Maths HUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.8 Times tables: 3, 6 and 9, and the relationship between them

Teacher guide | Year 3

Teaching point 1:

Counting in multiples of three can be represented by the three times table. Adjacent multiples of three have a difference of three. Facts from the three times table can be used to solve multiplication and division problems with different structures.

Teaching point 2:

Counting in multiples of six can be represented by the six times table. Adjacent multiples of six have a difference of six. Facts from the six times table can be used to solve multiplication and division problems with different structures.

Teaching point 3:

Products in the six times table are double the products in the three times table; products in the three times table are half of the products in the six times table.

Teaching point 4:

Counting in multiples of nine can be represented by the nine times table. Adjacent multiples of nine have a difference of nine. Facts from the nine times table can be used to solve multiplication and division problems with different structures.

Teaching point 5:

Products in the nine times table are triple the products in the three times table. Products that are in the three, six and nine times tables share the same factors.

Teaching point 6:

Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by three, six or nine.

Overview of learning

In this segment children will:

- skip count in threes/sixes/nines and build up the three/six/nine times tables
- explore links between the three and six times tables, applying their knowledge of doubling and halving
- explore links between the three and nine times tables, applying their knowledge of multiplying by three, and using the language of 'tripling'
- explore links between products in the three, six and nine times tables
- learn and apply divisibility rules for three, six and nine.

The teaching points in this segment (with the exception of *Teaching point 5*) follow a similar progression to that used when learning the two, four and eight times tables, and exploring the links between them (segment *2.7 Times tables: 2, 4 and 8, and the relationship between them*). By now, children should be gaining confidence in linking skip counting, grouping and multiplication to build up times tables. Through this segment, they will also be developing a greater sense of connections between times tables and the 'families' of related times tables (five and ten; two, four and eight; three, six and nine).

Teachers are encouraged to continue building up the class multiplication chart as each times table is covered (first introduced in segment *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1*):

×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42		56	63	70		
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66		88	99	110		
12	0	12	24	36	48	60	72		96	108	120		

Key: new facts in this segment
previously learnt facts
relevant previously learnt facts (commutativity)

The chart, along with the relationships between the times tables, should be used to help children to see that there are very few truly 'new' facts to be learnt in the three, six and nine times tables. However, in order for children to become fluent with these times tables, as well as using the connections between them (and with previously learnt tables), regular practice will be needed both in reciting the times tables (for example, '*One six is six, two sixes are twelve…*') and with isolated multiplication facts (for example, *'I know that seven times six is equal to forty-two.'*)

As in segment *2.7*, since children have already been introduced to division (segment *2.6 Structures: quotitive and partitive division*), and calculation of quotients using multiplication facts, division is embedded in the times table practice steps of this segment. Teachers should ensure that contextual division practice encompasses both the quotitive and partitive structures of division. Similarly, children have already been introduced to the 'one equation, two interpretations' concept of commutativity (segment *2.5 Commutativity (part 2), doubling and halving*) where, for example, 7×3 can represent either seven groups of three or three groups of seven. As such, practice also includes application of three/six/nine times table facts to solve problems about three/six/nine equal groups (distinct from problems about groups of three/six/nine).

Teaching point 5 explores the relationship between the three and nine times tables, so differs slightly from the relationships that children have learnt so far, since it is a *tripling* relationship rather than a *doubling* relationship. Aside from that difference, the progression is very similar to that used to explore the relationship between the three and six times tables (*Teaching point 3*).

As well as learning, applying and connecting the three, six and nine times tables, children will add to their set of divisibility rules (*Teaching point 6*).

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast*. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.*

Teaching point 1:

Counting in multiples of three can be represented by the three times table. Adjacent multiples of three have a difference of three. Facts from the three times table can be used to solve multiplication and division problems with different structures.

Steps in learning

1:1

Guidance

This teaching point follows the same progression for the three times tables as that for the four and eight times tables in segment *2.7 Times tables: 2, 4 and 8, and the relationship between them.*

Children will not have prior experience of skip counting in groups of three, so begin by looking at some groups of three, linking enumerating objects in groups of three with counting in threes, and writing the associated multiplication equations (write two equations for each example, as shown opposite). Use contexts that children already associate with three; for example, wheels on a tricycle. Use three-value counters alongside each context to support the idea of unitising in threes, and use a number line with the multiples of three highlighted for skip-counting support.

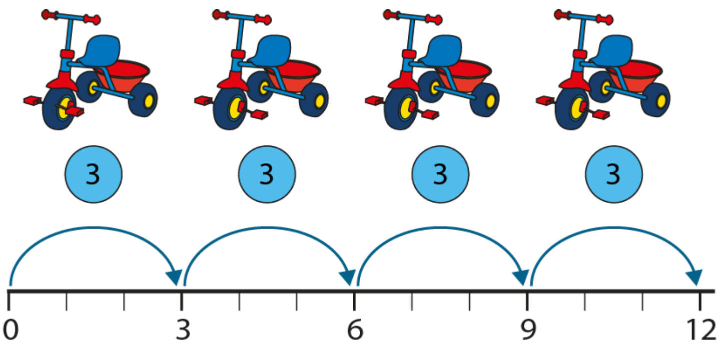
For each example, ask children to describe what each number in the equation represents:

- *'What does the "4" represent?'*
  'The "4" represents the number of tricycles.'
- *'What does the "3" represent?'*
  'The "3" represents the number of wheels on each tricycle.'
- *'What does the "12" represent?'*
  'The "12" represents how many wheels there are altogether.'

Representations

Example 1:

'How many wheels are there? Count in groups of three.'



- *'Three, six, nine, twelve. There are twelve wheels.'*
- *'There are four groups of three; there are twelve altogether.'*
- *'There are three, four times; there are twelve altogether.'*

$4 \times 3 = 12$ $\qquad$ $3 \times 4 = 12$

- *'Four is a factor.'*
- *'Three is a factor.'*
- *'The product of four and three is twelve'*
- *'Twelve is the product of four and three.'*

Example 2:

'Show me five groups of three.'

- *'How many dots are there? Count in groups of three.'*
- *'Three, six, nine, twelve, fifteen. There are fifteen dots.'*
- *'There are five groups of three; there are fifteen altogether.'*
- *'There are three, five times; there are fifteen altogether.'*

$5 \times 3 = 15$ $\qquad$ $3 \times 5 = 15$

	Remember, when describing a multiplication equation such as $4 \times 3 = 12$ use the language '*four times three is equal to twelve.*' Avoid saying '*times <u>by</u>*' or '*multiplied by*'. For more on this, see segment *2.2 Structures: multiplication representing equal groups, Overview of learning*. Also continue to use the language of factors and products to describe the multiplication equation: • ***'___ is a factor.'*** • ***'___ is a factor.'*** • ***'The product of ___ and ___ is ___.'*** • ***'___ is the product of ___ and ___.'*** Work through several examples in this way, varying the representations used. You can ask children to suggest other examples of groups of three that they know (for example, leaves on a three-leaf clover).	• *'Five is a factor.'* • *'Three is a factor.'* • *'The product of five and three is fifteen.'* • *'Fifteen is the product of five and three.'*

1:2

Briefly consider zero threes. Remind children of the generalisation from segment *2.4 Times tables: groups of 10 and of 5, and factors of 0 and 1*: ***'When zero is a factor, the product is zero.'***

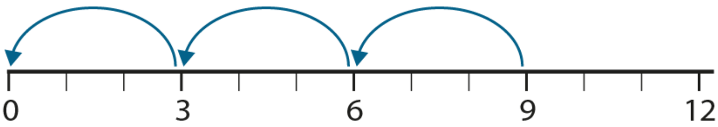
Use a number line, with backward jumps, to illustrate that this is also the case for zero groups of three. Then write and describe the pair of multiplication equations with '0' and '3' as factors.

Reminder of the generalisation:

$0 \times 2 = 0$	$0 \times 5 = 0$	$0 \times 10 = 0$
$2 \times 0 = 0$	$5 \times 0 = 0$	$10 \times 0 = 0$

'When zero is a factor, the product is zero.'

