Secondary Magazine Issue 74

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 74 of the Secondary Magazine.

Contents

Contributors to this issue include: Peter McOwan, Mary Pardoe, Richard Perring and Peter Ransom.

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for Excellence in the
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From the editor

We hope you will find something of interest in Issue 74 of the Secondary Magazine. This issue is not intended to be an illusion! However it does focus on optical illusions that may help to engage, and possibly enlighten, students. And the subject of The Interview is well known for his fascinating demonstrations and explanations of some illusions.

The 'illusions' that feature in discussions about mathematics education are usually described as illusions ***of*** something.

The *illusion of linearity* appears to the many secondary school students who treat all relationships between mathematical entities as if they were linear even when they are not. This illusion can be a significant obstacle to learning unless it is revealed and dealt with.

But another quite different phenomenon is also called the *illusion of linearity*. This is the perception of mathematics as essentially a linear activity and a linear body of knowledge. Many mathematicians believe that the general public is taken in by this illusion because school mathematics curricula create an illusory impression that learning mathematics is like climbing a ladder – having mastered each step completely you can start to learn the next step completely. They also believe that non-mathematicians think that professional mathematicians proceed linearly, always progressing along a well-defined route to a final conclusion. But mathematicians' experiences of mathematics are actually more like explorations of tangled bushes – with many roots and branches that sometimes join up, and buds that appear and sprout new twigs.

Some mathematicians believe that the *illusion of certainty* in mathematics is another aspect, related to the previous illusion, of the public perception of mathematics, and of the way most students view mathematics. Many students perceive mathematics as something that has already been 'cut and dried' – finished off – by others. But professional mathematicians believe that there will never be an end to responding to some unanswered questions in mathematics. Are all students developing a true appreciation of the fundamental nature of mathematics?

Are we as teachers sometimes taken in by another illusion – the *illusion of following an intended curriculum*? Standards, curriculum maps and textbooks lay out an intended curriculum. But what happens on a daily basis in the classroom is a result of many factors, such as our own past experiences and those of our students. What we 'follow' in the classroom is the implemented curriculum – into which the intended curriculum is transformed.

What other *illusions of*… have you come across in your mathematics teaching and learning?

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It's in the News!
Antimatter

The fortnightly *It's in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but as a framework which you can personalise to fit your classroom and your learners.

The Large Hadron Collider at Cern is rarely out of the news! Recently, in a story that could be taken from *Star Trek*, it was announced that scientists there had trapped antimatter!

When matter and antimatter meet they destroy each other and, in this *It's in the News!*, this context is used to offer an image for students to develop their understanding of addition and subtraction with negative numbers.

This issue's activity is more focussed and less open-ended than usual but is intended as the catalyst for student discussion, not just for answering the questions posed in the slides.

Download this *It's in the News!* resource - in PowerPoint format

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The Interview

Name: Peter McOwan

About you: I'm a Professor of Computer Science at Queen Mary, one of the largest colleges in the University of London. I spend my time teaching and trying to understand how the brain works, and how to apply this knowledge to build artificial intelligence and robots that interact with humans. I also enjoy doing a bit of magic now and again!

The most recent use of mathematics in your job was…
I've just finished writing a paper with colleagues in Psychology at UCL and in Japan on using the Taylor series to try and explain how we perceive motion. I love Taylor series - if you know what's happening at x, from a Taylor series you can predict what's happening at x+h, and that's a very useful thing for a person or a robot to be able to do. I've also been using some clever statistical techniques to create the female version of a male face and vice versa, which, as well as being useful in face perception experiments, is also great fun.

Why mathematics?
I've loved solving puzzles since I was a kid – which is the way most mathematicians start. When I was at school I discovered that maths could let you 'see into the future' (well, predict what was going to happen). I can still remember when I was able to use maths to predict where a beam of light would go after it went through a glass block or two – there was my pencil dot on the paper, I switched on the light beam… and Bingo! Later on, I realised that maths can also help us understand and predict how our brains work – once you have that mathematics right you know more about people's brains and can transfer that same maths into a computer on a robot to give the robot human-like sight.

