MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.6 Structures: quotitive and partitive division

Teacher guide | Year 2

Teaching point 1:

Objects can be grouped equally, sometimes with a remainder.

Teaching point 2:

Division equations can be used to represent 'grouping' problems, where the total quantity (dividend) and the group size (divisor) are known; the number of groups (quotient) can be calculated by skip counting in the divisor. (quotitive division)

Teaching point 3:

Division equations can be used to represent 'sharing' problems, where the total quantity (dividend) and the number we are sharing between (divisor) are known; the size of the shares (quotient) can be calculated by skip counting in the divisor. (partitive division)

Teaching point 4:

Strategies for finding the quotient, that are more efficient than skip counting, include using known multiplication facts and, when the divisor is two, using known halving facts.

Teaching point 5:

When the dividend is zero, the quotient is zero; when the dividend is equal to the divisor, the quotient is one; when the divisor is equal to one, the quotient is equal to the dividend.

Overview of learning

In this segment children will:

- explore different ways of dividing a quantity of objects into equal groups, describing the action using the language of division
- explore how quantities can be split into equal groups with some left over, describing the action using the language of division and remainders
- represent division with equations and identify the dividend, divisor and quotient (dividend ÷ divisor = quotient)
- be introduced to two structures of division – quotitive and partitive
- use the strategy of skip counting in multiples of the divisor to find the quotient, and interpret the skip counting and resulting quotient in both quotitive and partitive division contexts
- make connections between multiplication and division equations
- use known, or given, multiplication facts to carry out division calculations
- use known halving facts and strategies to find the quotient when the divisor is two
- explore the following special cases of division:
  - When the dividend is zero, the quotient is zero, e.g.
    $0 \div 2 = 0$ $\quad$ $0 \div 5 = 0$ $\quad$ $0 \div 10 = 0$
  - When the dividend is equal to the divisor, the quotient is one, e.g.
    $2 \div 2 = 1$ $\quad$ $5 \div 5 = 1$ $\quad$ $10 \div 10 = 1$
  - When the divisor is equal to one, the quotient is equal to the dividend, e.g.
    $2 \div 1 = 2$ $\quad$ $5 \div 1 = 5$ $\quad$ $10 \div 1 = 10$

In segment *2.2 Structures: multiplication representing equal groups*, children learnt about making equal groups. This segment begins, again, with making equal groups from a quantity of objects, but now children will begin to describe the action of splitting the total quantity of objects into groups using the language of division; for example, *'Six is divided into two groups of three.'* Children will briefly explore what happens when the total quantity can't be exactly divided into equal groups, using the language of remainders to describe the left-over quantity; for example, *'Nine is divided into four groups of two with a remainder of one.'* Remainders will be explored fully in segment *2.12 Division with remainders*, but for now the exploration simply serves to make children aware that it may not always be possible to divide a quantity neatly into equal groups.

The language and action of dividing a number of objects into equal groups in *Teaching point 1* prepares children for *Teaching point 2*, in which they will be introduced formally to division for the first time. *Teaching point 2* introduces the *quotitive* structure of division, which is sometimes referred to as 'division as grouping'. Quotitive division problems are those where both the total quantity (represented by the dividend) and the group size (represented by the divisor) are known, while the number of groups (represented by the quotient) is unknown. Children already have some informal experience of quotitive division; for example, after building up the two times table, they were asked questions such as: '*If ten children line up in twos, how many twos will there be?*' In *Teaching point 2*, children will connect quotitive division problems (with a divisor of two, five or ten) with their knowledge of skip counting (in multiples of two, five or ten). They will represent division problems with division equations and learn the meaning of the terms 'dividend', 'divisor' and 'quotient'. The strategy of skip counting in multiples of the divisor links to children's understanding of grouping and prepares them for applying known multiplication facts to solve division problems (*Teaching point 4*). The generalisation reached – we can skip count in the divisor to find the quotient – will apply to partitive division (*Teaching point 3*), but the interpretation of both the strategy and the result will differ.

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When describing quotitive division problems, the phrase 'divided by' is avoided; instead, the language used reflects the action of making equal groups of a known size, so the phrase 'divided into groups of' is used. For example, the question *'There are 15 biscuits. If I put them into bags of five, how many bags will I need?'* is answered with *'Fifteen divided into groups of five is equal to three.'*

Teaching point 3 introduces the *partitive* structure of division, which is sometimes referred to as 'division as sharing'. In partitive division, the total quantity is partitioned/divided between a known number of equal shares, as indicated by the divisor; for example, *'I have twenty conkers, and I share them equally between five children. How many conkers does each child get?'* Many children will have experiential familiarity with partitive division problems in terms of sharing items fairly. The aim, in *Teaching point 3*, is to move children away from the inefficient 'one for you, one for me' sharing approach in which they deal out the objects one at a time. The strategy of skip counting in multiples of the divisor is used as a developmental step to help children move from counting in ones to counting in other units; this serves to introduce partitive division problems and as preparation for children to use their known multiplication facts (*Teaching point 4*). In this strategy, skip counting according to the divisor represents distributing multiples of the divisor across the 'sharees'.

The language of 'grouping' is avoided when describing partitive division, and the language of 'sharing' or 'dividing between' is used instead to draw attention to this structure. For example, the solution to the above problem of twenty conkers shared between five children is described as *'Twenty divided between five is equal to four each.'* Each child gets an equal share; the size of the equal shares is four.

Teaching point 4 moves children on from using skip counting to using known multiplication facts to solve division problems. Children will focus on the links between multiplication and division equations, and apply their two, five and ten times tables. They will also formally connect the use of halving facts and strategies (discussed in segment *2.5 Commutativity (part 2), doubling and halving*) with division problems with a divisor of two.

Finally, *Teaching point 5* looks at some special cases of division, with children generalising about calculations with a dividend of zero, calculations with the dividend equal to the divisor, and calculations with the divisor equal to one.

The goal of the segment is for children to be able to approach any contextual division problem (quotitive or partitive) by:

- representing it with a division expression
- using the associated multiplication fact, as indicated by the dividend and the divisor, to find the quotient (or, as a backup, to skip count in multiples of the divisor)
- interpreting what each number in the completed division equation represents with respect to the original contextual problem (in quotitive division, the quotient will represent the number of groups; in partitive division, the quotient will represent the size of the equal shares).

All division problems in this segment have a divisor of two, five or ten, since these are the times tables already covered. Throughout, the two, five and ten times table charts should be made available so that children can focus on the structures and on identifying which facts they need to use, rather than being fully dependent on fluency in the facts at this early stage.

Note that some of the concepts in this segment are challenging to illustrate in static images; teachers are encouraged to read this guide alongside the accompanying animated PowerPoint slides in *2.6 Representations* (for best results, please view these in 'Slideshow' view).

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Language summary

Throughout this segment, there is a strong focus on careful use of language to accurately describe division and to reflect the different structures of division.

	Quotitive division contexts	Partitive division contexts	Division calculations with no associated context
Example problem	*'There are fifteen biscuits. If I put them into bags of five, how many bags will I need?'*	*'I have twenty conkers and I share them equally between five children. How many conkers does each child get?'*	30 ÷ 10 = $\square$
Key language	***'…divided into groups of…'*** *e.g.* *'Fifteen divided into groups of five is equal to three.'*	***'…divided between…'*** *e.g.* *Twenty divided between five is equal to four each.'*	***'…divided by…'*** *e.g.* *'Thirty divided by ten is equal to three.'*

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

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Teaching point 1:

Objects can be grouped equally, sometimes with a remainder.

Steps in learning

Guidance

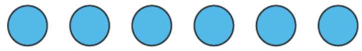
1:1 This teaching point builds on segment *2.2 Structures: multiplication representing equal groups* where children learnt about taking a given number of objects and grouping them equally or unequally. Here, where the full number of objects can't be grouped equally, equal groups are formed and the extra objects comprise a remainder.

The teaching point serves to remind children about splitting a quantity or measure into equal groups, now explicitly using language to focus on dividing quantities into groups, before they begin formal work on division.

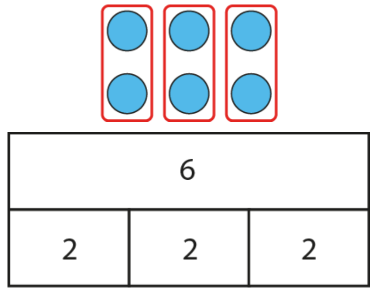
Here, children will learn that quantities cannot always be neatly divided into equal groups; however, in *Teaching points 2–5*, while children are learning the structures of division, the dividend will always be an integer multiple of the divisor (i.e. no remainders). Children will learn how to formally record and interpret remainders in segment *2.12 Division with remainders*.

Throughout this segment, examples and contexts will be based on the times tables already covered (two, five and ten times tables). Begin by chanting these times tables and reviewing skip counting forwards and backwards in multiples of two, five and ten. Use familiar representations from segments *2.3 Times tables: groups of 2 and commutativity (part 1)* and *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1*.

Representations



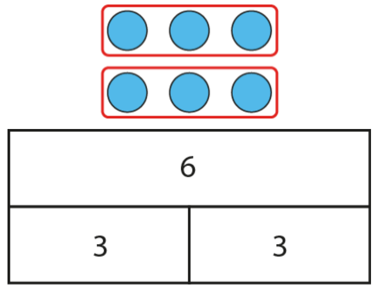
'There are six counters.'



$6 = 2 + 2 + 2$

$6 = 3 \times 2$ $\quad$ $6 = 2 \times 3$

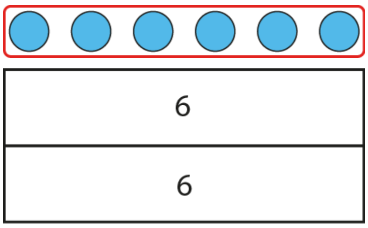
- *'There are three groups of two; there are six altogether.'*
- *'Six is divided into groups of two. There are three groups.'*
- *'Six is divided into three groups of two.'*



$6 = 3 + 3$

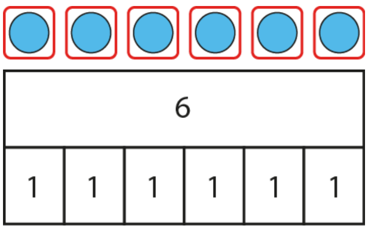
$6 = 2 \times 3$ $\quad$ $6 = 3 \times 2$

- *'There are two groups of three; there are six altogether.'*
- *'Six is divided into groups of three. There are two groups.'*
- *'Six is divided into two groups of three.'*



$6 = 1 \times 6$ $\quad$ $6 = 6 \times 1$

'There is one group of six; there are six altogether.'



$6 = 1 + 1 + 1 + 1 + 1 + 1$

$6 = 1 \times 6$ $\quad$ $6 = 6 \times 1$

- *'There are six groups of one; there are six altogether.'*
- *'Six is divided into groups of one. There are six groups.'*
- *'Six is divided into six groups of one.'*

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Then explore grouping:

- Present six counters.
- Ask how many counters there are altogether.
- Ask children to make equal groups with the six counters.
- As children suggest each different grouping, ask them to represent it with a repeated addition equation, two multiplication equations (factors in either order) and a part–part–whole representation (bar model or cherry diagram).
- Ask children to describe the situations and equations using the same language as in previous segments: ***'There are ___ groups of ___; there are ___ altogether.'***
- Then use language that connects to the action of dividing the counters into groups, in preparation for division:
  - ***'___ is divided into groups of ___. There are ___ groups.'***
  - ***'___ is divided into ___ groups of ___.'***

Note that, opposite (previous page), we have avoided using the division sentences for one group of six. The special case of division where the divisor is equal to the dividend (e.g. $6 \div 6$) is considered in *Teaching point 5*.

It is worth drawing children's attention to the fact that, with six counters, we can make equal groups of one, two, three and six, but not of four or five.

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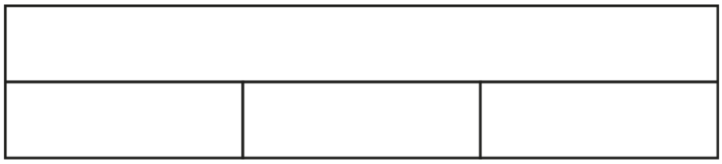
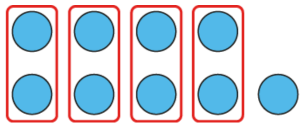
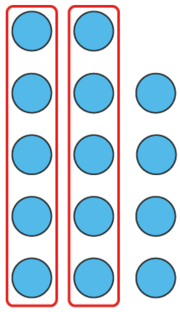
1:2	Repeat with some other multiples of two, five or ten, practising dividing the total number of counters into equal groups and describing the grouping with the stem sentences from step *1:1*. Then use a dòng nǎo jīn problem like the one opposite to assess and promote depth of understanding.	Dòng nǎo jīn: *'True or false? This bar model can only be used to represent six divided into three groups of two.'*  • *'Lily says that it can also represent fifteen divided into three groups of five. Is she right? Explain. '* • *'Jung Hee says that it represents twenty-five divided into three groups of ten, ten and five. Is he right? Explain.'*
1:3	Now explore situations with a remainder. Show, for example, nine counters divided into groups of two, and model how we can represent this using both an addition equation and a combined multiplication and addition equation, as shown opposite. Model the use of the word 'remainder' to describe the 'left over' counter, using the stem sentence: '___ ***is divided into*** ___ ***groups of*** ___ ***with a remainder of*** ___***.'*** Repeat with other groups of two, five or ten with remainders, including remainders greater than one. Note that, in segment *2.12 Division with remainders*, children will learn that the remainder is always smaller than the divisor; for now there is no need to explicitly draw attention to this fact. Note that, although no brackets are needed in the expression '$2 \times 5 + 4$' (since multiplication comes before addition in the order of operations), some children may see this as '$2 \times (5 + 4)$'. Ensure you make clear connections between the equation and the representation, by asking children to describe what each part of the equation represents, as shown in the second example opposite.	Describing remainders – example 1:  $9 = 2 + 2 + 2 + 2 + 1$ $9 = 4 \times 2 + 1$ *'Nine is divided into four groups of two with a remainder of one.'* Describing remainders – example 2:  $14 = 5 + 5 + 4$ $14 = 2 \times 5 + 4$ *'Fourteen is divided into two groups of five with a remainder of four.'* • *'What does the "14" represent?'* *'The "14" represents the total number of counters.'* • *'What does the "2 × 5" represent?'* *'The "2 × 5" represents the two groups of five.'* • *'What does the "4" represent?'* *'The "4" represents the remaining four counters.'*

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1:4

Keeping to equal groups of two, five or ten, with remainders, provide children with some practice recording equations to match different representations and drawing/making representations to match equations. Encourage them to continue describing each scenario using the stem sentence in step *1:3*.

Making representations to match equations:

'Use counters to represent the following equations.'

$17 = 8 \times 2 + 1$

$17 = 3 \times 5 + 2$

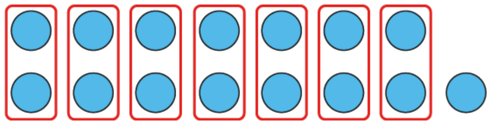
$17 = 1 \times 10 + 7$

Drawing representations to match equations:

'Iniko started drawing some mittens. Complete his drawing to match the equation.'