Counting backwards to zero groups of three:



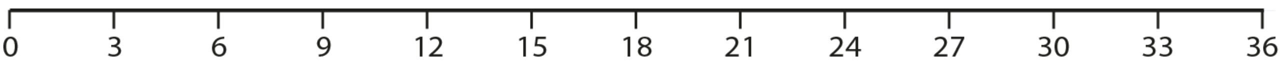
- *'Three threes, two threes, one three, zero threes.'*
- *'Three, three times; three, two times; three, one time; three, zero times.'*
- *'Nine, six, three, zero.'*

$0 \times 3 = 0$ $3 \times 0 = 0$

1:3

Practise skip counting, forwards and backwards in threes between 0 and 36, regularly outside the main maths lesson, so that children begin to develop fluency with this counting sequence before moving onto the next step. Use familiar representations such as a number line and the Gattegno chart.

Number line:



Gattegno chart:

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1:4

Now, using a familiar context, bring together the learning from steps *1:1–1:3*, working systematically to construct the three times table, beginning with zero threes.

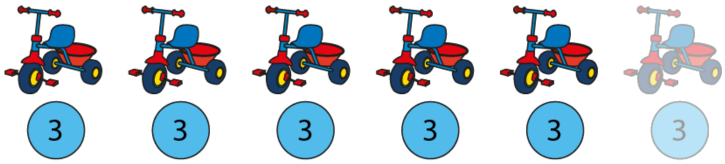
As with the other times tables, use a ratio chart to record the number of groups and the product. As you complete the ratio chart, also write the multiplication equations; write *pairs* of equations for each times table fact, using the following form of language:

- $6 \times 3 = 18$
  'Six groups of three is equal to eighteen.'
  'Six times three is equal to eighteen.'
- $3 \times 6 = 18$
  'Three, six times is equal to eighteen.'
  'Three times six is equal to eighteen.'

At each stage:

- encourage children to describe what each equation represents, for example:
  - *'There are six groups of three wheels.'*
  - *'There are eighteen wheels altogether.'*
  - *'The product of six and three is eighteen.'*
- then add another tricycle, and work with children to complete the next column of the table, using their knowledge of what comes next in the counting sequence when skip counting in threes.

Building up the three times table:



$0 \times 3 = 0$	$3 \times 0 = 0$
$1 \times 3 = 3$	$3 \times 1 = 3$
$2 \times 3 = 6$	$3 \times 2 = 6$
$3 \times 3 = 9$	$3 \times 3 = 9$
$4 \times 3 = 12$	$3 \times 4 = 12$
$5 \times 3 = 15$	$3 \times 5 = 15$
$\mathbf{6 \times 3 = 18}$	$\mathbf{3 \times 6 = 18}$

Number of tricycles	Total number of wheels
0	0
1	3
2	6
3	9
4	12
5	15
6	**18**

<table>
<tr>
<td>1:5</td>
<td>Once the ratio chart and full set of equations are complete, ask children questions, encouraging them to use the chart/equations for support, for example:

'If there are nine tricycles, how many wheels are there altogether?'
'How many tricycles are there if there are twenty-one wheels?'
'If the product is thirty, what are the factors?'
'Why are eight times three and three times eight both equal to twenty-four?'

</td>
<td>Complete ratio chart and three times table:
<table>
<tr><th>Number of tricycles</th><th>Total number of wheels</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>4</td><td>12</td></tr>
<tr><td>5</td><td>15</td></tr>
<tr><td>6</td><td>18</td></tr>
<tr><td>7</td><td>21</td></tr>
<tr><td>8</td><td>24</td></tr>
<tr><td>9</td><td>27</td></tr>
<tr><td>10</td><td>30</td></tr>
<tr><td>11</td><td>33</td></tr>
<tr><td>12</td><td>36</td></tr>
</table>
<table>
<tr><td>$0 \times 3 = 0$</td><td>$3 \times 0 = 0$</td></tr>
<tr><td>$1 \times 3 = 3$</td><td>$3 \times 1 = 3$</td></tr>
<tr><td>$2 \times 3 = 6$</td><td>$3 \times 2 = 6$</td></tr>
<tr><td>$3 \times 3 = 9$</td><td>$3 \times 3 = 9$</td></tr>
<tr><td>$4 \times 3 = 12$</td><td>$3 \times 4 = 12$</td></tr>
<tr><td>$5 \times 3 = 15$</td><td>$3 \times 5 = 15$</td></tr>
<tr><td>$6 \times 3 = 18$</td><td>$3 \times 6 = 18$</td></tr>
<tr><td>$7 \times 3 = 21$</td><td>$3 \times 7 = 21$</td></tr>
<tr><td>$8 \times 3 = 24$</td><td>$3 \times 8 = 24$</td></tr>
<tr><td>$9 \times 3 = 27$</td><td>$3 \times 9 = 27$</td></tr>
<tr><td>$10 \times 3 = 30$</td><td>$3 \times 10 = 30$</td></tr>
<tr><td>$11 \times 3 = 33$</td><td>$3 \times 11 = 33$</td></tr>
<tr><td>$\mathbf{12 \times 3 = 36}$</td><td>$\mathbf{3 \times 12 = 36}$</td></tr>
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1:6

Now practise chanting the three times table, with the written times table for support, using a variety of representations, including:

- stacked number lines (as shown opposite)
- the Gattegno chart
- concrete representations
- pictorial representations.

Use the following language:

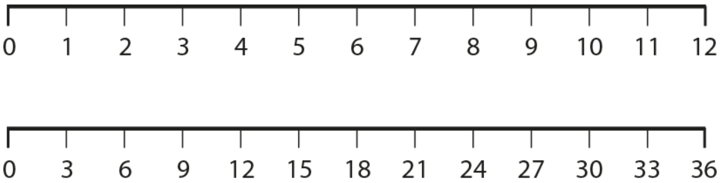
- *'One group of three is equal to three.' 'Two groups of three is equal to six…'*
- *'One times three is equal to three. Two times three is equal to six…'* then shortening to *'One three is three, two threes are six…'*

and

- *'Three, one time is equal to three…' 'Three, two times is equal to six…'*
- *'Three times one is equal to three…' 'Three times two is equal to six…'*

Regular practice should be undertaken, including outside the main maths lesson, until children are fluent.

Number line:



Gattegno chart:

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1:7

At this point, provide practice, including:

- completing/writing multiplication equations for contextual examples
- drawing/making contextual representations to match multiplication equations
- missing-number sequences and problems
- true-false style questions
- word problems, including measures contexts, for example:
  - *'What is the product of "9" and "3"?'*
  - *'Juice cartons come in packs of three. How many juice cartons are there in four packs?'*
  - *'I pick eight three-leaf clovers. How many leaves are there altogether?'*
  - *'A farmer has seven three-metre long fence panels. What length of fence can he build with these?'*

  Children should write a multiplication equation for each problem, rather than simply writing the product.

For word problems, ensure that some examples give three as the second piece of information, while others give it first (compare the second and third examples above). However, for now, all practice should be in the context of groups of three. The three times table will be applied to three equal groups in step *1.10*.

Completing multiplication equations:

'For each picture, complete the equations to show how many leaves there are altogether.'

$5 \times 3 = \square$ $\qquad$ $3 \times 5 = \square$

$\square \times 3 = \square$ $\qquad$ $3 \times \square = \square$

$\square \times \square = \square$ $\qquad$ $\square \times \square = \square$

Representing multiplication facts:

'Eloise wrote this in her book'

This shows 4 × 3 = 12

'Draw a picture like this to show:'

$7 \times 3 = 21$

Missing-number sequences/problems:

'Fill in the missing numbers.'

0	3	6	9	12							

36	33	30									

	At this stage, children can recite the three times table up to the number they need to find the answers, or use the multiplication chart for reference. Plenty of practice will be needed over an extended period until children are fluent in the isolated multiplication facts (for example, just knowing that seven threes are twenty-one, rather than having to recite the times table up to seven threes).	$3 \times \boxed{1} = \boxed{}$ $3 \times \boxed{3} = \boxed{}$ $3 \times \boxed{5} = \boxed{}$ $3 \times \boxed{7} = \boxed{}$ $3 \times \boxed{9} = \boxed{}$ $3 \times \boxed{11} = \boxed{}$ $\boxed{0} \times 3 = \boxed{}$ $\boxed{2} \times 3 = \boxed{}$ $\boxed{4} \times 3 = \boxed{}$ $\boxed{6} \times 3 = \boxed{}$ $\boxed{8} \times 3 = \boxed{}$ $\boxed{10} \times 3 = \boxed{}$ $\boxed{12} \times 3 = \boxed{}$