Some mathematics that amazed you is…
Some of the maths behind professional magicians' card tricks and mathematical-based con tricks (scroll down) is really amazing, but I can't tell you about it, sorry!

A significant mathematics-related incident in your life was…
I was working at UCL on a mathematical model for human motion perception, and the maths kept on indicating that the static background in a particular pattern would be moving. I checked and checked for my mistake, and eventually someone suggested I programme up the pattern and have a look at it on a computer display. In one of those 'OMG' moments, there it was, the static background looked as if it was moving, and my maths had predicted a brand new optical illusion. Awesome maths moment!

The best book you have ever read is…
Tough one! Up there would need to be Mathematics, Magic and Mystery, by Martin Gardner - a classic that I constantly re-read. I also really enjoyed Hiding the Elephant: How Magicians Invented the Impossible by famous magic illusion-creating engineer Jim Steinmeyer, a book that shows that without maths and science there would be no magic.

Who inspired you?
My parents and grandparents, they all loved learning new things, and that gave me a great start. I also had some amazing teachers at school who took the time to inspire me with science tales from beyond the

school curriculum. Historically, one of my heroes is Leonardo da Vinci - he combined maths, science and art and had some amazing inventive ideas. Today, I'm inspired by people like video games guru and space adventurer Richard Garriott. Again, he blends science, maths and art along with a passion for telling science stories. He's also a fellow magician and we worked together on some ideas for science-based magic tricks that he performed when he flew up to the international space station in 2008. I helped get Our Space off the ground, where there's some magic in space. You can say maths doesn't get you places! And, finally and fictionally, TV's timelord The Doctor, splendid fellow, all of them!

If you weren't doing this job you would…
I almost went to film school, I'd have loved to be a film director, but now I'm glad I'm a scientist, because I have far more interesting stories to tell. I also help TV companies with science ideas for shows, and do a lot of maths magic shows round the country – so my life is a little bit showbiz after all.

Can you teach us a short mathematical magic trick?
Better than that, if you follow this link, presto. There are a couple of free-to-download books I've co-written on maths, computer science and how the maths that makes the magic possible also lets us build some clever computer software. Enjoy...and for my next trick…

Focus on…illusions

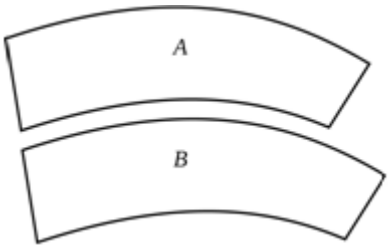
Because illusions are deceptive or counterintuitive phenomena, displaying them and encouraging discussion about them can sometimes be a fascinating and effective way to engage students in thinking about mathematical ideas. Therefore a collection of illusions may be a useful, as well as an intriguing, resource.

Below is a brief look at some kinds of illusion that may be illuminating.

Misleading diagrams

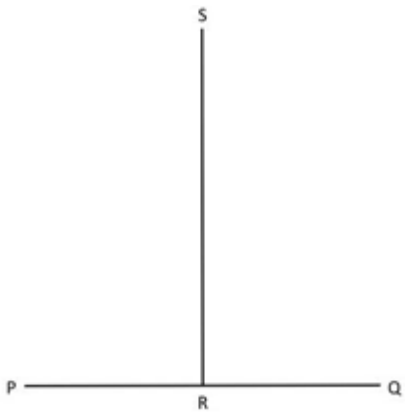
Students need to be aware that reasoning about a situation can rarely depend for its validity on what they merely 'see immediately' in a sketch representing the situation. They also need to take care when creating their own diagrams. Illusions that may help stimulate discussion about misleading diagrams, and prompt students to think of their own examples, include:

- the Jastrow illusion…which shape, A or B, is larger?



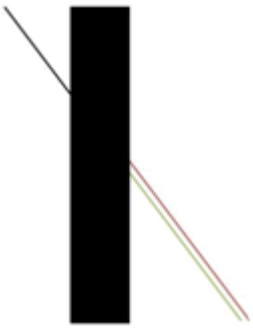
The two shapes, A and B, are actually identical – you could display the diagram on the whiteboard then move one shape to fit exactly over the other.