$13 = 6 \times 2 + 1$

Writing equations to match representations:

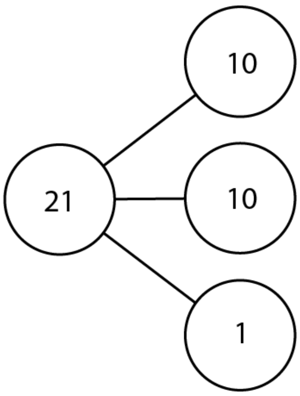


- *'Complete the equation to describe the counters.'*

  $15 = \boxed{} \times \boxed{} + \boxed{}$

- *'If I add another two counters what change do I need to make to the equation?'*

- *'Write an equation to match each representation.'*



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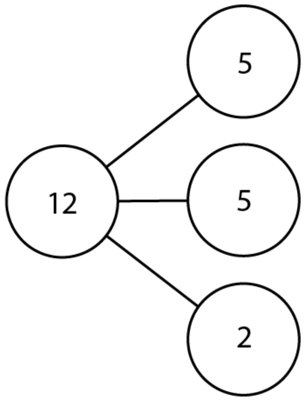
- *'Which equations match the representation?'*



	Do match (✓) or don't match (✗)
$2 \times 5 + 2 = 12$	
$5 \times 2 + 2 = 12$	
$6 \times 2 = 12$	
$5 + 2 \times 5 = 12$	
$12 = 5 + 5 \times 2$	

Dòng nǎo jīn:

- *'Aisha says this represents five groups of five.'*
- *'Ben says this represents two groups of ten with a remainder of five.'*
- *'Felicity says this represents four groups of five, with a remainder of five.'*
- *'Who is correct? Why?'*

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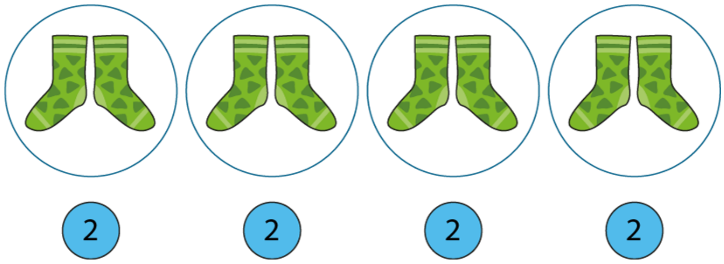
Teaching point 2:

Division equations can be used to represent 'grouping' problems, where the total quantity (dividend) and the group size (divisor) are known; the number of groups (quotient) can be calculated by skip counting in the divisor. (quotitive division)

Steps in learning

	Guidance	Representations
2:1	Quotitive division problems are those where the total quantity and the group size are both known, while the number of groups is unknown. Children already have some experience of division as 'grouping' (quotitive division) from segments *2.3 Times tables: groups of 2 and commutativity (part 1)* and *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1*; for example, after building up the two times table, they were asked questions such as: *'If ten children line up in twos, how many twos will there be?'* This teaching point explores the structure of quotitive division in detail, introducing the language and mathematical notation of division, and using skip counting with the group size (divisor) as a strategy to find the number of groups (quotient). Throughout this teaching point, only use groups of two, five or ten so that children can use their knowledge of unitising, skip counting and times tables from previous segments, and focus more fully on the *structure* of the problems rather than calculation. Begin by presenting a quotitive division problem, such as: *'There are eight socks. If I put them into pairs, how many pairs will there be?'* Show the total number of socks separately, and then work through the process of forming groups of two. As you form each group of two, count *'one pair is two, two pairs are four…'* until	 *'There are eight socks. If I put them into pairs, how many pairs will there be?'*  • *'Eight is divided into groups of two. There are <u>four</u> groups.'* • *'There are <u>four</u> groups of two in eight.'* • *'Eight is the total number of socks.'* • *'Two is the number of socks in each group/pair.'* • *'Four is the number of pairs of socks.'*

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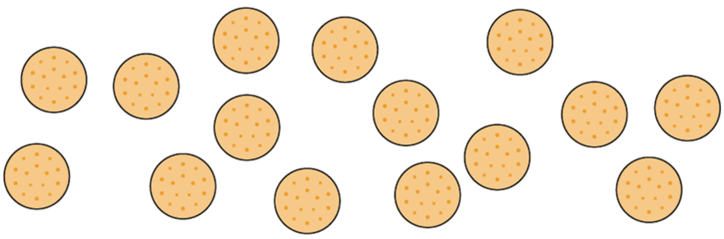
	you reach the required number of socks (eight). You can use a number line for support. For the first few examples (steps *2:1*, *2:2* and *2:6*), use the language of the context as you count, i.e. *'one pair is two'* rather than the more general *'one group of two is two'*. Then represent the solution with two-value counters and use the following stem sentence (from *Teaching point 1*) to describe the action of dividing the total number of socks into groups: ***'___ is divided into groups of ___. There are ___ groups.'*** The longer form of the stem sentence is used here to prepare children for writing the division equation. Draw further attention to the calculated quantity by asking the question *'How many groups of two are there in eight?'* and encouraging children to answer in full sentences *'There are <u>four</u> groups of two in eight.'* Make sure that children connect the numbers to the context; ask them to describe, in full sentences, what each number represents (as shown opposite, previous page).	
2:2	Now begin to look in more detail at how we can represent the act of grouping. Repeat step *2:1* for another context, such as: *'There are fifteen biscuits. If I put them into bags of five, how many bags will I need?'* Make sure the act of forming the groups is made clear, as you count out the groups of five biscuits (either using real biscuits and bags, or moving the pictures on the whiteboard): • *'One bag of five is five.'* • *'Two bags of five is ten.'* • *'Three bags of five is fifteen.'*	 • *'There are fifteen biscuits. If I put them into bags of five, how many bags will I need?'*

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As in step 2:1, represent the solution with counters and describe the solution in full sentences.
Then ask children to represent the solution (the bags of five biscuits) with a multiplication equation, encouraging them to describe what each number in the equation represents. Then remind children of the original question, and ask them to identify which number in the equation we didn't know at the beginning. Replace the unknown number (the '3') with a missing-number box to form a missing-number multiplication equation. Ask children again to describe what each part of the equation represents.
This process is animated in 2.6 Representations, slides 17–18.
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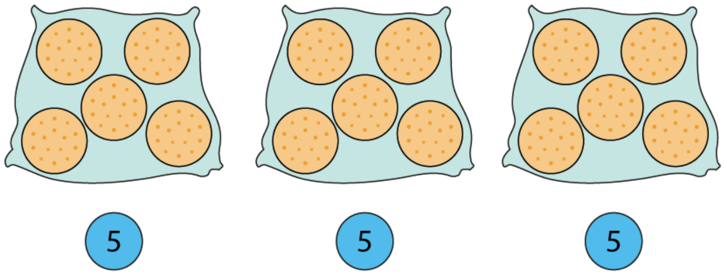

'Fifteen is divided into groups of five. There are three groups.'
'There are three groups of five in fifteen.'

'Fifteen is the total number of biscuits.'
'Five is the number of biscuits in each group/bag.'
'Three is the number of bags of biscuits.'

'How can we represent the solution?'

$3 \times 5 = 15$

'The "3" represents the number of bags we need.'
'The "5" represents the number of biscuits in each bag.'
'The "15" represents the total number of biscuits.'

'How can we represent the original problem?'

$\boxed{} \times 5 = 15$

'The missing number represents the number of bags we need.'
'The "5" represents the number of biscuits in each bag.'
'The "15" represents the total number of biscuits.'

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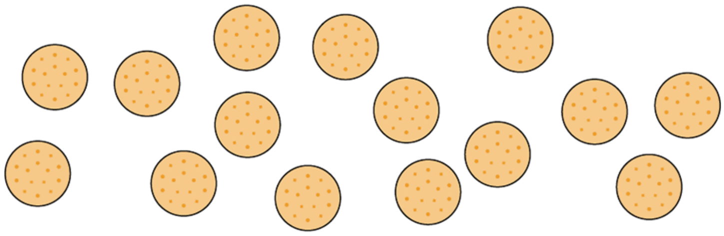
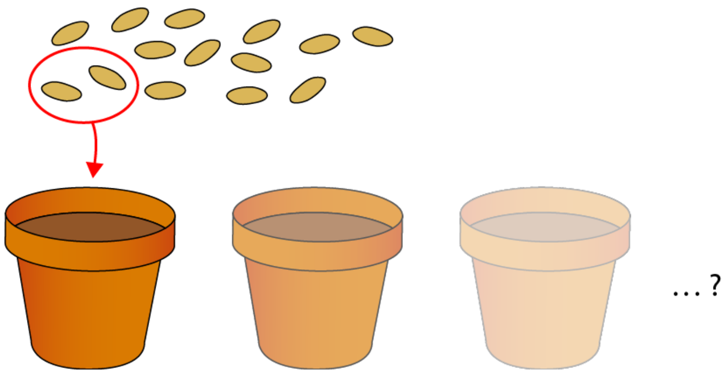
2:3	Now represent the problem from step *2:2* with a division expression (*15 ÷ 5*). Use the following language to describe the equation: *'fifteen divided into groups of five'* (see *2.6 Representations*, slide *19*). As before, ask children to describe what each number represents. The responses will be the same sentences used in the previous step.	 *'There are fifteen biscuits. If I put them into bags of five, how many bags will I need?'* • *'We can represent this as fifteen divided into groups of five.'* 15 ÷ 5 • *'The "15" represents the total number of biscuits.'* • *'The "5" represents the number of biscuits in each group/bag.'*
2:4	Present some other quotitive division problems (keeping to groups of two, five or ten), representing each with a division expression. Use the following stem sentence to describe each context and expression: ***'___ divided into groups of ___.'*** Continue to ask children to describe what each number represents, using full sentences: • ***'The ___ represents the total number of ___.'*** • ***'The ___ represents the number of ___ in each group.'*** For now keep the focus on the structure of the problems, writing division *expressions* only, not *equations* (i.e. not including the answer/quotient).	Representing quotitive division – dividing into groups of two: *'There are fourteen seeds. Two seeds are planted in each pot. How many pots are needed?'*  • *'Fourteen divided into groups of two.'* 14 ÷ 2 • *'The "14" represents the total number of seeds.'* • *'The "2" represents the number of seeds in each group/pot.'*

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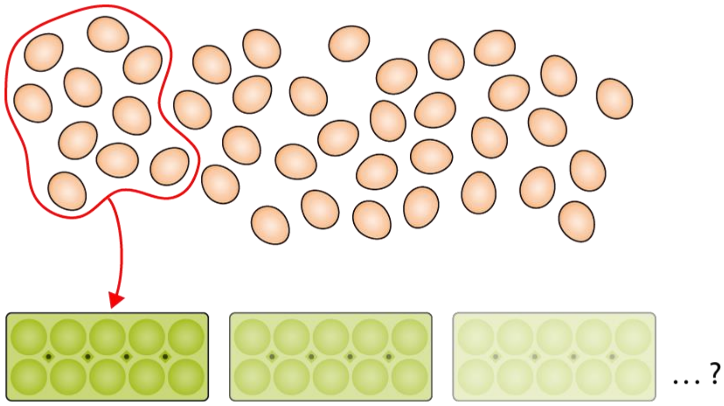
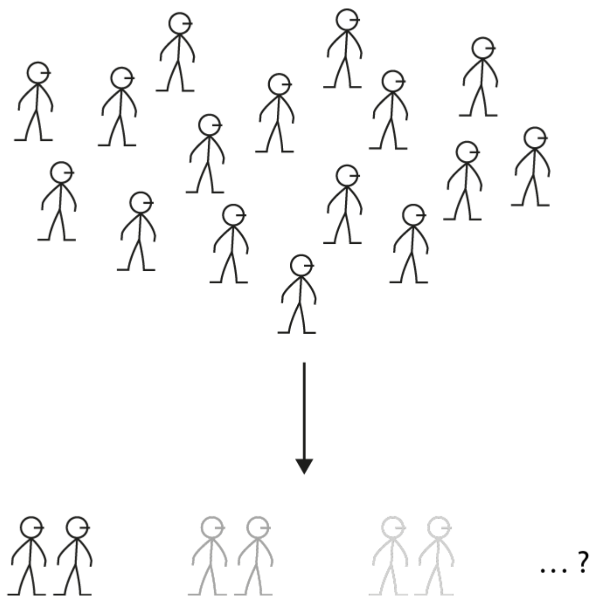
		Representing quotitive division – dividing into groups of ten: *'A farmer has forty eggs. She can fit ten eggs in a box. How many boxes does she need?'*  • *'Forty divided into groups of ten.'* 40 ÷ 10 • *'The "40" represents the total number of eggs.'* • *'The "10" represents the number of eggs in each group/box.'*
2:5	Provide children with some practice, including: • writing/completing division expressions to represent a quotitive division problem • writing/describing quotitive division problems to match a given division expression • describing a division expression. At this stage, use contexts where the cardinality of each group can be seen. Use sequences of related problems so that children can see links between the quantities in the contexts and the numbers in the expressions.	Writing/completing division expressions to represent a quotitive division problem: *'Complete the sentences and expressions to represent each problem.'* • *'There are sixteen children. The children get into pairs. How many pairs will there be?'* 

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We can represent this as 16 divided into groups of ____.

$16 \div \square$

- *'There are <u>eighteen</u> children. The children get into pairs. How many pairs will there be?'*

  We can represent this as ____ divided into groups of ____.

  $\square \div \square$

- *'There are <u>twenty</u> children. The children get into pairs. How many pairs will there be?'*

  We can represent this as ____ divided into groups of ____.

Writing quotitive division problems to match a complete division expression:

'Complete the questions to match the expressions.'

- $45 \div 5$
  Jasmine wants to draw <u>45</u> fingers. There are ____ fingers on each hand. How many hands should she draw?
- $40 \div 5$
  Jasmine wants to draw ____ fingers. There are <u>5</u> fingers on each hand. How many hands should she draw?
- $35 \div 5$
  Jasmine wants to draw ____ fingers. There are ____ fingers on each hand. How many hands should she draw?