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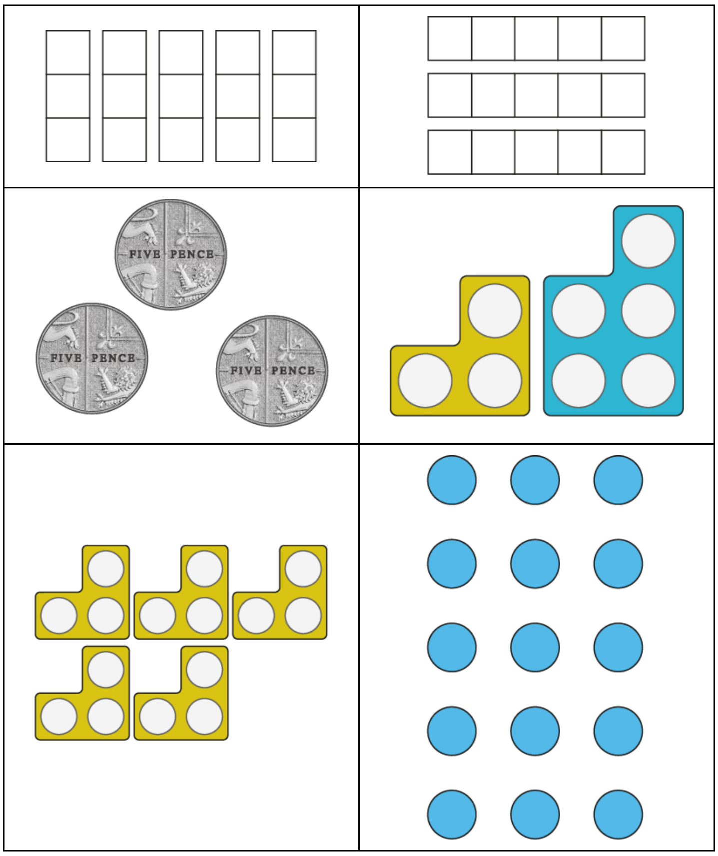
<td>
Dòng nǎo jīn:

'Which of these pictures could represent groups of three? Write two multiplication equations for each picture that represents groups of three.'



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1:8

Now ask children what patterns they can see in the three times table, prompting for the following:

- The products alternate between odd and even.
- Working down the list, the product increases by three each time.

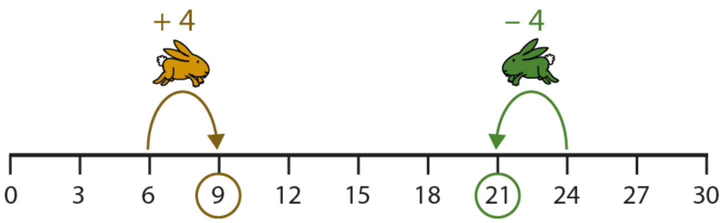
Focus on the fact that adjacent multiples of three have a difference of three, and that this knowledge can be used to find the next or previous multiple of three from a given multiple. Use the same representations as in earlier segments to illustrate this (ratio chart, mixed-operation equations, number line and arrays).

Finding adjacent multiples – ratio chart and number line:

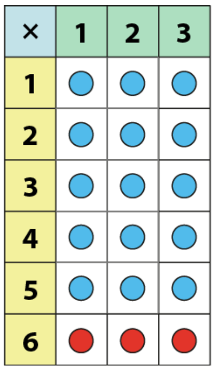
	$\times 3$
0	0
1	3
2	6
3	
4	12
5	15
6	18
7	
8	24
9	27
10	30
11	33
12	36

$\downarrow +3$ $\quad 3 \times 3 = 2 \times 3 + 3$

$\uparrow -3$ $\quad 7 \times 3 = 8 \times 3 - 3$



Finding adjacent multiples – array chart:



$6 \times 3 = 5 \times 3 + 3$

	Then challenge children to build the three times table from facts that they already know, using the rule about adjacent multiples. Before beginning, discuss why we already know the given facts/where we know them from, through applying the commutative law. For each missing fact, encourage children to write a mixed-operation equation relating it to the next/previous fact.	Building the three times table from known facts: *'Build up the three times table from facts we already know.'* $0 \times 3 =$ 0 $1 \times 3 =$ 3 $2 \times 3 =$ 6 $3 \times 3 =$ ☐ $4 \times 3 =$ 12 $5 \times 3 =$ 15 $6 \times 3 =$ ☐ $7 \times 3 =$ ☐ $8 \times 3 =$ 24 $9 \times 3 =$ ☐ $10 \times 3 =$ 30 $11 \times 3 =$ ☐ $12 \times 3 =$ ☐

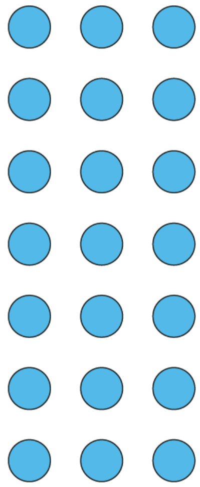
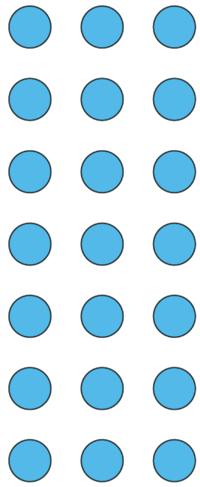
1:9	Provide children with varied practice based on their knowledge that adjacent multiples of three have a difference of three, including: • mixed-operation missing number/symbol problems • contextual problems, for example: • *'There are twelve tricycles outside the school.'* • *'How many wheels are there?'* • *'How many wheels would there be altogether if another tricycle arrived?'* • *'I had seven vases with three flowers in each one, but I dropped one vase of flowers. How many flowers do I have now?'* • *'I had eight rows of three cupcakes, but I ate one row. How many cupcakes are left?'*	Missing-number/symbol problems: *'Fill in the missing numbers.'* $10 \times 3 = 9 \times 3 + \square$ $6 \times 3 = \square \times 3 + 3$ $10 \times 3 - \square = 9 \times 3$ $6 \times 3 - 3 = \square \times 3$ *'Fill in the missing symbols (<, > or =).'* $9 \times 3 \bigcirc 8 \times 3$ $9 \times 3 \bigcirc 8 \times 3 + 3$ $9 \times 3 \bigcirc 9 \times 3 + 3$ $9 \times 3 \bigcirc 10 \times 3 - 3$ Dòng nǎo jīn: *'Fill in the missing numbers.'* $20 \times 3 = 60$ $3 \times 18 = 54$ so so $21 \times 3 = \square$ $3 \times 19 = \square$ $15 \times 3 = 45$ $3 \times 17 = 51$ so so $14 \times 3 = \square$ $3 \times 16 = \square$ $124 \times 3 = \square$ $125 \times 3 = 375$ $126 \times 3 = \square$

1:10

To complete this teaching point, provide general practice that includes extending three times table facts to:

- multiplication problems about three equal groups (as opposed to those about groups of three), as shown opposite and here: *'There are seven days in a week. How many days are there in three weeks?'*
- non-contextual division problems (writing equations; linking multiplication and division equations; missing-number problems); use intelligent practice as shown opposite
- contextual division problems, for example:
  - *'If the tricycle factory had thirty-three wheels, how many tricycles could they make?'* (quotitive division)
  - *'My ribbon is thirty-six centimetres long. If I split it into three equal pieces, how long will each piece be?'* (partitive division)
  - *'I have another ribbon that is thirty-nine centimetres long. Ted says that I can't divide it equally into three pieces. Do you agree with him? Why/why not?'* (partitive division)
- multi-step contextual problems, for example:
  - *'Ajay buys five packs of three pencils and two more pencils. How many pencils does he have altogether?'*
  - *'A tricycle factory made twelve tricycles. Two wheels fell off one tricycle. How many wheels were left in total?'*
  - *'Three children share twenty-four stickers between them. Each child is then given one more sticker. How many stickers does each child have?'*

Multiplication problems about three equal groups:

'Circle the groups of three and complete the sentence.'  There are ___ groups of three.	*'Circle the three equal groups and complete the sentence.'*  There are three groups of ___.

$3 \times \square = 21$

$\square \times 3 = 21$

Non-contextual division problems:

- *'Fill in the missing numbers.'*

$4 \times 3 = \square$ $\qquad$ $9 \times 3 = \square$

$\square \times 3 = 12$ $\qquad$ $\square \times 3 = 27$

$12 \div 3 = \square$ $\qquad$ $27 \div 3 = \square$

True/false question:

'True or false?'