- the vertical-horizontal illusion…which line segment, PQ or RS, is shorter?



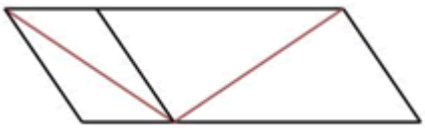
The horizontal line segment, PQ, and the vertical line segment, RS, are exactly the same length. Again you might rotate and then shift one line segment to fit on the other.

- the Poggendorff illusion…is the black line segment a continuation of the green line or the red line?



It is the green line continued.

- the parallelogram illusion…in each diagram how are the lengths of the two red line segments related?



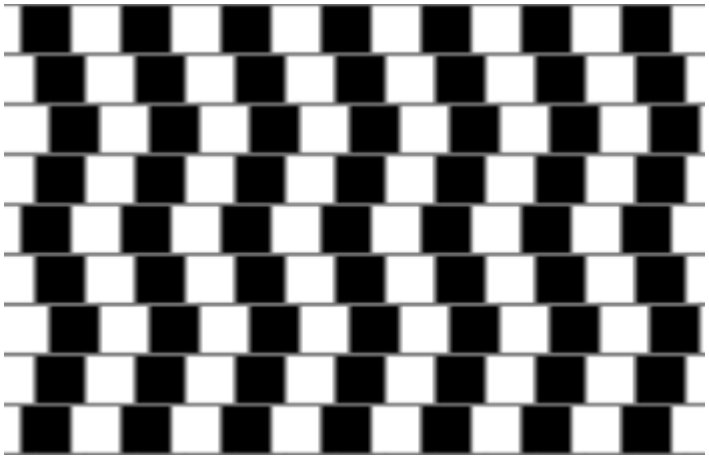
In both diagrams, the two red line segments are equal in length.

The Mülller-Lyer Illusion and Ponzo's Illusion are other well known illusions in which one line segment looks longer than another line segment that is actually the same length.

Parallel lines

Some illusions can encourage students to put into words their own explanations of the meaning of 'parallel'. For example:

- the café wall illusion…which lines in this diagram are parallel?



Café wall illusion image by Fibonacci

- in the Hering Illusion are the two bold lines parallel, and in the Wundt Illusion are the two red lines parallel?

  Could your students design other arrangements of line segments that create illusions in which parallel lines do not look parallel? They might, for example create illusions that are rather like Orbison's Illusion or Zöllner's Illusion.

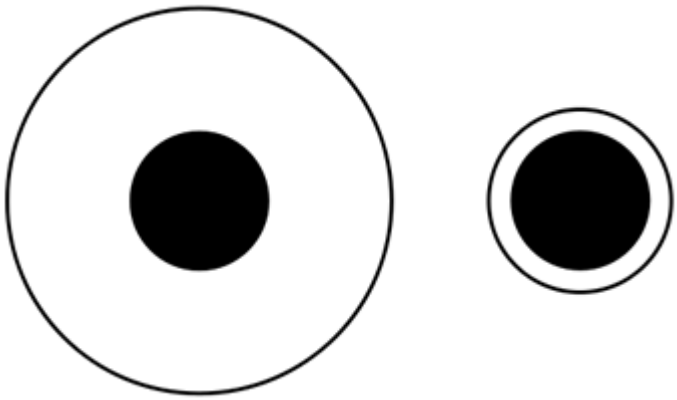
- the Ehrenstein Illusion is quite similar, but the bent lines illusion is created using concentric circles rather than straight-line segments.

Circles

Illusions that involve the areas of circles can focus students' attentions on relations between lengths, such as the radii of circles and widths of rings, and the areas of circles and rings. For example:

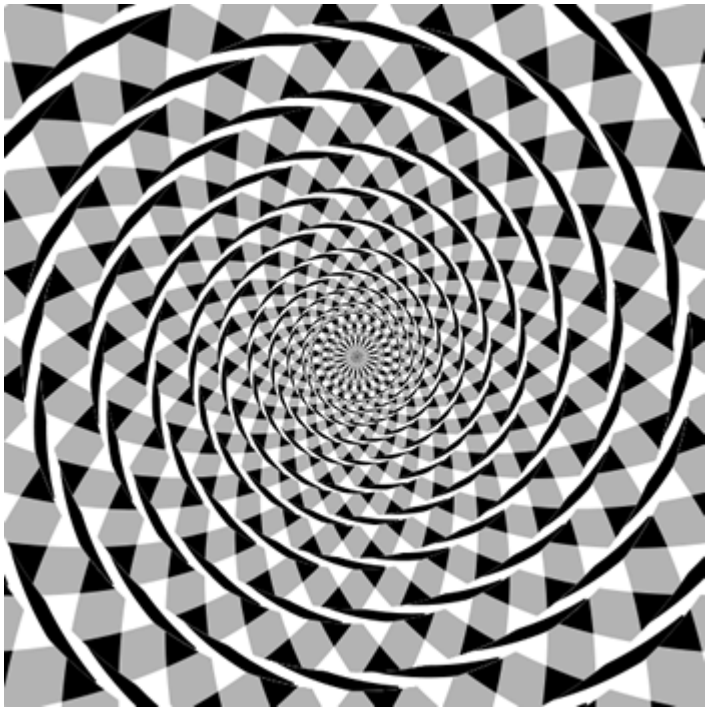
- the Bullseye Illusion immediately demands that students attend to the relation between the radius and area of any circle, and to calculating the area of a ring

- and in the Delboeuf Illusion…do the black circles appear to be the same size?



Delboeuf illusion image by Washiucho

- the Fraser's Spiral illusion also involves concentric circles, but where are the rings?



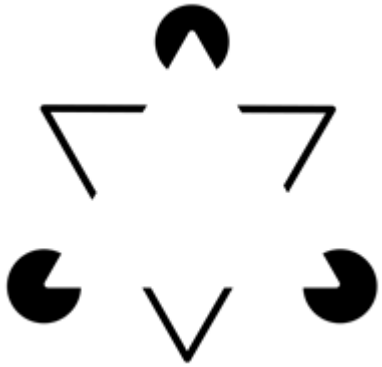
Fraser's Spiral illusion image by Mysid

- Arcs, rather than whole circles, feature in the Benham's Wheel.

Reification

Reification, in the context of visual images, is the phenomenon in which the mind 'brings into being' something that is not actually present in an image. For example:

- in the Kanizsa Triangle illusion how many triangles do you see?



Kanizsa triangle illusion image by JrPol

- this is an illusory cube:

Illusory cube image by Bernard Ladenthin

Could your students create their own illusory contour figures?

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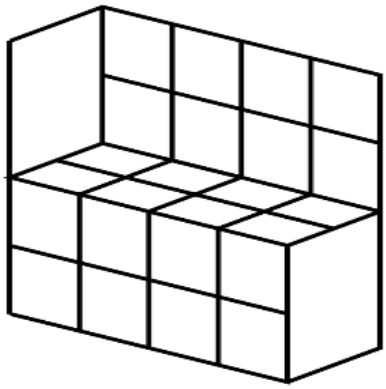
Multi-stability in 2-D sketches of 3-D objects

It is sometimes possible to interpret in more than one way a 2-D image that is intended to represent a 3-D structure or object – to 'flip' between seeing one object then another, or between one view then another of the same object. For example:

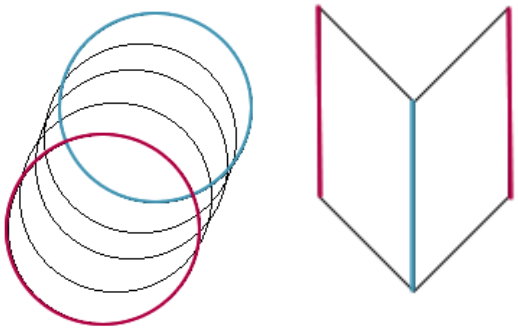
- many illusions that use the Necker Cube can be created, such as this 2-D representation of a 3-D object – challenge students to describe the object!



Necker cube illusion image by Alex Turner

- the Schroeder Stairs are similar – are you looking at the stairs from above or below?

- which is the front rim of the 'cylinder' – the red circle or the blue circle? And which is nearer to you – the blue edge joining the two pages of a book or the red edges?