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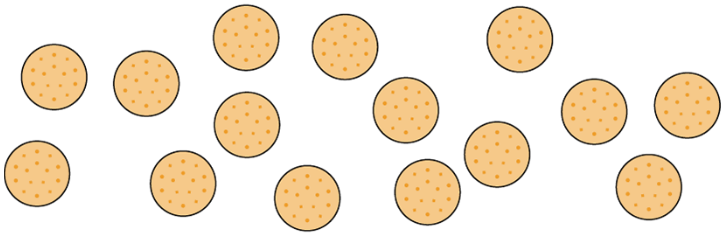
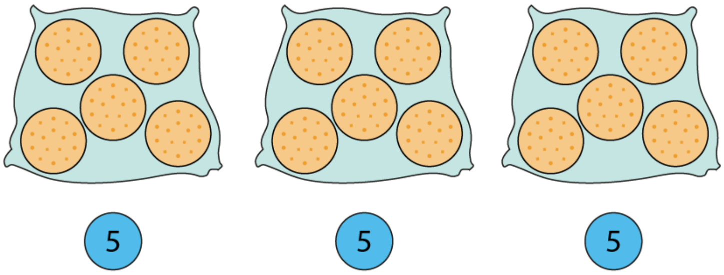
		Dòng nǎo jīn: • *'Write a question about eggs and boxes to go with this expression.'* 50 ÷ 10 • *'Write a question about pairs of socks to go with this expression.'* 12 ÷ 2
2:6	Return to the problem with the biscuits and bags from step *2:3*. Now connect enumerating the number of groups using skip counting (skip counting in fives until 15 is reached, as in step *2:2*) with representing the problem with a division expression (*15 ÷ 5*, as in step *2:3*) (see *2.6 Representations*, slide *23*). Model how the division *expression* can be turned into an *equation* showing the quotient. Continue to use the language of 'divided into groups of' to describe the resulting equation.	*'There are fifteen biscuits. If I put them into bags of five, how many bags will I need?'*  15 ÷ 5 • *'One bag of five is five.'* • *'Two bags of five are ten.'* • *'<u>Three</u> bags of five are fifteen.'*  • *'Fifteen is divided into groups of five. There are <u>three</u> groups.'* 15 ÷ 5 = 3 • *'Fifteen divided into groups of five is equal to <u>three</u>.'* • *'So, we need three bags.'*

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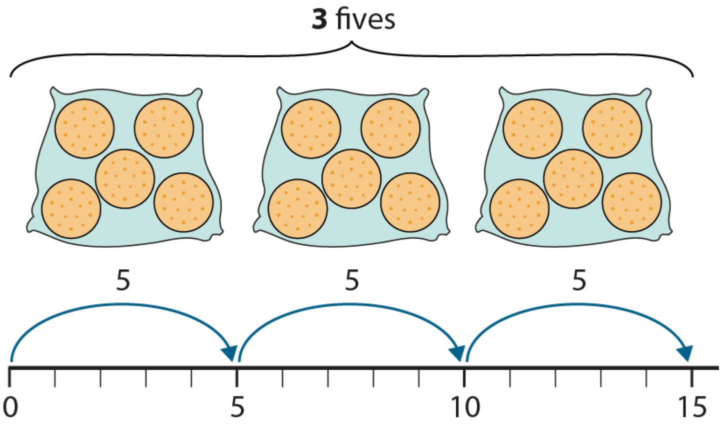
<table>
<tr>
<td>2:7</td>
<td>
Demonstrate how the skip-counting strategy can be represented on a number line using:

forward jumps to 15, alongside a repeated addition equation, to represent skip counting forwards, from the point of view of the filled bags (see 2.6 Representations, slide 24)
backward jumps from 15, alongside a repeated subtraction equation, to represent skip counting backwards, removing biscuits from the original fifteen as each bag is filled (see 2.6 Representations, slide 25).

The latter interpretation links division to repeated subtraction (as multiplication was linked to repeated addition). Counting forwards represents adding equal groups of five (multiplication) until we get to 15; we know to stop at 15 because we know we have just 15 biscuits to put in the bags. Counting backwards represents starting with 15 and removing equal groups of five (division) until we reach zero*; we stop at zero because we have ‘run out’ of biscuits.

* or, when there is a remainder (segment 2.12 Division with remainders), removing equal groups until we can’t make any more full groups.
</td>
<td>
Making groups of five:

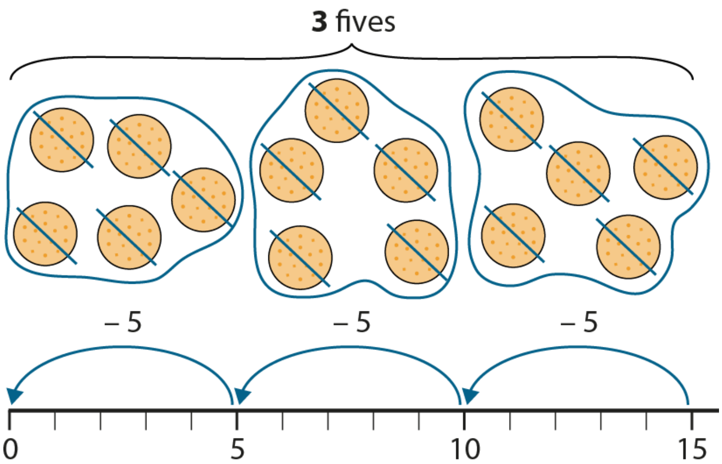


$5 + 5 + 5 = 15$

$15 \div 5 = 3$

‘Fifteen divided into groups of five is equal to three.’

Removing groups of five:



$15 - 5 - 5 - 5 = 0$

$15 \div 5 = 3$

‘Fifteen divided into groups of five is equal to three.’
</td>
</tr>
</table>

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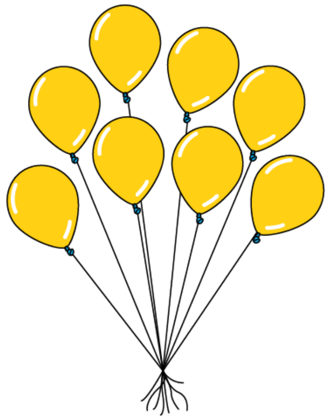
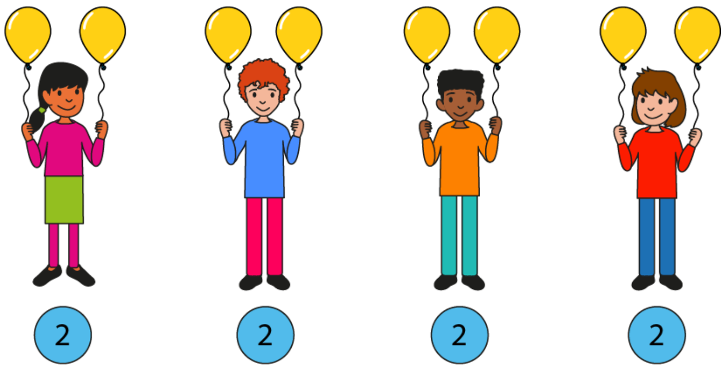
<table>
<tr>
<td>2:8</td>
<td>Now repeat for some other quotitive division contexts, keeping to groups of two, five or ten. Work through each problem as a class using the following steps:

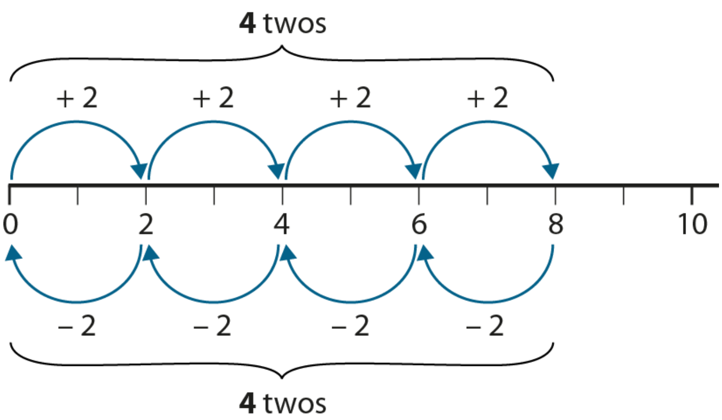
Represent the question with a division expression.
Skip count in the group size to find the number of groups, supported by a number line. You can begin to generalise the counting language as shown opposite (for example, 'one two is two, two twos are four…'), rather than tailoring each count to the context (for example, 'one child has two balloons, two children have four balloons…').
Also encourage children to use their fingers to keep a tally of the number of groups counted; this is useful while children are still skip counting to calculate, but unnecessary later on when they become fluent with the multiplication facts.
Represent the forwards and backwards skip counting with repeated addition and repeated subtraction equations respectively (see 2.6 Representations, slides 27–28).
Complete the division equation, and describe it using the language exemplified in step 2:7.

</td>
<td>'I have 8 balloons. I give 2 balloons to each child. How many children get balloons?'


$8 \div 2$

'One two is two.'
'Two twos are four.'
'Three twos are six.'
'<u>Four</u> twos are eight.'



$2 + 2 + 2 + 2 = 8$


$8 - 2 - 2 - 2 - 2 = 0$
</td>
</tr>
</table>

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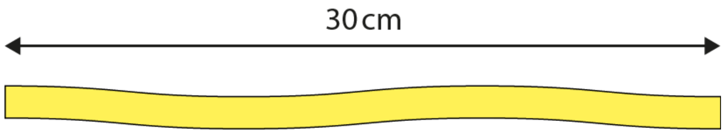
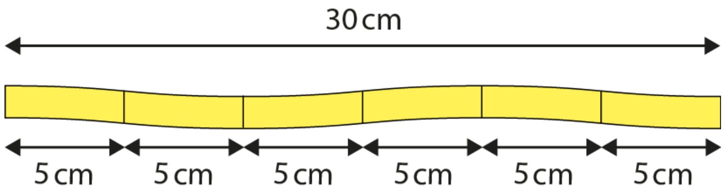
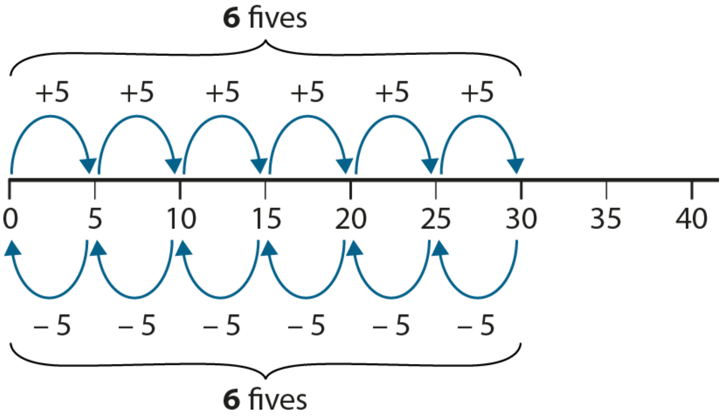
		• *'Eight is divided into groups of two. There are four groups.'* $8 \div 2 = 4$ • *'Eight divided into groups of two is equal to four.'* • *'So, four children get balloons.'*
2:9	As children gain confidence, include measures contexts as well (here the cardinality cannot be seen like it can in the previous examples). For these, ensure that the problems require the measure to be divided into 'groups' (equal parts), rather than scaled down (division as scaling is a separate structure, which will be covered in segment *2.17 Structures: using measures and comparison to understand scaling*). For the first measures example, use length, for example: *'A dressmaker has a ribbon that is thirty centimetres long. How many five-centimetre lengths can she make?'* Children no longer have discrete objects that they can manipulate, instead: • use skip counting forwards with the number line, as you mark the ribbon into equal lengths (see *2.6 Representations*, slide *30*) • use skip counting backwards with the number line as you remove equal lengths from the ribbon, until none is left (see *2.6 Representations*, slide *31*). To begin with, count using language that connects to the context, for example, *'One length is five centimetres…'* rather than *'One five is five…'*. As children gain confidence with measures contexts, you can begin to simplify to the latter.	*'A dressmaker has a ribbon that is thirty centimetres long. How many five-centimetre lengths can she make?'*  $30 \div 5$ Counting forwards:  • *'One length is five centimetres.'* • *'Two lengths are ten centimetres…'* • *'…Six lengths are thirty centimetres.'* Counting forwards or backwards – number line: $5 + 5 + 5 + 5 + 5 + 5 = 30$  $30 - 5 - 5 - 5 - 5 - 5 - 5 = 0$

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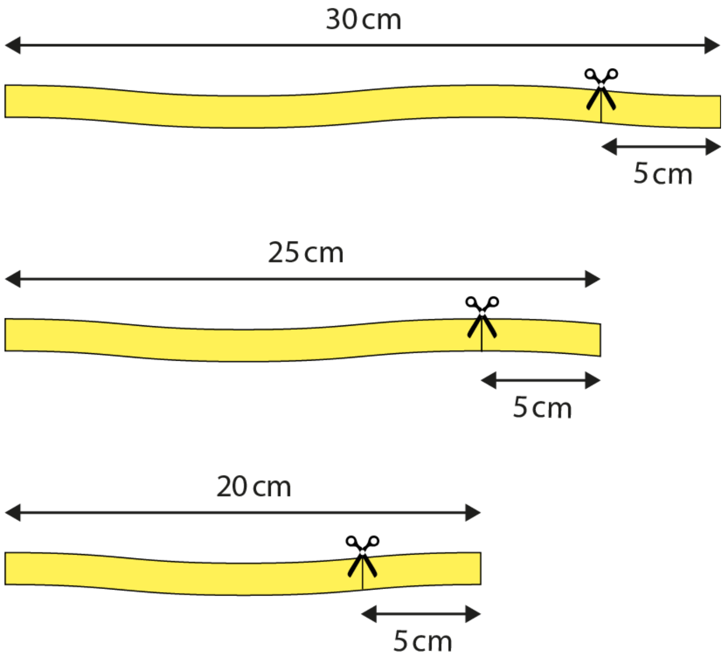
<table>
<tr>
<td></td>
<td></td>
<td>Counting backwards:


⋮
• 'One length is five centimetres.
I have twenty-five centimetres left.'
• 'Two lengths are ten centimetres.
I have twenty centimetres left.'
• 'Three lengths are fifteen centimetres.
I have fifteen centimetres left…'
• '… <u>Six lengths</u> are thirty centimetres.
I have none left.'

Summary:
• 'Thirty is divided into groups of five. There are <u>six</u> groups.'
30 ÷ 5 = 6
• 'Thirty divided into groups of five is equal to <u>six</u>.'
• 'So, the dressmaker gets <u>six</u> five-centimetre lengths of ribbon.'</td>
</tr>
<tr>
<td>2:10</td>
<td>Now, taking the most recent division equation you have worked with together as a class, introduce the following mathematical language to describe the elements of a division equation:
dividend ÷ divisor = quotient</td>
<td><table><tr><td>30</td><td>÷</td><td>5</td><td>=</td><td>6</td></tr><tr><td>dividend</td><td>÷</td><td>divisor</td><td>=</td><td>quotient</td></tr></table></td>
</tr>
</table>

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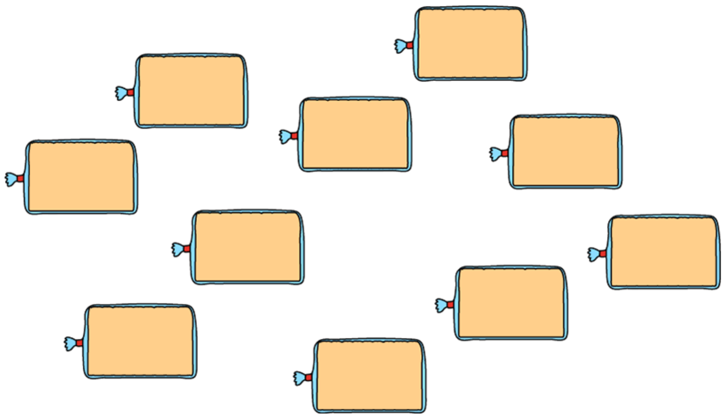
<table>
<tr>
<td></td>
<td>For now, connect the names to the numbers in the standalone equation (not to the context), as shown opposite.