$27 \div 3 - 3 = 3 \times 2$

	- *'Juice cartons come in packs of three. I have some packs and Jordan gives me one more pack. I now have thirty cartons of juice. How many packs did I have to begin with?'* - *'Tom brings ten sweets to a party, Nahla brings seven sweets and Sue brings ten sweets. They share the sweets equally. How many sweets does each child get?'* - Dòng nǎo jīn: *'If £54 is divided equally between three children, they each receive £18. How much money would they have altogether if they each received £19 instead?'*	

Teaching point 2:

Counting in multiples of six can be represented by the six times table. Adjacent multiples of six have a difference of six. Facts from the six times table can be used to solve multiplication and division problems with different structures.

Steps in learning

<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr><td>2:1</td><td>
Once all children are confident at recalling the three times table, the same teaching sequence (Teaching point 1) can be repeated for the six times table. Guidance is kept brief here; for more detail, refer back to Teaching point 1.
Begin by looking at some groups of six, linking enumerating objects in groups of six with counting in sixes, and writing the associated multiplication equations (write two equations for each example, as shown opposite). Use contexts that children already associate with six; for example, a die showing six dots or six legs on a bug. Note that familiarity with the arrangement of the six dots on a die will be useful in Teaching point 3 where the relationship between the three and six times tables is explored. Use six-value counters alongside each context, to support the idea of unitising in six, and use a number line with the multiples of six highlighted for skip-counting support.
For each example, ask children to describe what each number in the equation represents, and to use the language of factor and product to describe the equations.
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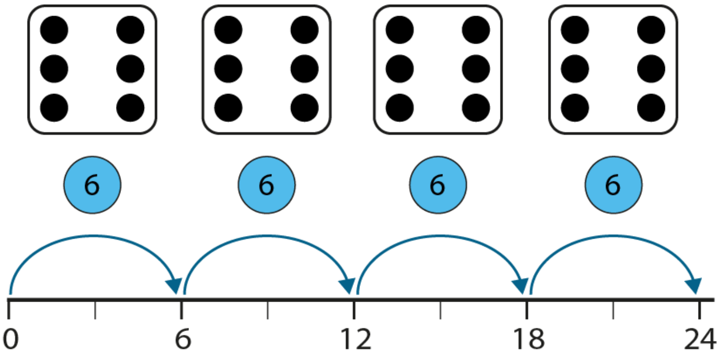
Example 1:
'How many dots are there? Count in groups of six.'


'Six, twelve, eighteen, twenty-four. There are twenty-four dots.'
'There are four groups of six; there are twenty-four altogether.'
'There is six, four times; there are twenty-four altogether.'

$4 \times 6 = 24$ $6 \times 4 = 24$

'Four is a factor.'
'Six is a factor.'
'The product of four and six is twenty-four.'
'Twenty-four is the product of four and six.'

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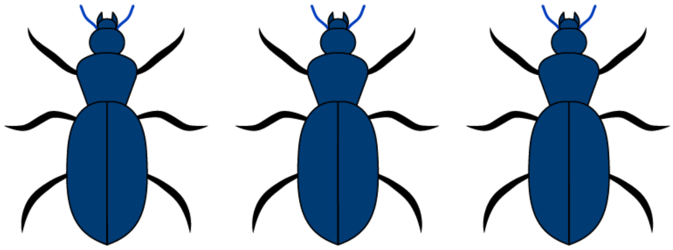
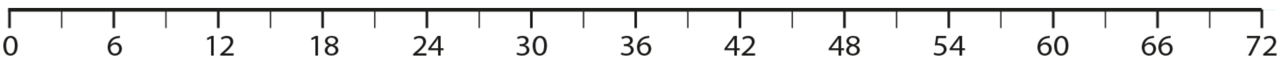
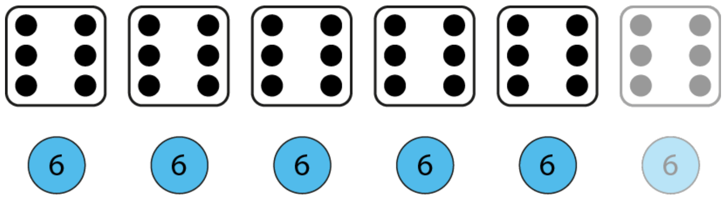
		Example 2: *'Show me three groups of six legs.'*  *'How many legs are there altogether? Count in groups of six.'* • *'Six, twelve, eighteen. There are eighteen legs.'* • *'There are three groups of six; there are eighteen altogether.'* • *'There are six, three times; there are eighteen altogether.'* $3 \times 6 = 18$ $6 \times 3 = 18$ • *'Three is a factor.'* • *'Six is a factor.'* • *'The product of three and six is eighteen.'* • *'Eighteen is the product of three and six.'*
2:2	Include writing the equations $0 \times 6 = 0$ and $6 \times 0 = 0$. By this point, children will know that when one of the factors is zero, the product will be zero. Refer to step *1:2* for more guidance if necessary.	
2:3	Practise skip counting, forwards and backwards in sixes between 0 and 72, regularly outside the main maths lesson, so that children begin to develop fluency with this counting sequence before moving onto the next step. Use familiar representations such as a number line and the Gattegno chart.	
	Number line: 	

2:4

Now, using a familiar context, work systematically to construct the six times table, beginning with zero sixes and working up to twelve sixes. Use a ratio chart to record the number of groups and the product as you go, and also write the multiplication equations (two equations for each times-table fact). Use the same form of language as described in step *1.4*, for example:

- $5 \times 6 = 30$
  - *'Five groups of six is equal to thirty.'*
  - *'Five times six is equal to thirty.'*
- $6 \times 5 = 30$
  - *'Six, five times is equal to thirty.'*
  - *'Six times five is equal to thirty.'*

Building up the six times table:



$0 \times 6 = 0$	$0 \times 6 = 0$
$1 \times 6 = 6$	$1 \times 6 = 6$
$2 \times 6 = 12$	$2 \times 6 = 12$
$3 \times 6 = 18$	$3 \times 6 = 18$
$4 \times 6 = 24$	$4 \times 6 = 24$
$5 \times 6 = 30$	$5 \times 6 = 30$
$\mathbf{6 \times 6 = 36}$	$\mathbf{6 \times 6 = 36}$

Number of six-value dice	Total number of dots
0	0
1	6
2	12
3	18
4	24
5	30
6	**36**

2:5

Once the ratio chart and full set of equations are complete, ask children questions, encouraging them to use the chart/equations for support, for example:

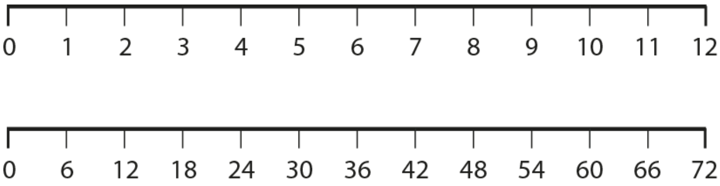
- *'How many dots would there be on seven dice each showing six dots?'*
- *'If I counted forty-eight dots, how many six-value dice would there be?'*
- *'If the product is forty-two, what are the factors?'*
- *'Why are eight times six and six times eight both equal to forty-eight?'*

Dòng nǎo jīn: *'Jon says that he can add one to all the products in the five times table to create the six times table because there is one more in each group. Is he right? '*

Complete ratio chart and six times table:

Number of six-value dice	Total number of dots
0	0
1	6
2	12
3	18
4	24
5	30
6	36
7	42
8	48
9	54
10	60
11	66
12	72

$0 \times 6 = 0$	$6 \times 0 = 0$
$1 \times 6 = 6$	$6 \times 1 = 6$
$2 \times 6 = 12$	$6 \times 2 = 12$
$3 \times 6 = 18$	$6 \times 3 = 18$
$4 \times 6 = 24$	$6 \times 4 = 24$
$5 \times 6 = 30$	$6 \times 5 = 30$
$6 \times 6 = 36$	$6 \times 6 = 36$
$7 \times 6 = 42$	$6 \times 7 = 42$
$8 \times 6 = 48$	$6 \times 8 = 48$
$9 \times 6 = 54$	$6 \times 9 = 54$
$10 \times 6 = 60$	$6 \times 10 = 60$
$11 \times 6 = 66$	$6 \times 11 = 66$
$12 \times 6 = 72$	$6 \times 12 = 72$

2:6	Now practise chanting the six times table, with the written times table for support, using a variety of representations, including: • stacked number lines (as shown opposite) • the Gattegno chart • concrete representations • pictorial representations. Use the following language: • *'One group of six is equal to six.'* *'Two groups of six is equal to twelve…'* • *'One times six is equal to six.'* *'Two times six is equal to twelve…'* then shortening to *'One six is six, two sixes are twelve…'* and • *'Six, one time is equal to six…'* *'Six, two times is equal to twelve…'* • *'Six times one is equal to six…'* *'Six times two is equal to twelve…'* Regular practice should be undertaken, including outside the main maths lesson, until children are fluent.	