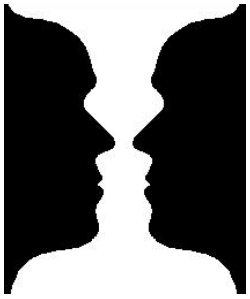
Tube of circles based on image by Fastily
and Isometric pages based on image by Carsrac

Students may enjoy designing their own 'ambiguous' images – perhaps incorporating the Necker Cube, the 'two-way' cylinder or the 'two-way' pages.

Multi-stability in other images

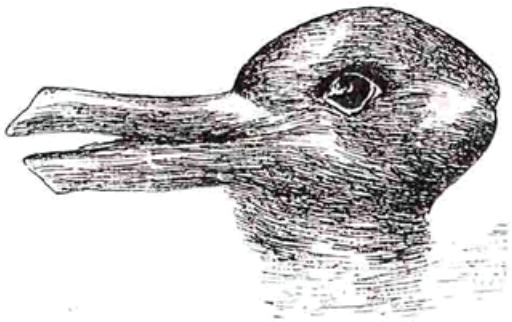
The act of moving from one perception of a multi-stable image to another is sometimes called a *Gestalt Switch*. Students usually find gestalt switches fascinating. Well known gestalt switch illusions include:

- many versions of the Goblet illusion – do you see two black faces or a white goblet on a black background?



Goblet illusion image by mikkalai

- various versions of the Duck-Rabbit illusion



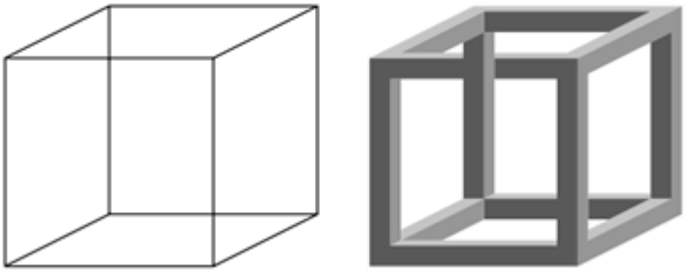
Duck rabbit illusion image by J. Jastrow

- the Young Girl-Old Woman Illusion.

3-D impossible shapes

Multi-stability is exploited to create images of impossible shapes that cannot be built – such as impossible buildings in the artwork of M. C. Escher.

- the impossible Freemish Crate, which is in Escher's 1958 lithograph Belvedere, is derived from the Necker Cube illusion.

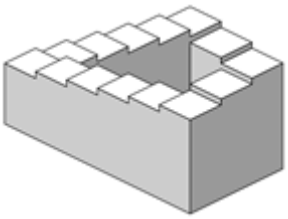


Freemish Crate/Necker cube illusions - image by Boivie based on the work of RTCNCA

- as is the Penrose Rectangle.
- other images of impossible shapes by Professor Sir Roger Penrose include the Penrose Triangle

Penrose Triangle image by Metoc

and the Penrose Stairway.