Present a range of equations, and ask children to identify the parts of the equation by name:
'___ is the dividend.'
'___ is the divisor.'
'___ is the quotient.'
Avoid generalising that the divisor is the group size, and the quotient is the number of groups, since this is only the case in quotitive division.</td>
<td></td>
</tr>
<tr>
<td>2:11</td>
<td>Now present another contextual quotitive division problem, and draw attention to the fact that we skip count using the divisor (here, the group size) to find the quotient (here, the number of groups).

Also use an array to represent the contextual problem: collect the total number of counters (ten for Example 1 opposite) and move them to create one row as you count each group (rows of five for Example 1 opposite); if working pictorially, cover the array, then reveal one row at a time. This representation will be used again in Teaching point 3, once children have a contextual understanding of partitive division. There we will still be skip counting using the divisor to find the quotient, but there the divisor will represent the number of groups, not the group size.

Work towards the following generalisation: 'We can skip count using the divisor to find the quotient.'

As you work through further examples, get children in the habit of identifying what information they are given. For all of these quotitive problems say:
'We know the total amount.'
'We know the group size.'</td>
<td>Example 1:
'I go into a bakery to buy ten loaves of bread.
I can fit five loaves of bread into each bag.
How many bags will I need?'



$10 \div \mathbf{5}$
'The divisor is five, so we skip count in fives.'
'The divisor represents the number of loaves of bread that will fit in each bag.'
<table><tr><td>Bag 1: </td><td>'One five is five.'</td></tr><tr><td>Bag 1: 
Bag 2: </td><td>'<u>Two</u> fives are ten.'</td></tr></table>$10 \div 5 = 2$
'Ten divided into groups of five is equal to two.'
'So, I need two bags.'</td>
</tr>
</table>

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Also get children in the habit of connecting the final division equation to the context, describing what each number in the division equation represents; for *Example 1* on the previous page:

$10 \div 5 = 2$

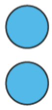
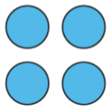
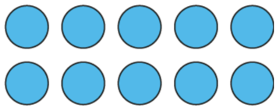
- *'The "10" represents the total number of loaves.'*
- *'The "5" represents the number of loaves in each bag.'*
- *'The "2" represents the number of bags we need.'*

Example 2:

'I go into a bakery to buy ten loaves of bread.
I can fit two loaves of bread into each bag.
How many bags will I need?'

$10 \div \mathbf{2}$

- *'The divisor is two, so we skip count in twos.'*
- *'The divisor represents the number of loaves of bread that will fit in a bag.'*

Bag: 1 	• *'One two is two.'*
Bag: 1 2 	• *'Two twos are four…'*
⋮	⋮
Bag: 1 2 3 4 5 	• *'<u>Five</u> twos are ten.'*

$10 \div 2 = 5$

- *'Ten divided into groups of two is equal to five.'*
- *'So, I need five bags.'*

2:12 Now provide children with varied practice, including:

- contextual (grouping apples, biscuits etc.) and abstract (grouping counters) quotitive division problems where the cardinality is apparent, including word problems
- quotitive division measures problems, including word problems
- true/false style questions
- matching multiplication and division equations, and missing-number problems (in these cases there is no 'structure' to the division problems – i.e. the divisor does not necessarily represent the group size, but

Contextual problem:

'Fill in the missing numbers.'

Number of hands	Number of fingers	Number of fingers on each hand
0	0	5
1		5
	10	5
3	15	5
	20	5
5		5
	30	5

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children know to skip count using the divisor).

Encourage children to write a division equation for each problem and to use their fingers to keep a tally of the number of groups counted. For word problems, include examples where the total number is not always the first number given in the sentence.

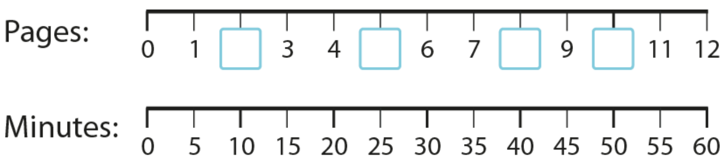
Example word problems:

- *'There are thirty-five oranges. I put five oranges in each bag. How many bags will I need?'*
- *'There are fourteen shoes. The shoes are put into pairs. How many pairs of shoes are there?'*
- *'Two children can fit into each carriage of a fairground ride. How many carriages are needed for twenty people?'*
- *'Ten people can fit in a minibus. How many minibuses are needed for fifty people?'*
- *'A bath holds eighty litres of water, how many ten-litre buckets of water are needed to fill the bath?'*
- *'A fairground ride cost £2. I spent £6, how many times did I ride?'*
- *'I have some packages that weigh 8 kg. Each package weighs 2 kg. How many packages do I have?'*
- *'There are five school days each week. How many weeks is fifteen school days?'*

(Note that the group size is always given, so all missing numbers are found either through multiplication or quotitive division.)

Contextual measures problem:

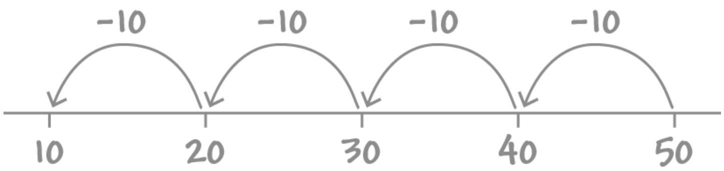
- *'It takes five minutes to type one page. How many pages can be typed in twenty-five minutes?'*
- *'Fill in the missing numbers to show how many pages could be typed in different amounts of time.'*



- *'How many pages can be typed in one hour?'*

True/false style problems:

'Jake wrote this in his book.'



50 is divided into groups of 10.
There are 4 groups.

'Is he correct? Explain.'

Matching multiplication and division equations:

'Draw a line to match each multiplication equation to the correct division equation.'

Multiplication equations	**Division equations**
$? \times 2 = 6$	$8 \div 2 = ?$
$8 = ? \times 2$	$60 \div 10 = ?$
$? \times 5 = 10$	$6 \div 2 = ?$
$60 = ? \times 10$	$10 \div 5 = ?$

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Missing-number problems:

- *'Fill in the missing numbers.'*

$\square \times 10 = 30$ $30 \div 10 = \square$

$\square \times 10 = 40$ $40 \div 10 = \square$

$\square \times 10 = 50$ $50 \div 10 = \square$

$4 \div 2 = \square$ $60 \div 5 = \square$

$6 \div 2 = \square$ $55 \div 5 = \square$

$8 \div 2 = \square$ $50 \div 5 = \square$

$10 \div 2 = \square$ $45 \div 5 = \square$

$12 \div 2 = \square$ $40 \div 5 = \square$

Dòng nǎo jīn:

'I cut a twenty-three centimetre ribbon into five-centimetre lengths.'

- *'How many five-centimetre lengths are there?'*
- *'How much is left over?'*
- *'Which equation represents the problem?'*

$23 = 4 \times 5$

$23 = 4 \times 5 + 3$

$20 = 4 \times 5 + 3$

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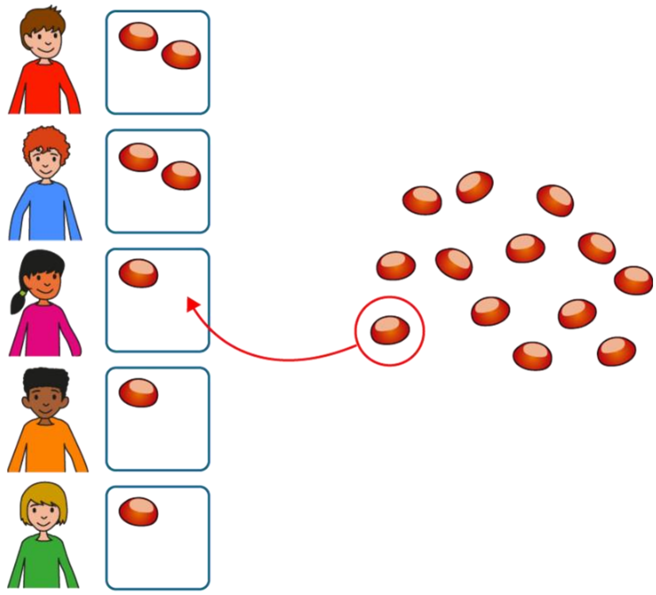
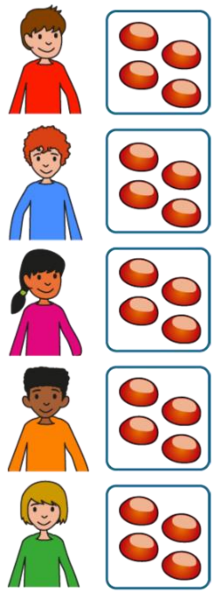
Teaching point 3:

Division equations can be used to represent 'sharing' problems, where the total quantity (dividend) and the number we are sharing between (divisor) are known; the size of the shares (quotient) can be calculated by skip counting in the divisor. (partitive division)

Steps in learning

	Guidance	Representations
3:1	In partitive division, the total quantity is partitioned/divided into a known number of equal shares as indicated by the divisor. Children are likely to have some everyday experience of division as 'sharing' (partitive division). For example, they are probably used to sharing items equally between friends, using the inefficient 'one for you, one for me' approach, dealing the objects out one at a time until there are none left. This teaching point explores the structure of partitive division in detail, using the language and mathematical notation of division introduced in *Teaching point 2*, and using skip counting as indicated by the divisor. As discussed in the *Overview of learning*, using the strategy of skip counting according to the divisor represents distributing multiples of the divisor across the 'sharees'. The language of 'grouping' is avoided, and the language of 'sharing' or 'dividing between' is used instead, to draw attention to this structure. For example, if there are twenty conkers shared between five children, we have twenty divided between five; each child gets an equal share, the size of the equal shares is four. Throughout this teaching point, keep the divisor to two, five or ten, so that children can focus more fully on the	*'I have twenty conkers, and I share them equally between five children. How many conkers does each child get?'*  • *'Twenty conkers are shared equally between five children. Each child gets four conkers.'* 

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	structure of the problems rather than calculation. Begin by presenting a partitive division problem, such as: *'I have twenty conkers, and I share them equally between five children. How many conkers does each child get?'* Note that the problem involves sharing objects between *people*, as this is likely to be familiar to children. The total number of objects in this first example should also be large enough to make sharing them out one-by-one time-consuming. Go through the process of 'dealing out' one conker per child until all of the conkers have been shared out (ideally, work with concrete resources). Highlight that this method of sharing is time-consuming and has many chances for error (you could model making a mistake, perhaps 'accidentally' dealing to one child twice in a row, or missing a child). When you have finished sharing, count how many conkers each child has, to check that they each have the same number. Then summarise the outcome: '*Twenty conkers are shared equally between five children. Each child gets four conkers.*' Finally, stress again that, with this method, many steps were needed to make sure that each child ended up with the same number of conkers.	
3:2	Work through another example, now introducing the language and notation of division, then sharing the objects one-by-one: • Introduce the problem. • Describe the problem using the following stem sentence: ***'We can represent this as ___ divided between ___.'**** • Represent the problem with a division *expression*, and ask children	

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to describe what each number represents.

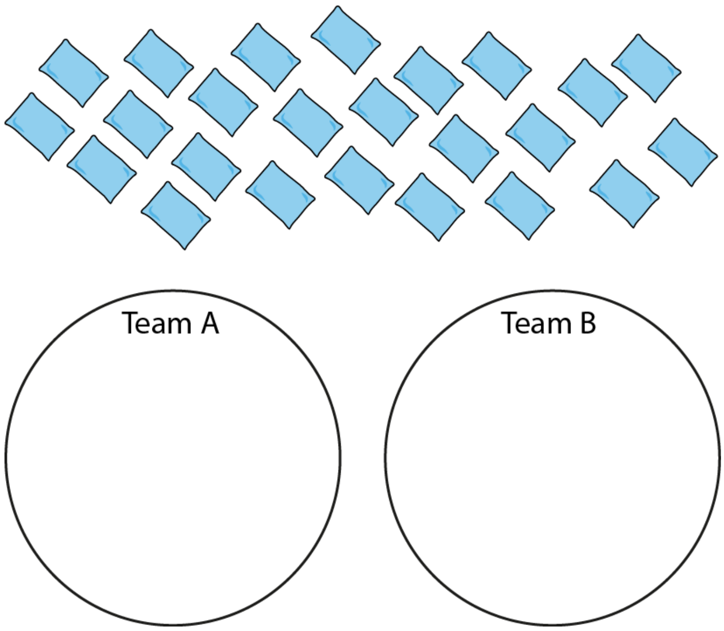
- Go through the process of sharing out the objects one-by-one, emphasising how long it takes.
- Complete the division *equation*, and again ask children to describe what each number represents.
  Summarise using the following stem sentence: ***'___ divided between ___ is equal to ___ each.'***
- Connect the numerical answer to the context.

Work through a variety of partitive division examples until children are confident using the division language and notation, and are getting used to how inefficient the 'one-by-one' sharing method is. Include some examples that involve sharing objects between something other than people/teams; for example: *'I have sixty apples and ten boxes. If I share the apples equally between the boxes, how many apples will there be in each box.'*

*Note: this language ('*divided between* ___') contrasts with the language used when describing quotitive division contexts ('*divided into groups of* ___').

Describing the problem:

'There are twenty-four bean bags. If they are shared equally between two teams, how many bean bags does each team get?'