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<td>2:7</td>
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Provide practice, similar to that in step 1.7. For now, all practice should be in the context of groups of six. The six times table will be applied to six equal groups in step 2.10.

Example word problems:

'What is the product of "5" and "6"?'
'If six children can fit into one carriage of a fairground ride, how many children are there in four full carriages?'
'Toby has bought three, six litre containers of water for a camping trip. How much water does he have altogether?'
'Some children are making bunting for a school fete. They need eight six-metre lengths. What is the total length of bunting they need to make?'

Children should write a multiplication equation for each problem, rather than simply writing the product.
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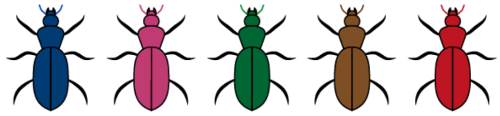
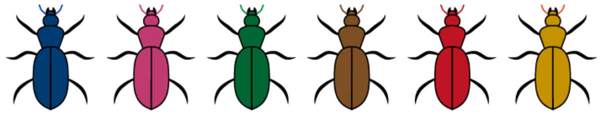
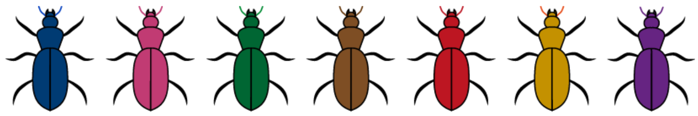
Completing multiplication equations:

'For each picture, complete the equations to show how many legs there are altogether.'


$5 \times 6 = \square$ $6 \times 5 = \square$

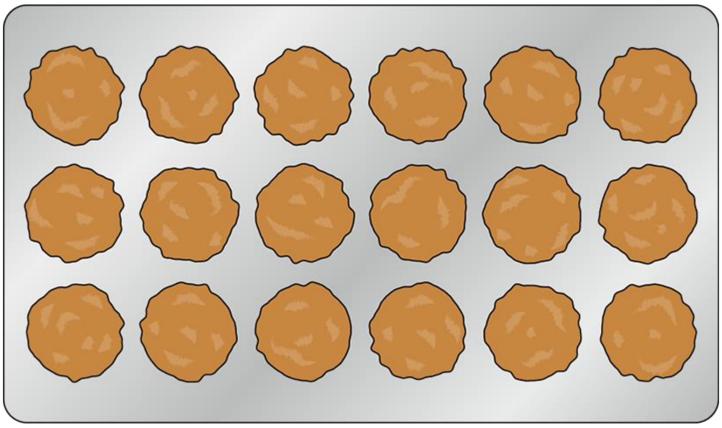

$\square \times 6 = \square$ $6 \times \square = \square$


$\square \times \square = \square$ $\square \times \square = \square$

Representing multiplication facts:

'The tray of biscuits represents:'

$3 \times 6 = 18$


'Draw a picture like this to represent:'

$7 \times 6 = 42$
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At this stage, children can recite the six times table up to the number they need to find the answers, or use the multiplication chart for reference. Plenty of practice will be needed over an extended period until children are fluent in the isolated multiplication facts (for example, just knowing that seven times six is forty-two, rather than having to recite the times table up to seven sixes).

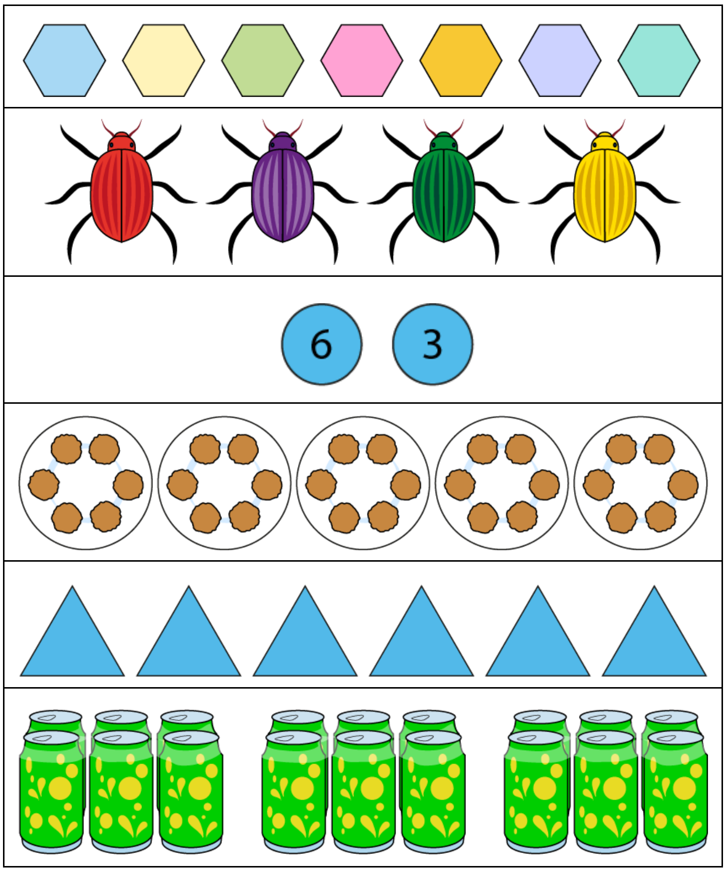
Missing-number sequences/problems:
'Fill in the missing numbers.'

0	6	12	18	24							

72	66	60									

6 ×	1	=	
6 ×	3	=	
6 ×	5	=	
6 ×	7	=	
6 ×	9	=	
6 ×	11	=	

0	× 6 =	
2	× 6 =	
4	× 6 =	
6	× 6 =	
8	× 6 =	
10	× 6 =	
12	× 6 =	

		Dòng nǎo jīn *'Which of these pictures could represent groups of six? Write two multiplication equations for each picture that represents groups of six.'* 

2:8

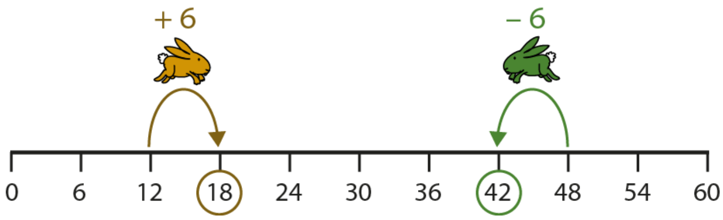
Similarly to step *1.8*, ask children what patterns they can see in the six times table, prompting for the following:

- The products are all even numbers.
- Products in the six times table are also products in the three times table (this will be covered in more detail in *Teaching point 3*).
- Working down the list, the product increases by six each time.

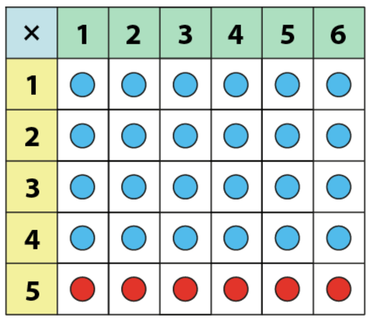
Focus in on the fact that adjacent multiples of six have a difference of six, and that this knowledge can be used to find the next or previous multiple of six from a given multiple. Use the same representations as before to illustrate this (ratio chart, mixed-operation equations, number line and arrays).