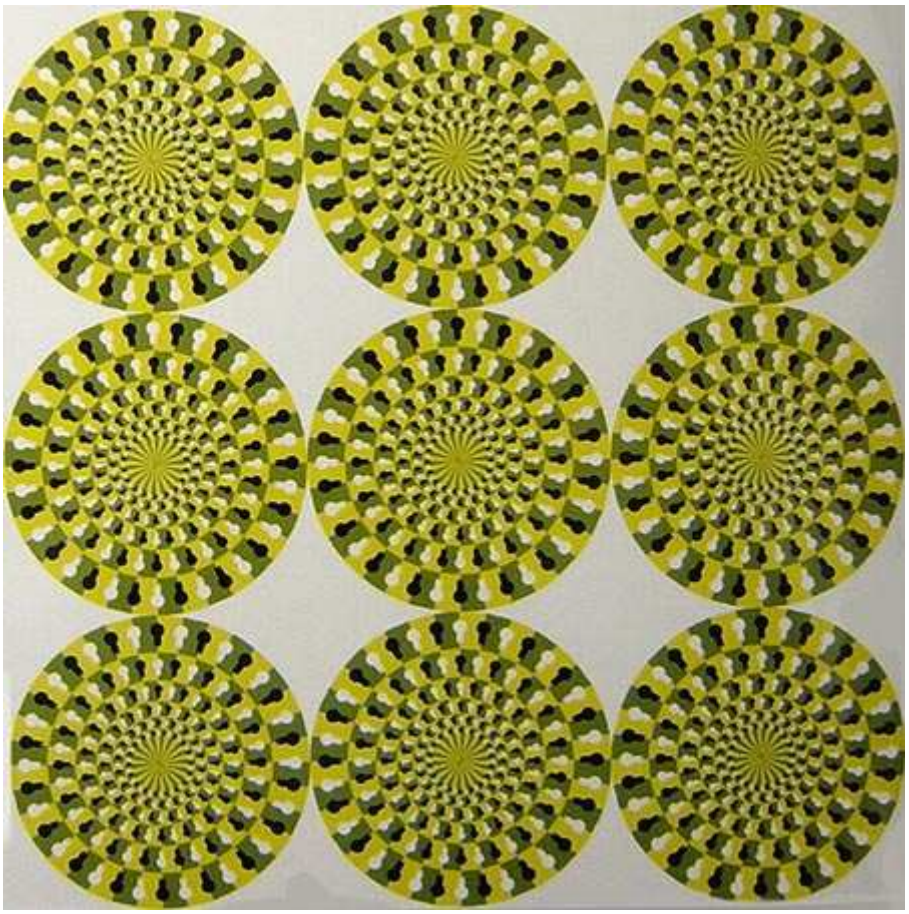


Impossible stairs image by Sakurambo

Illusions of movement

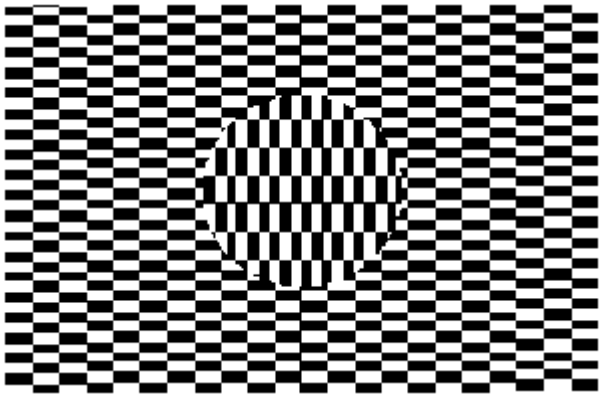
Many beautiful optical illusions appear to show movement even though the images are actually static. Such images might enhance students' explorations of the mathematics of rotation – in a way that students do not expect.

- this image is similar to the first of the many amazing images that you can see on Akiyoshi's illusion pages.



Moving circles image by Master Hand

- some illusory motion is not rotation – as in these anomalous motion illusions.
- in many illusions of movement a part of the image that is inset appears to move.



Ouchi's pattern image by RobinH

Good places to start looking for illusions that will focus the attention of students on particular themes or questions – while fascinating the students – are WolframMathWorld, Akiyoshi's illusion pages, and Cut the Knot.

Illusions are shown and explained in the context of scientific explorations in The Perception Deception: maths made optical illusions pages of CS4N Computer Science For Fun.

You might also enjoy some of the featured Mighty Optical Illusions.

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5 things to do this fortnight

- Would you like to witness an amazing magic show, and sneak behind the scenes to explore the maths and computing powering the tricks? You can do just that by going to a free public lecture on 25 January 2011 from 5pm at the Manchester Museum at the University of Manchester. This Computer Science Magic Show will be performed by Peter McOwan, Professor of Computer Science at Queen Mary, University of London, and the subject of our interview in this issue. Places are limited so you will need to book online or by phone.

- At the Ringing and Schools page of the bell-ringers of All Saints church, Wokingham, you will find ideas for projects about the mathematics of bell-ringing, an invitation to discuss your questions and ideas, and offers of related lectures that can be arranged for your students. From the All Saints site you can also reach the informative and fascinating Bellringing and Ringing Shapes pages of John Harrison's website. John, who is the Tower Foreman of the All Saints ringers, has explored, and created models of, shapes obtained by mapping (in a particular way) the numbers in rows of change-ringing methods to coordinates in three-dimensional space. His original, and as yet un-published, ideas and findings are well worth investigating! (In Issue 70 of this magazine, the focus was on change-ringing).