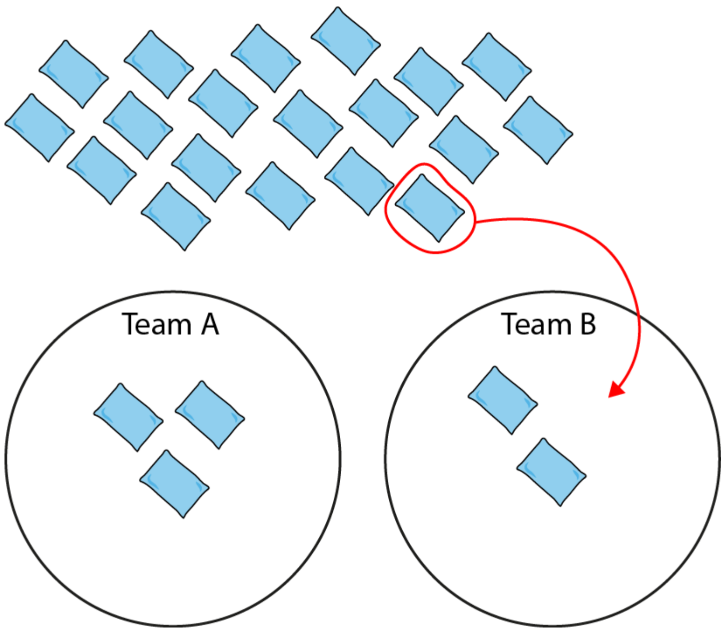


- *'We can represent this as twenty-four divided between two.'*

  $24 \div 2$

- *'The "24" represents the total number of bean-bags.'*
- *'The "2" represents the number of teams.'*

Sharing the objects one-by-one:



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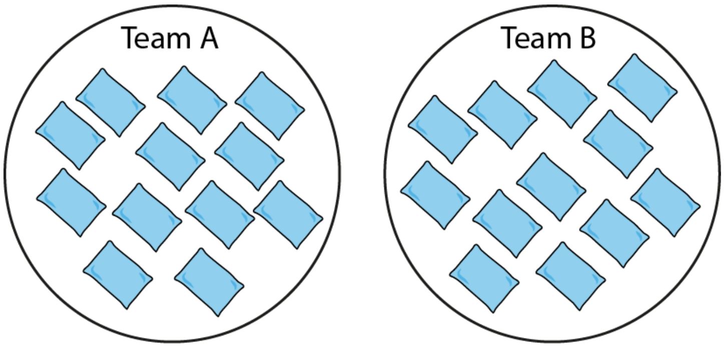
		Describing the solution:  24 ÷ 2 = 12 • *'Twenty-four divided between two is equal to twelve each.'* • *'So, each team gets twelve bean bags.'*
3:3	In this step, children's attention is drawn to the fact that they can use skip counting to solve problems that involve 'sharing' (partitive) in the same way they did for problems involving finding the 'number of groups' (quotitive). By repeatedly distributing a quantity equal to the divisor and sharing them 'in one go', the number that each 'sharee' gets (the size of each share) is equal to the number of times the divisor is removed/distributed. For example, the problem of fifteen balloons shared equally between five children is solved by repeatedly removing and equally distributing five balloons (one each); when five balloons have been distributed three times, all fifteen balloons have been shared and we can see that each child gets three. Begin with an example where balloons are shared between children, starting with a case where the children will only get one each, for example: '*There are five balloons. If they are shared equally between five children, how many balloons does each child get?'* Represent the problem with a division expression, and describe it as before. Then, rather than dealing out the	Five objects shared between five – presenting the problem: *'There are five balloons. If they are shared equally between five children, how many balloons does each child get?'*   • *'We can represent this as five divided between five.'* 5 ÷ 5

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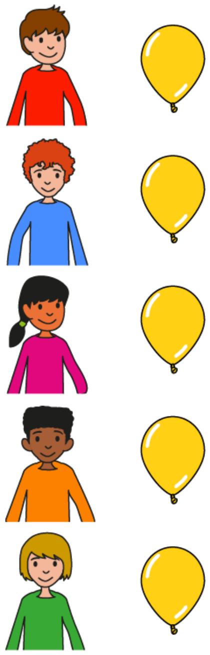
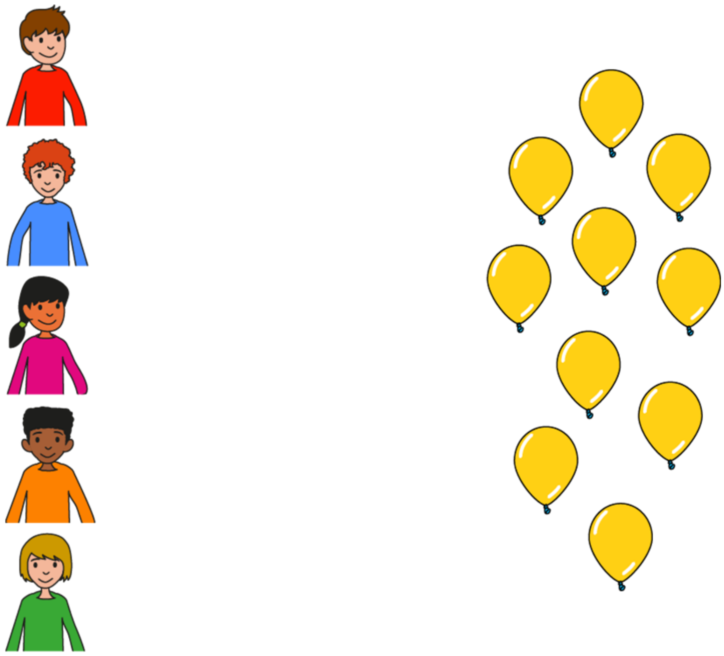
	balloons one at a time to each child in turn, show all five balloons being distributed simultaneously (see *2.6 Representations*, slide *38*). Summarise the result as before. Now, present a second question, this time dividing *ten* balloons between *five* children, so that they get two balloons each. Show the first five balloons being distributed evenly, resulting in one balloon for each child; then show the remaining five balloons being distributed evenly, resulting in a total of two balloons for each child (see *2.6 Representations*, slide *39*). Repeat for *fifteen* balloons shared between *five* children. Use the following language to track the count: • *'One five is one each. That's five.'* • *'Two fives is two each. That's ten.'* • *'Three fives is three each. That's fifteen.'*	Five shared between five – describing the solution:  1 each • *'One five is <u>one</u> each. That's five.'* $5 \div 5 = 1$ • *'Five divided between five is equal to <u>one</u> each.'* • *'So, each child gets one balloon.'* Ten shared between five – presenting the problem:  • *'We can represent this as ten divided between five.'* $10 \div 5$

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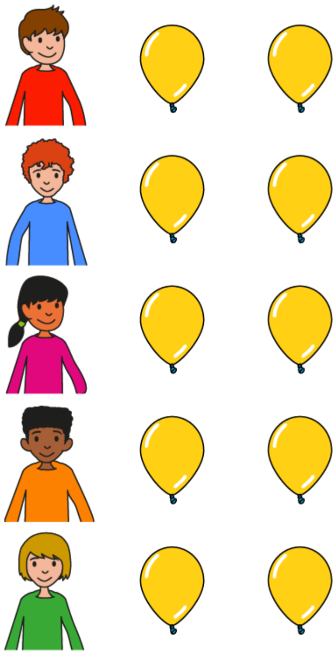
		Ten shared between five – describing the solution:  2 each • *'One five is one each. That's five.'* • *'Two fives is <u>two</u> each. That's ten.'* 10 ÷ 5 = 2 • *'Ten divided between five is equal to <u>two</u> each.'* • *So, each child gets two balloons.'*

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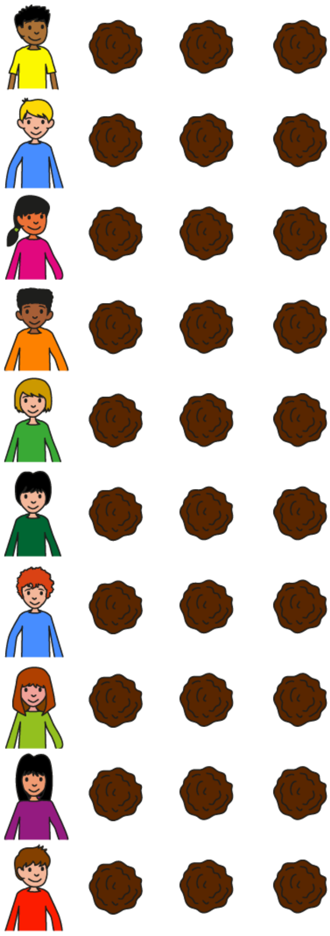
3:4	Repeat step *3:3*, now for a context where the dividend is divided into ten equal shares, for example:
	• *'There are ten children waiting for their lunch. There are ten meatballs in the saucepan. How many meatballs does each child get if they are shared equally?'*
	  ◦ *'If there are twenty meatballs, how many does each child get?'*
	  ◦ *'If there are thirty meatballs, how many does each child get?'*
	Ensure that the children can see that it is more efficient to skip count in tens, distributing ten meatballs at a time, than it is to deal out one at a time.
		Thirty shared between ten – summary:
		'There are ten children waiting for their lunch. If there are thirty meatballs, how many does each child get?'
		• *'We can represent this as thirty divided between ten.'*
		30 ÷ 10
		
		3 each
		• *'One ten is one each. That's ten.'*
		• *'Two tens is two each. That's twenty.'*
		• *'Three tens is three each. That's thirty.'*
		30 ÷ 10 = 3
		• *'Thirty divided between ten is equal to three each.'*
		• *'So, each child gets three meatballs.'*

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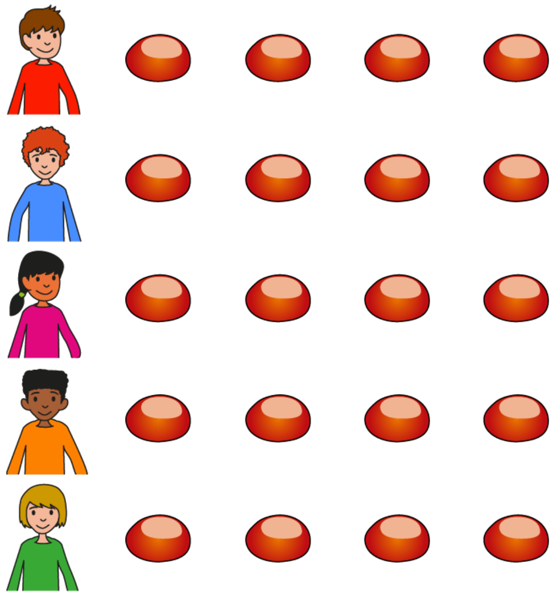
3:5	Work through the 'conkers' problem again (from step *3:1*), demonstrating how to use knowledge of skip counting in multiples of five. Then ask children which method is most efficient: dealing the conkers out one-by-one or skip counting in fives. Discuss as a class, prompting children to see that counting in fives is the quickest method.	*'I have twenty conkers and I share them equally between five children. How many conkers does each child get?'* • *'We can represent this as twenty divided between five.'* 20 ÷ 5  4 each • *'One five is one each. That's five.'* … • *'Four fives is four each. That's twenty.'* 20 ÷ 5 = 4 • *'Twenty divided between five is equal to four each.'* • *So, each child gets four conkers.'*

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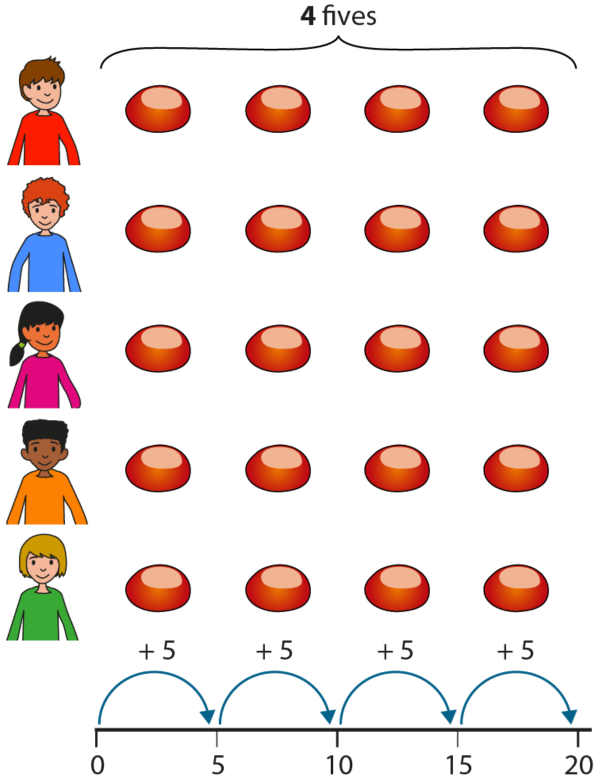
<table>
<tr>
<td>3:6</td>
<td>Continuing with the conkers context, demonstrate how the skip-counting strategy can be represented on a number line (in the same way as in step 2:7), using:

forward jumps to twenty to represent skip counting forwards, from the point of view of the children receiving the conkers (see ‘receiving a share each’ opposite and 2.6 Representations, slide 42)
backward jumps from twenty to represent skip counting backwards, removing conkers from the original twenty as they are shared out (see ‘removing a share each’ on the next page and 2.6 Representations, slide 43).

</td>
<td>Receiving a share each:


‘One five is one each. That's five.’
‘Two fives is two each. That’s ten.’
‘Three fives is three each. That’s fifteen.’
‘Four fives is <u>four</u> each. That’s twenty.’

$20 \div 5 = 4$

‘Twenty divided between five is equal to <u>four</u> each.’

</td>
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</table>

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<table>
<tr>
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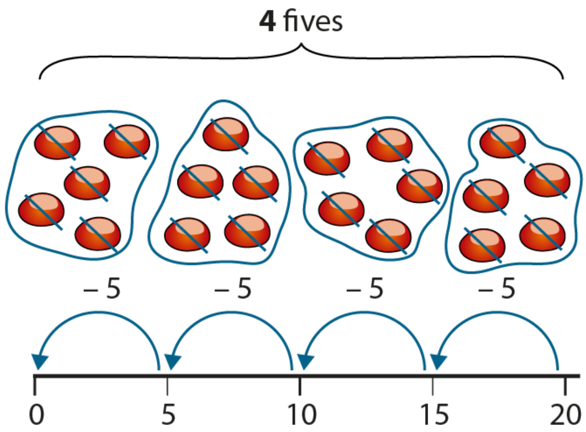
<td>
Removing a share each:



- *'One five is one each. I have fifteen left.'*
- *'Two fives is two each. I have ten left.'*
- *'Three fives is three each. I have five left.'*
- *'Four fives is <u>four</u> each. I have zero left.'*

$20 \div 5 = 4$

- *'Twenty divided between five is equal to <u>four</u> each.'*
</td>
</tr>
<tr>
<td>3:7</td>
<td>
Now repeat for some other partitive division contexts, keeping to examples with a divisor of two, five or ten (i.e. sharing between two, five or ten). Work through each problem as a class using the following steps:

- Represent the question with a division *expression* (dividend ÷ divisor).
- Ask children what number we are going to skip count in (they should identify the divisor in the expression).
- Skip count in multiples of the divisor, supported by a number line. Use the following stem sentences (as exemplified in steps *3:3–3:6*):
  - ***'One ___ is one each. That's ___.'***
  - ***'Two ___ is two each. That's ___…'***

  Also encourage children to use their fingers to keep a tally of the number of multiples of the divisor counted (the size of the shares).
- Complete the division equation, and describe it using the stem sentence:
</td>
<td></td>
</tr>
</table>

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<table>
<tr>
<td></td>
<td>

'___ divided between ___ is equal to ___ each.'

- Connect the quotient to the context. (You can ask children to describe what each number in the completed division equation represents.)

Include some examples that involve sharing objects between something other than people/teams (e.g. sharing pens between pots).

</td>
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<tr>
<td>

3:8

</td>
<td>

Now present another partitive division problem, and draw attention to the fact that we are skip counting using the divisor.

As in step *2.11*, also use an array to represent the contextual problem: collect the total number of counters (ten for *Example 1* opposite) and move them to create one row as you count each multiple of the divisor (rows of five for *Example 1* opposite); if working pictorially, cover the array, then reveal one row at a time.