Finding adjacent multiples – ratio chart and number line:

	× 6		
0	0		
1	6		
2	12		
3		↓ + 6	$3 \times 6 = 2 \times 6 + 6$
4	24		
5	30		
6	36		
7		↑ − 6	$6 \times 6 = 7 \times 6 - 6$
8	48		
9	54		
10	60		
11	66		
12	72		



Finding adjacent multiples – array chart:



$4 \times 6 = 5 \times 6 - 6$

	Then challenge children to build the six times table from facts that they already know, using the rule about adjacent multiples. Before beginning, discuss why we already know the given facts/where we know them from, through applying the commutative law.	Building the six times table from known facts: *'Build up the six times table from facts we already know.'* $0 \times 6 =$ 0 $1 \times 6 =$ 6 $2 \times 6 =$ 12 $3 \times 6 =$ 18 $4 \times 6 =$ 24 $5 \times 6 =$ 30 $6 \times 6 =$ $\boxed{}$ $7 \times 6 =$ $\boxed{}$ $8 \times 6 =$ 48 $9 \times 6 =$ $\boxed{}$ $10 \times 6 =$ 60 $11 \times 6 =$ $\boxed{}$ $12 \times 6 =$ $\boxed{}$
2:9	In the same way as in step *1:9*, provide children with varied practice based on their knowledge that adjacent multiples of six have a difference of six. Example word problems: • *'I had eight flowers, each with six petals. If I get one more flower, how many petals do I have altogether?'* • *'Jordan buys ten packs of six soft drinks for a party. One pack is lemonade and the rest of the packs are cola. How many cans of cola are there?'*	Missing-number/symbol problems: • *'Fill in the missing numbers.'* $7 \times 6 = 6 \times \boxed{} + 6$ $9 \times \boxed{} = 10 \times 6 - 6$ $6 \times 9 - 6 = \boxed{} \times 6$ $\boxed{} \times 6 + 6 = 10 \times 6$

- *'Fill in the missing symbols (<, > or =).'*

$4 \times 6 \bigcirc 5 \times 6 - 5$

$4 \times 6 \bigcirc 5 \times 6 - 6$

$4 \times 6 \bigcirc 3 \times 6 + 3$

$4 \times 6 \bigcirc 3 \times 6 + 6$

$4 \times 6 \bigcirc 6 \times 4$

$4 \times 6 \bigcirc 5 \times 6$

$6 \times 5 \bigcirc 4 \times 6$

True/false questions:

'True or false?'

$6 \times 10 - 6 = 11 \times 6$

$7 \times 6 + 6 = 6 \times 8 - 3$

Dòng nǎo jīn

'Fill in the missing numbers.'

$20 \times 6 = 120$
so
$21 \times 6 = \square$

$6 \times 18 = 108$
so
$6 \times 19 = \square$

$15 \times 6 = 90$
so
$14 \times 6 = \square$

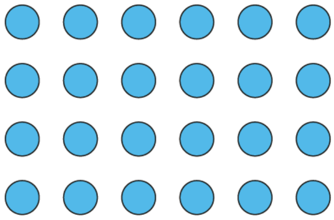
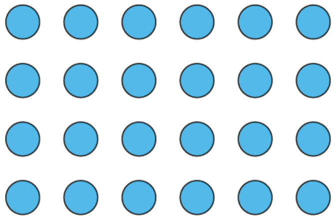
$6 \times 17 = 102$
so
$6 \times 16 = \square$

2:10

Provide children with general practice that includes extending six times table facts to:

- multiplication problems about six equal groups (as opposed to those about groups of six), as shown opposite and here: '*Six children each have eight stickers. How many stickers do they have altogether?*'
- non-contextual division problems (writing equations; linking multiplication and division equations; missing-number problems); use intelligent practice as shown opposite
- contextual division problems, for example:
  - '*There are six cans of drink in one multipack. Mr Smith needs sixty cans of drink for a party. How many multipacks must he buy?*' (quotitive division)
  - '*If forty-two cookies are shared equally between six children, how many does each child get?*' (partitive division)
  - '*If cookies are baked in rows of six, draw a representation of forty-eight cookies.*' (quotitive division)
- multi-step contextual problems, for example:
  - '*Apples come in bags of six. Lily buys four full bags of apples and three extra apples. How many apples does she have?*'
  - '*Kashvi buys six bags of six apples but four apples fall out of his bag on the way home. How many apples does he have left?*'
  - '*Six children share thirty sweets between them. They each put one sweet away to save for later and eat the rest now. How many sweets does each child eat straight away?*'

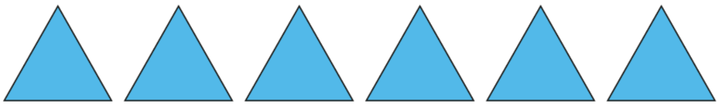
Multiplication problems about six equal groups:

'*Circle the <u>groups of six</u> and complete the sentence.*'  There are ___ groups of six.	'*Circle the <u>six equal groups</u> and complete the sentence.*'  There are six groups of ___.

$$6 \times \square = 24$$

$$\square \times 6 = 24$$

- '*Does this represent a fact in the six times table?*'



Division problems:

- '*Fill in the missing numbers.*'

$6 \times 2 = \square$ $\quad\quad$ $\square = 12 \times 6$

$2 \times 6 = \square$ $\quad\quad$ $\square = 6 \times 12$

$12 \div 6 = \square$ $\quad\quad$ $\square = 72 \div 6$

- *'Stan organises his toy cars into seven rows of six cars on his toy car park. He then takes one car away from each row. How many cars does he have left in the car park?'*
- *'Some children gather four red balls, six yellow balls and eight green balls. They share the balls out equally between six teams. How many balls does each team get?'*
- *'The school cook buys four six-kilogram bags of flour and one more two-kilogram bag. How much flour is this?'*

Dòng nǎo jīn: *'There are thirty-two children in a class and each table seats six children. How many tables does the class need to make sure everyone has a seat?'*

Number of bugs	0	1		3	4		6	
Total number of legs		6	12		24	30		42

- *'True or false?'*

  $54 \div 6 - 6 = 8 \times 6$

- *'What multiplication fact can be used to solve this division calculation?'*

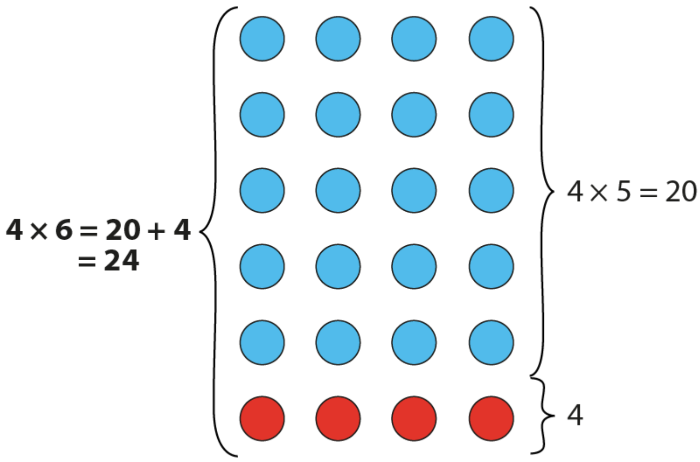
  $48 \div 6 = ?$

  I can use this multiplication fact: ___ × ___ = ___

2:11 To complete this teaching point, spend a little time exploring how facts in the six times table can be found using known facts in the five times table. Note that this strategy uses the distributive law, which will be explored in detail in segment *2.10 Connecting multiplication and division, and the distributive law*; for now, use arrays to make the link between the multiplication facts (as shown opposite), rather than writing out full mixed operation equations such as:

$4 \times 6 = 4 \times 5 + 4 \times 1$

Dòng nǎo jīn: '*Fiona and Jeremy have some flowers. Fiona's flowers each have five petals; she has thirty-five petals altogether. Jeremy's flowers each have six petals; if he has the same number of flowers as Fiona, how many petals does he have altogether?'*



Teaching point 3:

Products in the six times table are double the products in the three times table; products in the three times table are half of the products in the six times table.

Steps in learning

3:1 In this teaching point, the relationship between the three and six times tables is explored. Children have already made similar comparisons for other times tables, so links can be made to those comparisons.

Some children may already have mentioned some of the connections between the three and six times tables during work on *Teaching points 1* and *2*. Now, children should be given the opportunity to discuss the relationships together, beginning with 'double skip counting'.

First practise counting forwards from zero in multiples of three, then in multiples of six. Use representations such as:

- a number line with both multiples of three and multiples of six labelled
- the Gattegno chart.

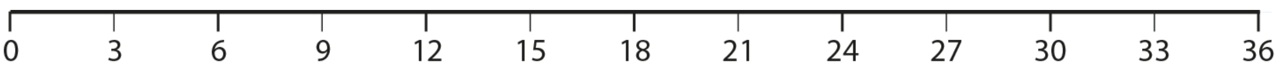
Then split the class in half, with one half counting in multiples of three and the other half counting in multiples of six, up to 36. The group counting in threes should count on every 'beat', while the group counting in sixes should count on every other 'beat', such that both groups will say the multiples of six at the same time.

Then ask children what they notice, prompting for the following:

- All of the numbers said by the 'sixes' group' are also said by the 'threes group'.
- Not all of the numbers said by the 'threes group' are also said by the 'sixes group'.
- For every number said by the 'sixes group', the 'threes group' says two numbers.