- Do you know any 12–18 year old students who would like to take part in the 3rd European Student Conference in Mathematics, which will take place in Athens from 30 March to 3 April 2011? This conference will consist of workshops and sessions on a wide variety of themes such as space, life and fractals. Students are encouraged to present their own work on projects on any of the many themes.

- Is there anything mathematically interesting about the paper sizes we use? Professor John D Barrow FRS may surprise you if you attend his free Gresham College public lecture, The Uses of Irrationality: Paper Sizes and the Golden Ratio at 1pm on 11 January in the Museum of London.

- Wednesday 22 Dec 2010 is the birth date of Srinivasa Ramanujan. You could celebrate it by watching the four-minute Indian film about his life, God, Zero and Infinity, seeing Singing Banana tell the taxicab story of GH Hardy and Ramanujan, and finding an interesting fact about every whole number to 9 999 at What's Special About This Number?.

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Subject Leadership Diary

What's the next number in this sequence: 0, 0, 1, 0, 0, 3, 4, 0, …, and why? (hint: drip, drip, drip,…).

My answer is 38. Why? It is because these are the millimetres of rain that have fallen in the first 9 days of November in my garden. I have to confess to being a weather enthusiast and have kept a record of the rain fallen in the garden over the last 7 years. I also have 53 aneroid barometers on the walls in the hall – just to check on what the weather is likely to do. I make no apologies for this fetish since I use the rainfall data in the classroom, and I enjoy bringing barometers back to pristine condition after rescuing them from charity shops – where I do not pay more than £5 for one.

Working with real data in the classroom is a must if we wish to motivate students to study statistics, which is probably one of the most useful topics of mathematics that they will use later in life. We are surrounded by statistics – and if we can make our students aware of how to use them sensibly while not getting taken in by misleading statistics or false claims, then at some point in the future they will appreciate what they have learnt.

My worry is the way I see GCSE Statistics going with controlled assessment. In the past, all 60+ of our students doing GCSE Statistics in Year 10 have chosen their own topic for coursework. Their personal choices have varied quite considerably – from carrying out a graveyard survey at a local church, comparing data gathered from that with data from a similar survey done while on holiday in another part of the country, and also examining local death records, to a monthly record of blood sugar levels, carbohydrate intake and insulin dosage kept over a period of time by a diabetic student. All the topics chosen by students were relevant to them, and motivated them to do what interested them (at least for the first week). Then, using and applying their statistical skills, they consolidated their findings by comparing their real primary data with secondary data (though at times it was very difficult for them to find appropriate secondary data). Producing questionnaires or surveys, and enticing people to complete them, raised awareness of how difficult some face-to-face tasks are, yet gave them some understanding of what they might have to do in their future careers.

So, let us return to the rainfall data. How can this be used in the classroom? In the past I have had students who are interested in doing a weather related topic make their own rain gauge (see details at the Met Office), where there is a stack of teacher resources, and collect data over a period of time. They can then use my record (secondary data) to compare two locations, or compare their results with conclusions from data gathered in different parts of the country. They can calculate moving averages (I have seven years of data), and see whether there is any climate change in the area – are we really getting more rain, or is it just the same but coming in fewer days? Of course I will not tell them what hypotheses to investigate or how to do it. This work links in with our status as an eco-school.

Some students love this freedom to investigate something that interests them; others hate it because they have to think for themselves. However we have students who later on in life tell us how useful the GCSE Statistics course has been – especially those who go on to take subjects which involve a good deal of statistics, such as geography. A couple of years ago I came across a student who had been in one of my GCSE Statistics groups in the late 80s. He was now a research archaeologist, and he came into school and talked to the 60 students on the current statistics course about how statistics were used in archaeology, and about how he was benefitting from having studied the subject at school.

Let us hope that the change to GCSE Statistics with controlled assessment does not kill the motivational momentum made by learners following their own interests!