Work towards the following generalisation: ***'We can skip count using the divisor to find the quotient.'***

Note that this is the same generalisation as that reached for quotitive division.

As you work through further examples, get children in the habit of identifying what information they are given; in all of these partitive problems say:

- *'We know the total amount.'*
- *'We know the number it is divided between.'*

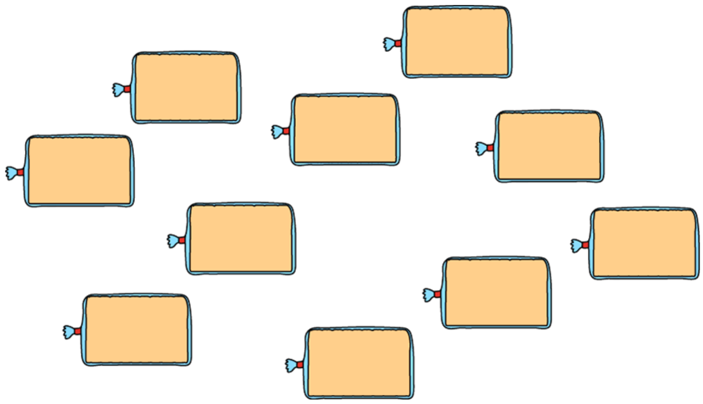
Also get children into the habit of connecting the final division equation to the context, describing what each number in the division equation represents; for *Example 1* opposite:

$10 \div 5 = 2$

</td>
<td>

Example 1:

'I go into a bakery to buy ten loaves of bread. I have five shopping bags. If I share the loaves of bread equally between the bags, how many will there be in each bag?'

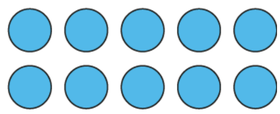


$10 \div \mathbf{5}$

- *'The divisor is five, so we skip count in fives.'*
- *'The divisor represents the number of bags.'*

<table>
<tr><td>Bag: 1 2 3 4 5
</td><td>

- *'One five is five.'*

</td></tr>
<tr><td>Bag: 1 2 3 4 5
</td><td>

- *'Two fives are ten.'*

</td></tr>
</table>

$10 \div 5 = 2$

- *'Ten divided between five is equal to two each.'*
- *'So, there will be two loaves of bread in each bag.'*

</td>
</tr>
</table>

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- *'The "10" represents the total number of loaves.'*
- *'The "5" represents the bags we are sharing the loaves between.'*
- *'The "2" represents the number of loaves we should put in each bag.'*

Example 2:

'I go into a bakery to buy ten loaves of bread. I have two shopping bags. If I share the loaves of bread equally between the bags, how many will there be in each bag?'

10 ÷ **2**

- *'The divisor is two, so we skip count in twos.'*
- *'The divisor represents the number of bags.'*

Bag 1: Bag 2:	*'One two is two.'*
Bag 1: Bag 2:	*'Two twos are four…'*
⋮	
Bag 1: Bag 2:	*'Five twos are ten.'*

10 ÷ 2 = 5

- *'Ten divided between two is equal to five each.'*
- *'So, there will be five loaves of bread in each bag.'*

3:9

Now provide children with varied partitive division practice, including:

- contextual (sharing conkers, apples etc.) and abstract (sharing counters) partitive division problems, including word problems
- true/false style questions
- matching multiplication and division equations, and missing-number problems (see step *2:12;* as noted before, there is no 'structure' to these missing-number problems – i.e. the divisor does not necessarily represent a group size or a number we are sharing between, but children know to skip count using the divisor; you can use the more general phrase *'___divided by___'* when speaking about these problems).

Encourage children to write a division equation for each problem and to use their fingers to keep a tally of the

Contextual problems:

- *'Fill in the missing numbers.'*

Number of sweets for each child	Total number of sweets	Number of children
0	0	2
	2	2
2		2
	6	2
	8	2
5		2
	12	2

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number of multiples counted. For word problems, include examples where the total number is not always the first number given in the sentence.

Example word problems:

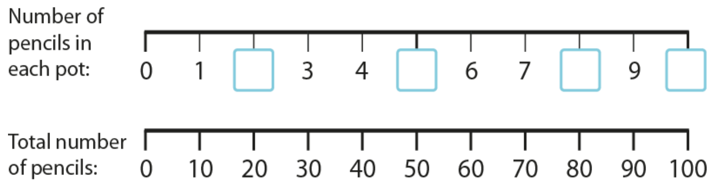
- *'Fourteen children are put into two equal teams. How many children are there in each team?'*
- *'There are five monkeys; thirty-five bananas are shared equally between them. How many bananas does each monkey get?'*
- *'If seventy tins of beans are shared equally between ten boxes, how many tins will there be in each box?'*
- Dòng nǎo jīn: *'Thirty-seven stickers are shared equally between ten children. '*
  - *'How many stickers does each child get?'*
  - *'How many stickers are left over?'*

Dòng nǎo jīn:

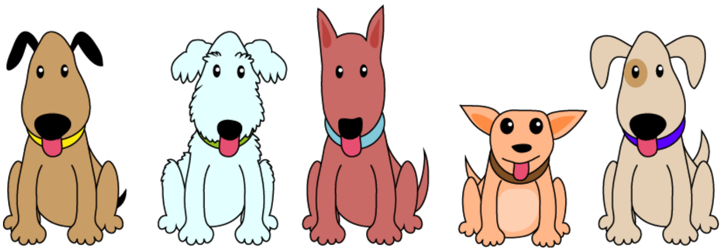
- *'Sean says:* "I have two packets of biscuits with the same number of biscuits in each packet. I have twenty biscuits altogether."*'*
- *'Simon says:* "I have four packets of biscuits with the same number of biscuits in each packet. I have the same number of biscuits as Sean."*'*

'Explain how this could be true.'

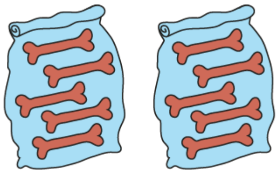
- *'Some pencils are shared equally into ten pots.'*
  - *'If there are twenty pencils, how many pencils are there in each pot?'*
  - *'Fill in the missing numbers to show how many pencils in each of the ten pots, for different quantities of pencils.'*



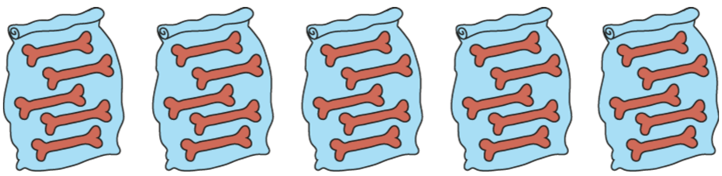
- *'There are five dogs and some packs of bones. Each pack contains five bones.'*



  - *'How many bones does each dog get if there are two packs?'*



  - *'How many bones does each dog get if there are five packs?'*



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<table>
<tr>
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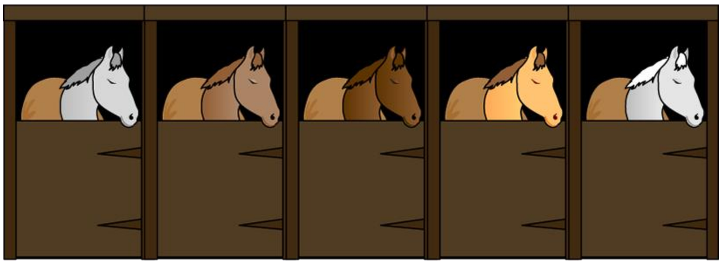
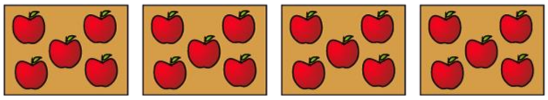
<td>
True/false style problems:


'The apples are shared equally between the horses. Ulyana says each horse gets five apples. Is she correct? Explain.'
</td>
</tr>
<tr>
<td>3:10</td>
<td>
To complete this teaching point provide mixed practice, including both quotitive and partitive division problems. Ensure that children can confidently:

represent each problem with a division expression
identify that they need to skip count in the divisor to find the quotient
describe what the quotient represents, i.e. link the 'answer' back to the contextual problem (in quotitive division problems, the quotient will represent the number of groups; in partitive division problems, the quotient will represent the size of the equal shares).

To assess understanding of the structures and strategies, you can present sorting, matching and true/false style questions such as those shown opposite and on the next page.
</td>
<td>
Sorting problem – identifying the strategy:
'Put each question in the correct box.'
<ol type="A">
40 eggs in boxes of 10.
How many boxes?
30 biscuits shared between 5 bags.
How many in each bag?
10 children share 30 pens.
How many pens each?
40 flowers in bunches of 5.
How many bunches?

<table>
<tr><th>Skip count in tens</th><th>Skip count in fives</th></tr>
<tr><td></td><td></td></tr>
</table>
</td>
</tr>
</table>

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<table>
<tr>
<td></td>
<td>Dòng nǎo jīn:

'There are forty stickers. The stickers are shared so that each child gets ten each. How many children get stickers? '
'If one more child joins, and stickers are shared equally, how many will each child get now?'
'Show this using counters.'

</td>
<td>Matching problem – interpreting the solution:

'Draw a line to match each question and equation with the correct answer.'

<table>
<tr><th>Questions and equations</th><th>Answers</th></tr>
<tr><td>50 eggs are grouped into boxes of 10.
$50 \div 10 = 5$</td><td>10 boxes are needed.</td></tr>
<tr><td>50 eggs are shared between 10 boxes.
$50 \div 10 = 5$</td><td>5 boxes are needed.</td></tr>
<tr><td>50 eggs are shared between 5 boxes.
$50 \div 5 = 10$</td><td>5 eggs go into each box.</td></tr>
<tr><td>50 eggs are grouped into boxes of 5.
$50 \div 5 = 10$</td><td>10 eggs go into each box.</td></tr>
</table>
</td>
</tr>
</table>

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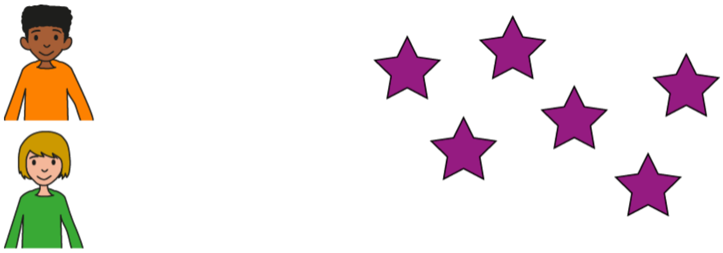
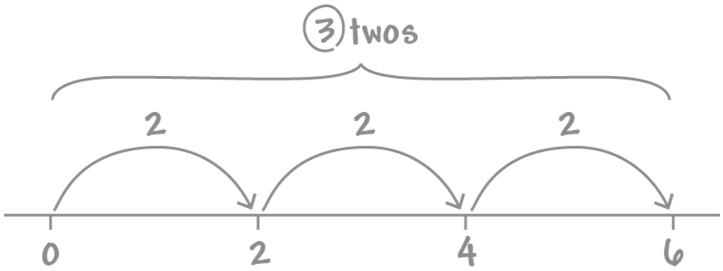
True/false style problem:

'Jacinta and Toby are solving this problem.'

6 stickers are shared between 2 children.
How many stickers does each child get?

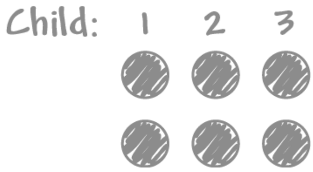


- *'Jacinta writes:'*



They get 3 each.

- *'Toby writes:'*



They get 2 each.

- *'Who is correct? Why?'*

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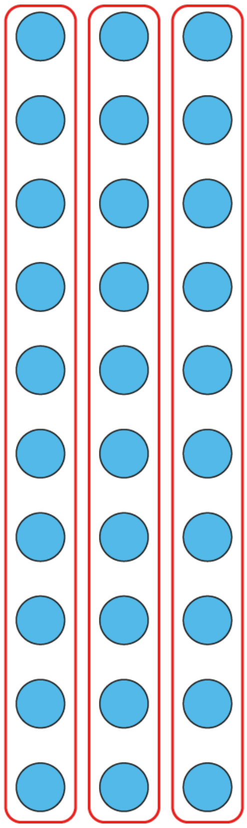
Teaching point 4:

Strategies for finding the quotient, that are more efficient than skip counting, include using known multiplication facts and, when the divisor is two, using known halving facts.

Steps in learning

	Guidance	Representations
4:1	So far in this segment, children have used knowledge of skip counting to find the quotient, and to support their understanding of the quotitive and partitive division structures. The connection between division and multiplication has been embedded throughout, through careful use of language, through developing understanding of division equations by first looking at missing-factor multiplication equations, and through connection to repeated addition and repeated subtraction supported by number lines. In this teaching point, the link between multiplication facts and division is made more explicit, and children explore how they can use known facts, rather than skip counting from/to zero or reciting the relevant times table until the required product is reached. Throughout this teaching point, make the two, five and ten times table charts available so that children can focus on identifying *which* facts they need to use, rather than being fully dependent on fluency in the facts at this early stage. Begin by presenting an array representing three groups of ten. Circle the groups of ten, and ask children to suggest equations that represent the image. Prompt the class until you have recorded *both* multiplication equations, as well as a division equation, then ask children to describe what each number represents (they represent the same thing in each equation). Note that only the division equation with a divisor of	Linking multiplication and division facts – array representation:  $3 \times 10 = 30$ $10 \times 3 = 30$ $30 \div 10 = 3$ • *'The "30" represents the total number of counters.'* • *'The "10" represents the size of the groups.'* • *'The "3" represents the number of groups.'* • *'There are three groups of ten in thirty, so thirty divided into groups of ten is three.'*

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<table>
<tr>
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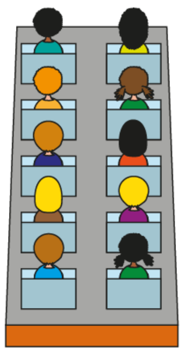
<td>ten is considered (30 ÷ 10 = 3), since the other possible equation (30 ÷ 3 = 10) links to skip counting in multiples of three / the three times table, which children have not yet learnt. Then model the language: 'There are three groups of ten in thirty, so thirty divided into groups of ten is three.' Highlight the '3' in each equation.

Repeat for some other groups of ten (four groups of ten, five groups of ten…), using the stem sentence: '___ tens are equal to ___, so ___ divided into groups of ten is equal to ___.'</td>
<td></td>
</tr>
<tr>
<td>4:2</td>
<td>Now work through a contextual quotitive division problem, asking children to represent the problem with a division expression and to then identify what multiplication fact they can use to help them find the quotient.

Then vary the problem a couple of times, increasing the dividend by ten to draw attention to the connection with the ten times table. As you work through, you can begin to shorten the language, as shown opposite for 40 ÷ 10 and 50 ÷ 10, for example: 'Four tens are forty, so forty divided into tens is four.'</td>
<td>Linking multiplication and division facts – contextual quotitive problem:

'A carriage on a fairground ride holds ten people.'