After discussion, double skip count again, recording the pattern in a table as shown below.

Number line:



Gattegno chart:

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Comparing counting in multiples of three and six:

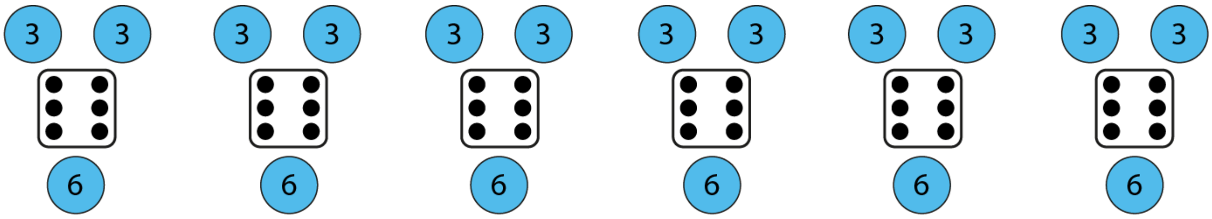
Number	**0**	1	2	**3**	4	5	**6**	7	8	**9**	10	11	**12**	13	14	**15**	16	17	**18**	19	20	**21**	22	23	**24**
Counting in 3s	✓			✓			✓			✓			✓			✓			✓			✓			✓
Counting in 6s	✓						✓						✓						✓						✓

3:2 Now discuss the relationship between the number of threes and the number of sixes, in terms of *groups* of three or *groups* of six, using a familiar pictorial representation. Present a die showing six dots, and ask:

- *'How many groups of three are there?'*
- *'How many groups of six are there?'*

Use counters to represent one group of three and two groups of six, as shown below. Then add another die, ask the questions again and add the next set of counters. Continue until there are about six dice, and use a bar model to summarise the relationships.

Work towards the generalisation: ***'For every one group of six, there are two groups of three.'***



3	3	3	3	3	3	3	3	3	3	3	3
6		6		6		6		6		6	

3:3

Now show the three and six times tables, side-by-side, and ask children:

- *'What's the same?'*
- *'What's different?'*

Encourage children to use the language of factors and product, and of doubling and halving. Prompt for the following observations:

- The first seven products in the six times table are also found in the three times table. Ask children, *'Would this continue to be the case if the three times table was continued beyond twelve threes?'* Then use the generalised sentence: ***'Products in the six times table are also in the three times table.'***
- Complete missing-number problems, as shown opposite, work as a class towards the generalisation: ***'The product of an even number and three is a product in the six times table.'***

As a class, sort some numbers into a Venn diagram, as shown opposite. Once the numbers are sorted, ask questions to draw children's attention to the patterns and connections:

- *'Which section does not have any numbers in it? Why?'*
- *'What do you notice about the numbers in the section where the two sets overlap?'*
- *'What do you notice about the numbers that don't go inside the circles?'*

Dòng nǎo jīn:

'Rishi says all multiples of six are multiples of three.'

'Emily says all multiples of three are multiples of six.'

'Are they right? Why/why not?'

Comparing the three and six times tables:

$0 \times 3 = 0$	$0 \times 6 = 0$
$1 \times 3 = 3$	$1 \times 6 = 6$
$2 \times 3 = 6$	$2 \times 6 = 12$
$3 \times 3 = 9$	$3 \times 6 = 18$
$4 \times 3 = 12$	$4 \times 6 = 24$
$5 \times 3 = 15$	$5 \times 6 = 30$
$6 \times 3 = 18$	$6 \times 6 = 36$
$7 \times 3 = 21$	$7 \times 6 = 42$
$8 \times 3 = 24$	$8 \times 6 = 48$
$9 \times 3 = 27$	$9 \times 6 = 54$
$10 \times 3 = 30$	$10 \times 6 = 60$
$11 \times 3 = 33$	$11 \times 6 = 66$
$12 \times 3 = 36$	$12 \times 6 = 72$

Missing-number problems:

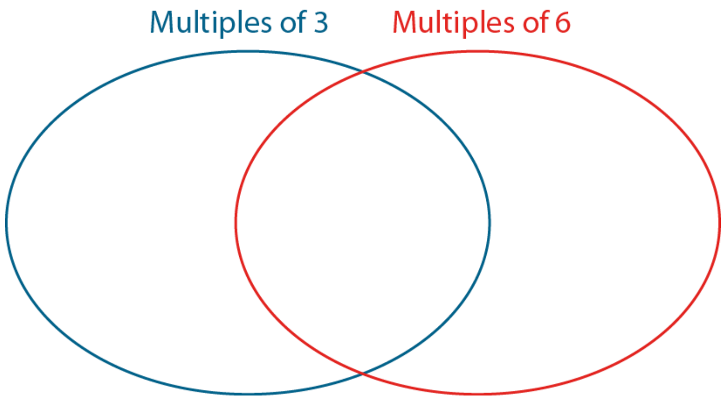
'Fill in the missing numbers.'

0		0		0
2		6		1
4				2
6	$\times 3 =$		$= 6 \times$	3
8				4
10				5
12				6

Sorting numbers:

'Place these numbers in the diagram. In the overlapping section, you should place the numbers that are both multiples of three and six. Numbers that are neither multiples of three nor six should go outside the circles.'

0	3	6	31	15	61	30	36	60



3:4

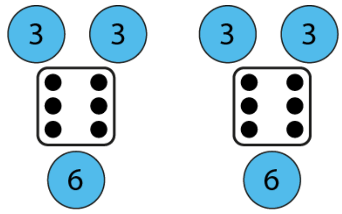
Now use the language of doubling and halving to compare two equations with the same product (e.g. $4 \times 3 = 12$ and $2 \times 6 = 12$). You can use the dice example from step *3.2*, along with a number line.

For a given number of dice (e.g. two):

- ask how many groups of three there are (four)
- represent the groups of three with three-value counters
- represent the jumps of three on a number line
- write a multiplication equation ($4 x 3 = 12$).

Then:

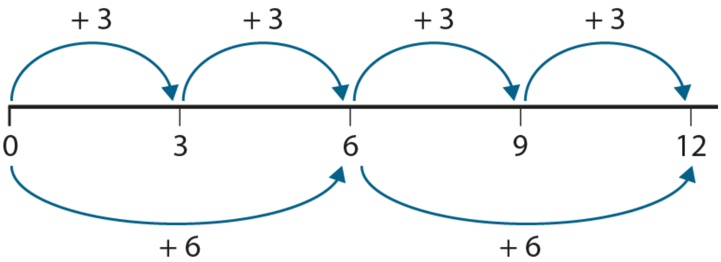
- ask how many groups of six there are (two)
- represent the groups of six with six-value counters
- represent the jumps of six on the same number line as the jumps of three
- write a multiplication equation ($2 x 6 = 12$).



$4 \times 3 = 12$

$2 \times 6 = 12$

4 threes



2 sixes

- *'Two times six is equal to twelve, so double-two times three is equal to twelve.'*
- *'Four times three is equal to twelve, so half-of-four times six is equal to twelve.'*

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<td>Then compare the equations, asking children what they notice about the factors and the products. Encourage them to use the language modelled opposite. You could use a bar model to summarise the relationships.

Repeat for some other quantities of dice, until the pattern is clear.</td>
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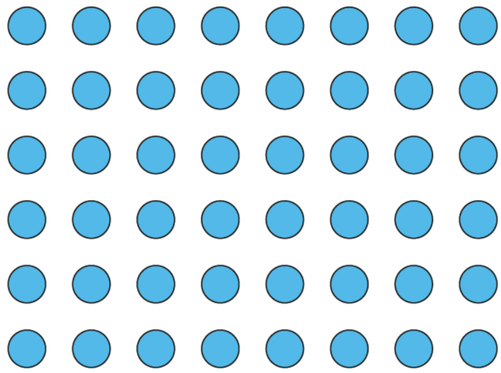
<td>3:5</td>
<td>Provide children with some intelligent practice and contextual problems related to pairs of three and six times-table facts that have the same product. Include related pairs of division facts, supporting children to think about dividing into equal groups of three or six, or between three or six. You can also provide questions about arrays, such as that shown opposite, to support children's understanding.

Throughout, encourage children to use a given multiplication (or division) fact and known doubling/halving strategies to find the answers; encourage them to reason about their thinking using the language in step 3.4. (For more on doubling/halving strategies, see segment 2.5 Commutativity (part 2), doubling and halving.)

