longer activity to develop reasoned argument. Are there other approaches that teachers are using, as identified in Section 1?

Developing Practice

5. Agree what aspects each teacher would like to develop in their own classroom and give time for each teacher to decide what actions they would like to take.

Embedding in Practice

6. Conclude by agreeing what teachers will bring back to a future staff meeting to discuss children's responses to what teachers have built into their lessons.

Further links

- Research Gateway: Reasoning