• 'If there are thirty people, how many carriages are needed?'

$30 \div 10 = \boxed{}$

'There are three groups of ten in thirty, so thirty divided into groups of ten is three.'

$3 \times 10 = 30$
so
$30 \div 10 = 3$

• 'If there are forty people, how many carriages are needed?'

$4 \times 10 = 40$ 'Four tens are forty…
so   …so…
$40 \div 10 = 4$ … forty divided into tens is four.'</td>
</tr>
</table>

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<tr>
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<td>

- *'If there are fifty people, how many carriages are needed?'*

$5 \times 10 = 50$ — *'Five tens are fifty…*
so — *…so…*
$50 \div 10 = 5$ — *… fifty divided into tens is five.'*

</td>
</tr>
<tr>
<td>4:3</td>
<td>Now look at the three pairs of multiplication and division equations explored so far, and prompt children to make the following generalisation:
'If the divisor is ten, we can use the ten times table to find the quotient.'</td>
<td>
<table>
<tr><td>$3 \times 10 = 30$

$30 \div 10 = 3$</td><td>$4 \times 10 = 40$

$40 \div 10 = 4$</td><td>$5 \times 10 = 50$

$50 \div 10 = 5$</td></tr>
</table>
</td>
</tr>
<tr>
<td>4:4</td>
<td>Provide children with opportunities to apply the generalisation from step 4:3, using intelligent practice and both quotitive and partitive contextual problems. Encourage children to identify the known multiplication fact that they can use to solve each problem, referring to the ten times table chart for support. You can now begin to include some partitive measures contexts, for example:

'Haseem shares eighty litres of water equally between ten buckets. How much water does he put in each bucket?'
'Chloe and Nina have a sixty-centimetre strawberry lace. They cut it into ten equal pieces. How long is each piece?'

For each word problem, encourage children to first think about, and write, a division equation to represent the context, before calculating using their times tables facts.

Work through the second dòng não jīn problem on the next page as a class, to consider both calculation efficiency and how to solve practical partitive measures problems.</td>
<td>
'Fill in the missing numbers.'
<table>
<tr><td rowspan="2">× 10 ↓</td><td>1</td><td>2</td><td></td><td>4</td><td>5</td><td></td><td rowspan="2">↑ ÷ 10</td></tr>
<tr><td>10</td><td></td><td>30</td><td>40</td><td></td><td>60</td></tr>
</table>

$5 \times 10 = 50$
so
$50 \div 10 = \square$

$10 \times 7 = 70$
so
$70 \div 10 = \square$

$6 \times 10 = \square$
so
$60 \div 10 = \square$

$10 \times 8 = 80$
so
$80 \div 10 = \square$

$10 \div 10 = \square$
$20 \div 10 = \square$
$30 \div 10 = \square$
$40 \div 10 = \square$

$120 \div 10 = \square$
$110 \div 10 = \square$
$100 \div 10 = \square$
$90 \div 10 = \square$
</td>
</tr>
</table>

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Dòng nǎo jīn:

- *'Fill in the missing numbers.'*

$\square \times 10 = 30$

so

$30 \div 10 = \square$

$\square \times \square = \square$

so

$70 \div 10 = \square$

$\square \times 10 = \square$

so

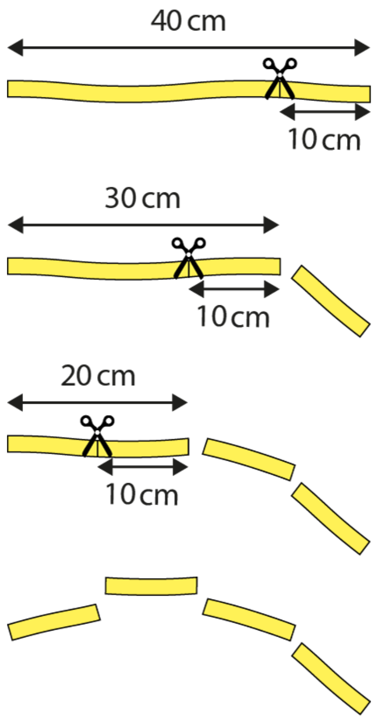
$50 \div 10 = \square$

$15 \times 10 = 150$

so

$150 \div 10 = \square$

- *'A dressmaker has some forty-centimetre lengths of ribbon. She wants to divide each of them into ten equal pieces. Dana and Hari both try to cut the ribbon into ten equal pieces. Whose method do think is best? Why?'*
  - *'Dana says,* "I'll count in tens." *She takes a forty-centimetre piece of ribbon and cuts off ten centimetres at a time.'*



- *'One ten is ten.'*
- *'Two tens are twenty.'*
- *'Three tens are thirty.'*
- *'Four tens forty.'*

'Has Dana divided the ribbon into 10 equal pieces?'

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<table>
<tr>
<td></td>
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<td>

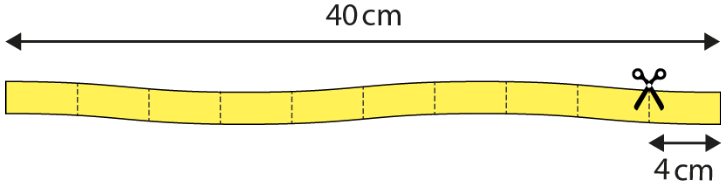
- *'Hari takes a forty-centimetre piece of ribbon and writes:'*

  $4 \times 10 = 40$

  so

  $40 \div 10 = 4$

  'He says, "Each piece needs to be four centimetres long." *He then cuts the ribbon into four-centimetre pieces.'*



'Has Hari divided the ribbon into 10 equal pieces?'

</td>
</tr>
<tr>
<td>4:5</td>
<td>

Repeat the process (steps *4:1– 4:3*) for a divisor of five. As children begin to see the patterns, you will probably need fewer repetitions before moving to the generalisation: ***'If the divisor is five, we can use the five times table to find the quotient.'***

Then provide practice similar to that in step *4:4*, now for a divisor of five. Encourage children to identify the known multiplication fact that they can use to solve each problem, referring to the five times table chart for support.

Example word problems:

- *'Hailey works in a shop that sells ping-pong balls. She needs to put forty balls into bags of five. How many bags does she need?'*
  (quotitive)
- *'It takes Joe five minutes to make a paper party hat. How many hats can he make in sixty minutes?'*
  (quotitive, measures)
- *'There are five horses in a field. Dalir brings forty-five apples to share equally between the horses. How many apples does each horse get?'*
  (partitive)

</td>
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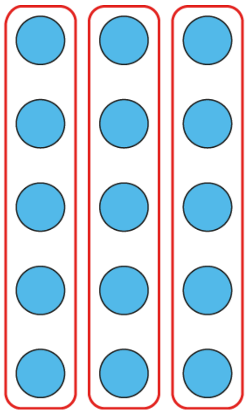
Linking multiplication and division facts – array representation:



$3 \times 5 = 15$ $5 \times 3 = 15$

$15 \div 5 = 3$

- *'The "15" represents the total number of counters.'*
- *'The "5" represents the size of the groups.'*
- *'The "3" represents the number of groups.'*
- *'There are three groups of five in fifteen, so fifteen divided into groups of five is three.'*

</td>
</tr>
</table>

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	• *'Thirty-five kilograms of potatoes are shared equally between five sacks. How many kilograms of potatoes are in each sack?'* (partitive, measures)	Missing-number problems: *'Fill in the missing numbers.'*

× 5 ↓	1		3	4		6	↑ **÷ 5**
	5	10		20	25		

$8 \times 5 = 40$
so
$40 \div 5 = \square$

$6 \times 5 = 30$
so
$30 \div 5 = \square$

$7 \times 5 = \square$
so
$35 \div 5 = \square$

$5 \times 5 = 25$
so
$25 \div 5 = \square$

$5 \div 5 = \square$
$10 \div 5 = \square$
$15 \div 5 = \square$
$20 \div 5 = \square$

$60 \div 5 = \square$
$55 \div 5 = \square$
$50 \div 5 = \square$
$45 \div 5 = \square$

Dòng nǎo jīn:
'Fill in the missing numbers.'

$\square \times 5 = 15$
so
$15 \div 5 = \square$

$\square \times \square = \square$
so
$45 \div 5 = \square$

$\square \times 5 = \square$
so
$30 \div 5 = \square$

$15 \times 5 = 75$
so
$75 \div 5 = \square$

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4:6

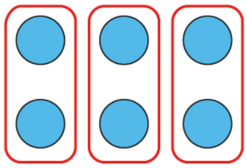
Repeat again for a divisor of two, using the generalisation: ***'If the divisor is two, we can use the two times table to find the quotient.'***

Encourage children to identify the known multiplication fact that they can use to solve each practice-problem, referring to the two times table chart for support.

Example word problems:

- *'There are eighteen bowling shoes on the shelf. How many pairs is this?'* (quotitive)
- *'In a relay race each person runs two kilometres. If the total distance is twelve kilometres, how many people need to be in the relay team?'* (quotitive, measures)
- *'Fabiana is playing a dice game. She needs to roll a total of eight to win. She also needs to roll equal numbers. If she rolls two dice, what number does she need to roll on each dice?'* (partitive)
- *'Diego needs to put twenty litres of water into buckets. If he has two buckets, how many litres of water does he need to pour into each bucket?'* (partitive, measures)

Linking multiplication and division facts – array representation:



$3 \times 2 = 6$ $\qquad$ $2 \times 3 = 6$

$6 \div 2 = 3$

- *'The "6" represents the total number of counters.'*
- *'The "2" represents the size of the groups. '*
- *'The "3" represents the number of groups.'*
- *'There are three groups of two in six, so six divided into groups of two is three.'*

Missing-number problems:

Fill in the missing numbers.

× 2 ↓	1		3	4		6	↑ **÷ 2**
	2	4		8	10		

$5 \times 2 = 10$
so
$10 \div 2 = \square$

$2 \times 7 = 14$
so
$14 \div 2 = \square$

$6 \times 2 = 12$
so
$12 \div 2 = \square$

$2 \times 8 = 16$
so
$16 \div 2 = \square$

$2 \div 2 = \square$

$4 \div 2 = \square$

$6 \div 2 = \square$

$8 \div 2 = \square$

$24 \div 2 = \square$

$22 \div 2 = \square$

$20 \div 2 = \square$

$18 \div 2 = \square$

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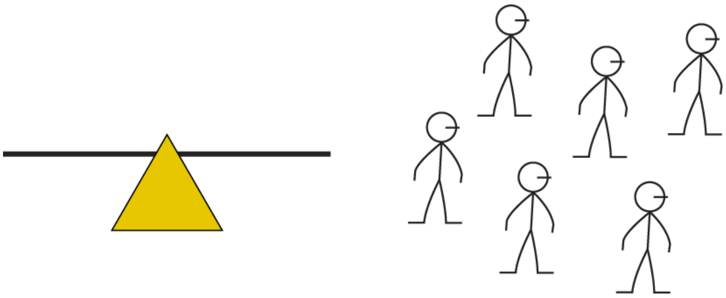
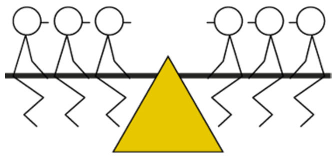
		Dòng nǎo jīn: *'Fill in the missing numbers.'* $\square \times 2 = 6$ so $6 \div 2 = \square$ $\square \times \square = \square$ so $18 \div 2 = \square$ $\square \times 2 = \square$ so $12 \div 2 = \square$ $15 \times 2 = 30$ so $30 \div 2 = \square$
4:7	In segment *2.5 Commutativity (part 2), doubling and halving*, children connected doubling and halving to the two times table. Children may already have noticed that when the divisor is two, the quotient is half of the dividend. In steps *4:7–4:9*, take some time to explore this in detail, linking problems with a divisor of two to known halving facts. Contextually, halving involves dividing a quantity between two, so is a partitive division problem; as such, begin by presenting a partitive division problem with a divisor of two, using a familiar halving context from segment *2.5*. Present the problem, and represent it with a division equation, then link to the language of halving, as shown opposite. Repeat with the same context, increasing the dividend by two each time. Work towards the generalisation: ***'If the divisor is two, the quotient is half of the dividend.'***	*'There are six stick-children who want to play on the seesaw; an equal number of them should sit on each side. How many stick-children should sit on each side?'*  $6 \div 2 = 3$ • *'Six divided between two is equal to three.'* • *'Half of six is three.'* 

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<table>
<tr>
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<td>
<table>
<tr><th>total number of stick-children</th><th>÷</th><th>2</th><th>=</th><th>number of stick-children on each side</th></tr>
<tr><td>6</td><td>÷</td><td>2</td><td>=</td><td>3</td></tr>
<tr><td colspan="5">half</td></tr>
<tr><td>8</td><td>÷</td><td>2</td><td>=</td><td>4</td></tr>
<tr><td colspan="5">half</td></tr>
<tr><td>10</td><td>÷</td><td>2</td><td>=</td><td>5</td></tr>
<tr><td colspan="5">half</td></tr>
</table>
</td>
</tr>
<tr>
<td>4:8</td>
<td>Now, in a similar way, look at a quotitive division problem with a divisor of two. Although the contextual problem can no longer be considered as halving, in the sense of splitting into two equal groups, draw attention to the fact that the equations are the same as those in the previous step, and we can still use our halving facts to find the quotient.</td>
<td>'There are six bowling shoes on a shelf. How many pairs is this?'

$6 \div 2 = 3$
• 'Six divided into groups of two is equal to three.'
• 'Half of six is three.'
</td>
</tr>
</table>

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total number of shoes	**÷**	**2**	**=**	**number of pairs of shoes**
6	÷	2	=	3
		half		
8	÷	2	=	4
		half		
10	÷	2	=	5
		half		

4:9	To assess children's understanding of the connection between halving and division with a divisor of two, present some problems that use a dividend greater than 24; in these problems, children should use a given halving fact to find the quotient, or vice versa, as exemplified opposite.	*'Fill in the missing numbers.'* Half of 56 is 23. 56 ÷ 2 = $\square$ 76 ÷ 2 = 38 Half of 76 is ____. Half of 82 is 41. 82 ÷ 2 = $\square$ 94 ÷ 2 = 47 Half of 94 is ____. 68 pencils are divided between 2 pots. There are 34 pencils in each pot. Half of 68 is ____. Half of 42 is 21. 42 socks are put into pairs. There are ____ pairs of socks.

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Dòng nǎo jīn:

'The table shows how many stickers some children have.'

	Number of stickers
Felicity	37
John	double Felicity's number
Barney	same as John
Barney	74

'Barney shares his stickers equally between two sticker books. Use the information in the table to work out how many stickers are in each of Barney's books.'

4:10

When linking multiplication and division equations, children may over-apply ideas of commutativity, and think that the dividend, divisor and quotient are all interchangeable. For example, they may use the multiplication fact $2 \times 5 = 10$ to write any of the following division equations:

- $10 \div 5 = 2$
- $10 \div 2 = 5$
- $5 \div 10 = 2$ ✗
- $2 \div 10 = 5$ ✗
- $5 \div 2 = 10$ ✗
- $2 \div 5 = 10$ ✗

While the divisor and quotient are interchangeable (as in the first two equations in the list above), the dividend cannot be swapped with the quotient or the divisor.

If children make such mistakes with contextual problems, this can be challenged by asking them to describe what each number in their equation represents. However, it is worth exploring this with all children, using intelligent practice, as exemplified opposite.

Intelligent practice:

'Draw a line to match each multiplication equation with the related division equation.'

Multiplication equations	Division equations
$3 \times 10 = 30$	$40 \div 10 = 4$
$20 = 2 \times 10$	$60 \div 10 = 6$
$6 \times 10 = 60$	$50 \div 10 = 5$
$50 = 5 \times 10$	$80 \div 10 = 8$
$80 = 8 \times 10$	$2 = 20 \div 10$
$4 \times 10 = 40$	$30 \div 10 = 3$

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<table>
<tr>
<td></td>
<td>These ideas will be explored further, and formalised, in segment 2.10 Connecting multiplication and division, and the distributive law.</td>
<td>
True/false problems:

'Use a tick or a cross to show whether each <u>pair</u> of equations is correct or not.'

<table>
<tr><th></th><th></th><th></th><th>Correct (✓) or incorrect (✗)</th></tr>
<tr><td>A.</td><td>$2 \times 5 = 10$</td><td>$10 \div 5 = 2$</td><td></td></tr>
<tr><td>B.</td><td>$5 \times 2 = 10$</td><td>$10 \div 2 = 5$</td><td></td></tr>
<tr><td>C.</td><td>$10 = 2 \times 5$</td><td>$5 \div 10 = 2$</td><td></td></tr>
<tr><td>D.</td><td>$10 = 5 \times 2$</td><td>$5 = 10 \div 2$</td><td></td></tr>
</table>
'Explain why the pairs of equations marked ✗ are not correct.'

'Jasmine is answering this question:
10 children share 40 stickers between them.
How many stickers does each child get?'<br>'Jasmine writes:'
$4 \times 10 = 40$
so
$10 \div 40 = 4$
'Is she correct? Explain.'

</td>
</tr>
<tr>
<td>4:11</td>
<td>In the remaining steps of this teaching point, explore the divisibility rules for divisors of two, five and ten. At this stage, do not include examples where the dividend is zero (this will be explored in the next teaching point).

Begin with divisibility by two. In segment 2.3 Times tables: groups of 2 and commutativity (part 1), children have already generalised that all multiples of two are even numbers. They also know from Spine 1: Number, Addition and Subtraction, segment 10, that a number is even if the ones digit is even. Remind children of these points by looking at the two times table chart.

Then examine some familiar contextual examples of dividing by two, presenting each problem along with</td>
<td>
'Sort these numbers according to whether they can be divided by two or not.'
<table>
<tr><td>4</td><td>62</td><td>2</td><td>7</td><td>100</td><td>5</td><td>31</td><td>34</td><td>48</td><td>99</td><td>43</td></tr>
</table>
<table>
<tr><th>Can be divided by 2</th><th>Can't be divided by 2</th></tr>
<tr><td></td><td></td></tr>
</table>
</td>
</tr>
</table>

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	the completed division equation; for example: • *'There are eight socks. If I put them into pairs, there will be four pairs.'* $8 \div 2 = 4$ • *'There are sixteen children. If they get into groups of two, there will be eight groups.'* $16 \div 2 = 8$ • *'I go into a bakery to buy ten loaves of bread. I can fit two loaves of bread into each bag, so I need five bags.'* $10 \div 2 = 5$ Ask children what they notice about the dividend in each of the equations, prompting them to identify that it is an even number in each case. Work towards the following generalisation: ***'A number is divisible by two if the ones digit is even.'*** Teachers should note that, in fact, any number can be divided by two; here the point is whether dividing by two gives a *whole number* or not. At this stage, however, children are working only within the context of integers, so the generalised statement is used within that framework. The divisibility rules in the upcoming steps involve the same implicit assumption. Provide children with some practice including: • sorting numbers between 1 and 100 according to whether they can be divided by two or not • solving contextual problems; for example, *'Can seventy-six socks be put into a whole number of pairs?'* • writing their own division problems.	

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<table>
<tr>
<td>4:12</td>
<td>
In Spine 1: Number, Addition and Subtraction, segment 1.8, children generalised that all multiples of ten 'end with a zero' (have a ones digit of zero). Explore this now in the context of division.

Examine some familiar contextual examples of dividing by ten; for example:
• 'If I put thirty eggs into boxes of ten, I will need three boxes.'
30 ÷ 10 = 3
• 'If forty cakes are shared equally between ten plates, there will be four cakes on each plate.'
40 ÷ 10 = 4
• 'If a dressmaker cuts a sixty-centimetre length of ribbon into ten-centimetre pieces, she will have six pieces.'
60 ÷ 10 = 6

Ask children what they notice about the dividend in each of the equations, prompting them to identify that the ones digit is equal to zero in each case, and work towards the following generalisation: 'A number is divisible by ten if the ones digit is zero.'

Provide practice similar to that described in step 4:11.
</td>
<td>
• 'Are there equal groups of 10 in 34? Use counters to explain your answer.'

• 'Which year group(s) can be put into teams of ten?'
<table>
<tr><th>Year group</th><th>Number of children</th></tr>
<tr><td>1</td><td>150</td></tr>
<tr><td>2</td><td>135</td></tr>
<tr><td>3</td><td>120</td></tr>
</table>
</td>
</tr>
<tr>
<td>4:13</td>
<td>
Finally, repeat for division by five. From segment 2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1, children are already familiar with the fact that products in the five times table have a ones digit of zero or five.

Examine some familiar contextual examples of dividing by five; for example:
• 'There are fifteen biscuits. If I put them into bags of five, I will need three bags.'
15 ÷ 5 = 3
</td>
<td>
• 'Stickers come in sheets of five. How many stickers could I have altogether? Circle the correct answers.'
40   105   52   5   75   90

• 'There are forty conkers.'
  • 'Cecily says that they can only be shared equally between ten children.'
  • 'Akish says they can only be shared equally <u>either</u> between ten children <u>or</u> between five children. '
  • 'Niran says they can only be shared equally <u>either</u> between ten children <u>or</u> between two children.
'Are any of them correct? Explain.'
</td>
</tr>
</table>

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	• *'If twenty conkers are shared equally between five children, each child will get four conkers.'* 20 ÷ 5 = 4 • *'A dressmaker has a ribbon that is thirty centimetres long. She needs five-centimetre lengths. She can make six five-centimetre lengths.'* • 30 ÷ 5 = 6 Work towards the generalisation: ***'A number is divisible by five if the ones digit is five or zero.'*** Again, provide practice similar to that described in step *4:11*.	

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Teaching point 5:

When the dividend is zero, the quotient is zero; when the dividend is equal to the divisor, the quotient is one; when the divisor is equal to one, the quotient is equal to the dividend.

Steps in learning

	Guidance	Representations
5:1	In segment *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1*, children learnt the following generalisations: • ***'When zero is a factor, the product is zero.'*** • ***'When one is a factor, the product is equal to the other factor.'*** In this teaching point, these concepts are linked to division. Begin by reminding children that when zero is a factor, the product is zero. Then present a multiplication equation where one of the factors is zero (e.g. $0 \times 5 = 0$). Describe the equation in terms of groups of five: *'I have nothing. I don't have any groups of five.'* Then connect to a quotitive division problem, for example, '*I am putting biscuits into bags of 5. I have zero biscuits. How many bags of 5 can I make?'* Write the associated division expression and, through discussion, encourage children to reason that since you don't have any biscuits, you can't make any bags of five biscuits. Work through some more examples like this, until children are familiar with the idea that if the total number is zero, the number of equal groups that can be made is zero. Include all three possible situations: • $0 \div 2 = 0$ • $0 \div 5 = 0$ • $0 \div 10 = 0$	Multiplication equation with a factor of zero: $0 \times 5 = 0$ • *'One of the factors is zero so the product is zero.'* • *'Zero groups of five is zero.'* Related quotitive division problem: *'I am putting biscuits into bags of five. I have zero biscuits. How many bags of five can I make?'* $0 \div 5 = \boxed{}$ • *'I don't have any biscuits. I can't make any bags of five biscuits.'* $0 \div 5 = \boxed{0}$ • *'Zero divided into groups of five is equal to zero.'*

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5:2	Now explore some *partitive* division problems with a dividend of zero, as exemplified opposite. As before, include cases with divisors of two, five and ten.	Multiplication equation with a factor of zero: $0 \times 2 = 0$ • *'One of the factors is zero so the product is zero.'* • *'Zero times two is equal to zero.'* Related partitive division problem: *'I am sharing apples between two people. I have zero apples. How many will each person get?'* $0 \div 2 = \boxed{}$ • *'I don't have any apples. Neither of the people get any apples.'* $0 \div 2 = \boxed{2}$ • *'Zero divided between two is equal to zero.'*
5:3	Write out the three division equations explored in the previous steps: • $0 \div 2 = 0$ • $0 \div 5 = 0$ • $0 \div 10 = 0$ Ask children: • *'What's the same?'* • *'What's different?'* working towards the generalisation: '***When the dividend is zero, the quotient is zero.'*** Note that, at this stage, children do not need to consider why we cannot divide by zero (i.e. why we cannot have a divisor of zero). However, it is worth *teachers* noting that the relationship between multiplication and division highlights why; for example, the calculation $12 \div 0 = ?$ is equivalent to writing $0 \times ? = 12$, which is clearly impossible.	

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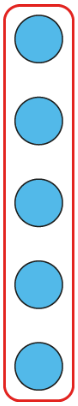
<table>
<tr>
<td>5:4</td>
<td>Now follow a similar progression for cases where the divisor is equal to the dividend, working through several quotitive and partitive examples as exemplified opposite and on the next page. You can use an array representation for support.

Then compare the three division equations:
• $2 \div 2 = 1$
• $5 \div 5 = 1$
• $10 \div 10 = 1$

working towards the generalisation:
‘When the dividend is equal to the divisor, the quotient is one.’</td>
<td>Quotitive structure:
• Multiplication equation with a factor of one:



$1 \times 5 = 5$ $5 \times 1 = 5$
• ‘One of the factors is one so the product is equal to the other factor.’
• ‘One five is equal to five.’
• ‘Five, one time is equal to five.’
(see 2.6 Representations, slide 65)

• Related quotitive division problem:
‘I am putting biscuits into bags of five. I have five biscuits. How many bags of five can I make?’
$5 \div 5 = \boxed{}$
• ‘I have five biscuits. I can put them all into one bag to make one bag of five.’
$5 \div 5 = \boxed{1}$
• ‘Five divided into groups of five is equal to one.’
(see 2.6 Representations, slide 66)</td>
</tr>
</table>

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Partitive structure:

- Multiplication equation with a factor of one:

$1 \times 5 = 5$ $5 \times 1 = 5$

  - *'One of the factors is one so the product is equal to the other factor.'*
  - *'One five times is equal to five.'*
  - *'Five ones is equal to five.'*

(see *2.6 Representations*, slide *67*)

- Related partitive division problem:
  'I am sharing apples between five people. I have five apples. How many will each person get?'

  $5 \div 5 = \boxed{}$

  - *'There are the same number of apples as there are people, so each person gets one apple.'*

  $5 \div 5 = \boxed{1}$

  - *'Five divided between five is equal to one.'*

(see *2.6 Representations*, slide *68*)

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5:5	Similarly, work through examples where the divisor is equal to one. Compare the resulting division equations: • $2 \div 1 = 2$ • $5 \div 1 = 5$ • $10 \div 1 = 10$ working towards the generalisation: ***'When the divisor is equal to one, the quotient is equal to the dividend.'***	Quotitive structure: • Multiplication equation with a factor of one  $5 \times 1 = 5$ $1 \times 5 = 5$ • *'One of the factors is one so the product is equal to the other factor.'* • *'Five ones is equal to five.'* • *'One, five times is equal to five.'* (see *2.6 Representations*, slide *69*) • Related quotitive division problem: *'I am putting biscuits into bags of one. I have five biscuits. How many bags of one can I make?'* $5 \div 1 = \boxed{}$ • *'I have five biscuits. I can put one in each bag, so I need one bag for every biscuit; I need five bags.'* $5 \div 1 = \boxed{5}$ • *'Five divided into groups of one is equal to five.'* (see *2.6 Representations*, slide *70*)

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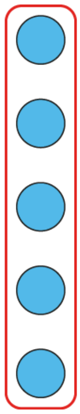
		Partitive structure: • Multiplication equation with a factor of one:  $5 \times 1 = 5$ $1 \times 5 = 5$ • *'One of the factors is one so the product is equal to the other factor.'* • *'Five, one time is equal to five.'* • *'One five is equal to five.'* (see *2.6 Representations*, slide *71*) • Related partitive division problem: *'I am sharing apples between one person. I have five apples. How many will the person get?'* $5 \div 1 = \boxed{}$ • *'There is one person, so they get all of the apples; the person gets five apples.'* $5 \div 1 = \boxed{5}$ • *'Five divided between one is equal to five.'* (see *2.6 Representations*, slide *72*)
5:6	Provide children with some missing-number problems, extending to examples where the divisor is something other than two, five or ten (so that children must apply the generalisation and reasoning around the values of zero and one, rather than using known facts from their times tables). Also provide further contextual problems, similar to those used throughout this teaching point.	*'Fill in the missing numbers.'* $\boxed{} \times 5 = 15$ $15 \div 5 = \boxed{}$ $\boxed{} \times 5 = 10$ $10 \div 5 = \boxed{}$ $\boxed{} \times 5 = 5$ $5 \div 5 = \boxed{}$ $\boxed{} \times 5 = 0$ $0 \div 5 = \boxed{}$

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		'Fill in the missing numbers.' $0 \times 10 = \square$ $\square \div 10 = 0$ $1 \times 10 = \square$ $\square \div 10 = 1$ $2 \times 10 = \square$ $\square \div 10 = 2$ $3 \times 10 = \square$ $\square \div 10 = 3$ $6 = \square \times 2$ $\square = 6 \div 2$ $4 = \square \times 2$ $\square = 4 \div 2$ $2 = \square \times 2$ $\square = 2 \div 2$ $0 = \square \times 2$ $\square = 0 \div 2$ $2 \times \square = 0$ $0 \div 2 = \square$ $4 \times \square = 0$ $0 \div 4 = \square$ $6 \times \square = 0$ $0 \div 6 = \square$ $1 = 1 \times \square$ $1 \div 1 = \square$ $3 = 3 \times \square$ $3 \div 3 = \square$ $5 = 5 \times \square$ $5 \div 5 = \square$ $7 = \square \times 1$ $7 \div 1 = \square$ $9 = \square \times 1$ $9 \div 1 = \square$ $11 = \square \times 1$ $11 \div 1 = \square$

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Dòng nǎo jīn:

'Fill in the missing numbers.'

	dividend	**divisor**	**quotient**
dividend			0
divisor			5
quotient	0	10	

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