Example word problems:
• 'Rishi rolls two dice and gets two sixes. Emily rolls four dice; she rolls all threes. Emily says she got a higher total than Rishi. Is she right? Why/why not?'
• 'There are twenty-four marbles.'
    • 'If the marbles are shared equally between three children, how many marbles does each child get?'
    • 'If the marbles are shared equally between six children, how many marbles does each child get?' (partitive division)</td>
<td>Array problem:

• 'How many groups of six are there?'
$\square \times 6 = \square$

• 'How many groups of three are there?'
$\square \times 3 = \square$

Missing-number problems:
'Fill in the missing numbers.'
$2 \times 3 = 1 \times 6$ $20 \times 3 = \square \times 6$
$4 \times 3 = 2 \times 6$ $40 \times 3 = \square$
$6 \times 3 = \square \times 6$ $60 \times 3 = \square \times 6$
$\square \times 3 = 4 \times 6$ $\square \times 3 = 40 \times 6$
$\square \times 3 = 400 \times 6$ $\square \times 3 = 90 \times 6$</td>
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- *'Thirty-six cupcakes are made in a bakery.'*
  - *'How many packets of <u>three</u> can be made?'*
  - *'How many packets of <u>six</u> can be made?'*
(quotitive division)
- Dòng nǎo jīn:
  - *'Three children shared some sweets; each child got ten sweets. If they then had to share all of the sweets equally with three extra children, how many sweets would each child have?'*
  - *'I cut one six-centimetre length from a twenty-four centimetre ribbon. How many three- centimetre lengths can I make from the remaining ribbon?'*

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$12 \div 3 = 4$ $12 \div 6 = \square$

$18 \div 3 = \square$ $18 \div 6 = \square$

$24 \div 3 = \square$ $24 \div 6 = \square$

$5 \times 3 + 3 = \square$

$4 \times 3 + 6 = \square$

$4 \times 3 + 3 + 3 = \square$

$7 \times 3 - 3 = 6 \times \square$

$9 \times 3 + 6 = \square \times 3$

True/false questions:

'True or false?'

$5 \times 3 + 3 = 6 \times 3$

$4 \times 3 + 6 = 5 \times 3$

$6 \times 3 + 3 = 7 \times 6$

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<td>3:6</td>
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Now, shift the focus onto pairs of facts where one factor is the same and the other is either three or six, e.g.:

$5 \times 3 = 15$

$5 \times 6 = 30$

Ask children what they notice, prompting them to describe how:

- six is <u>double</u> three

and

- five-times-six is <u>double</u> five-times-three

- three is <u>half of</u> six

and

- five-times-three is <u>half of</u> five-times-six.

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$5 \times 3 = 15$

half   double

$5 \times 6 = 30$

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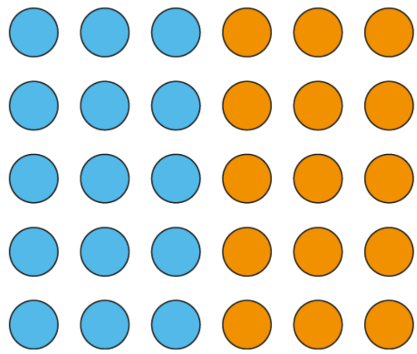
3:7

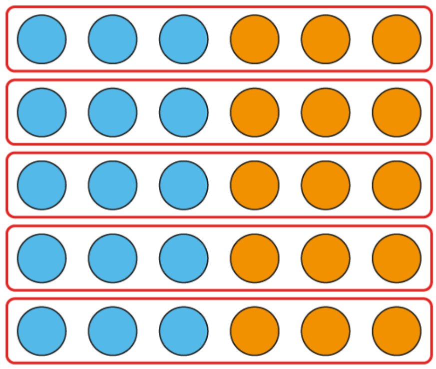
Use arrays with two different counter colours to examine *why* this is true, working through some multiplication facts in sequence, and using the following stem sentences to describe the relationships:

- ***'Six is double three, so ___ sixes are double ___ threes.***
- ***'Three is half of six, so ___ threes are half of ___ sixes.'***

First work through the example from the previous step (*5 × 3* and *5 × 6*), as shown opposite. Then work through subsequent multiplication facts (*6 × 3* and *6 × 6*, *7 × 3* and *7 × 6*…) until children are confident with the patterns and language.



- *'How many sixes are there?'*

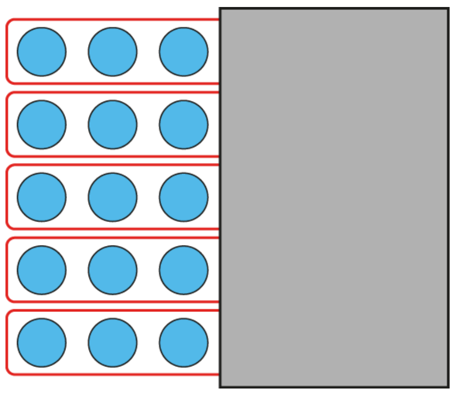


'There are five groups of six.'

$5 \times 6 = 30$ *'Five sixes are thirty.'*

$6 \times 5 = 30$ *'Six, five times is thirty.'*

- *'How many threes are there?'*



'There are five groups of three.'

$5 \times 3 = 15$ *'Five threes are fifteen.'*

$3 \times 5 = 15$ *'Three, five times is fifteen.'*

- *'Six is <u>double</u> three, so five sixes is <u>double</u> five-threes.'*
- *'Three is <u>half of</u> six, so five threes is <u>half of</u> five-sixes.'*

3:8

Provide children with some intelligent practice and contextual problems related to pairs of facts where one factor is the same and the other is either three or six.

Include related pairs of division facts, supporting children to think about dividing into equal groups of three or six, or sharing between three or six.

Example word problem:

'Stephan does three five-minute chores; they take him fifteen minutes. Cara does six five-minute chores. How long do Cara's chores take?'

Missing-number problems

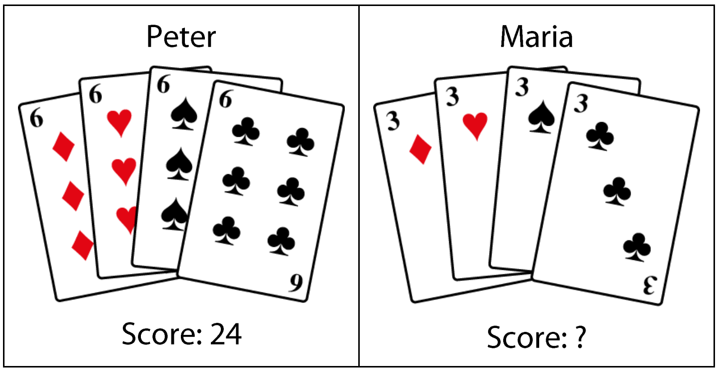
'Fill in the missing numbers.'

$4 \times 3 = \square$	$3 \times 5 = \square$	$3 \times 7 = \square$
$4 \times 6 = \square$	$6 \times 5 = \square$	$7 \times 6 = \square$

$30 \div 3 = 10$	$60 \div 6 = 10$
$24 \div 3 = \square$	$48 \div 6 = \square$
$18 \div 3 = \square$	$18 \div 6 = \square$

Example contextual problem:

'Peter and Maria are playing a card game. They need to collect four-of-a-kind; then they score the value of their cards.'

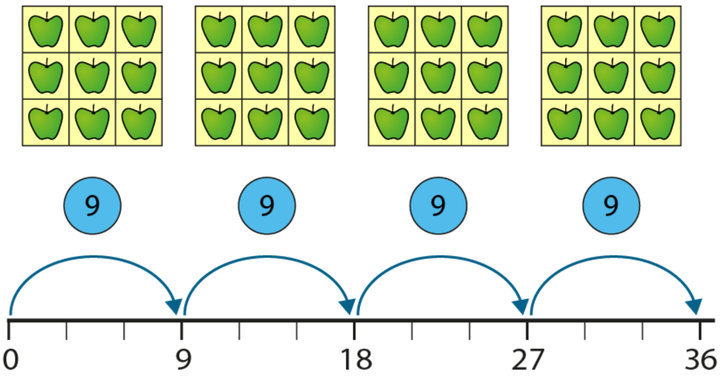


- *'Who has the highest score?'*
- *'What is Maria's score?'*

Teaching point 4:

Counting in multiples of nine can be represented by the nine times table. Adjacent multiples of nine have a difference of nine. Facts from the nine times table can be used to solve multiplication and division problems with different structures.

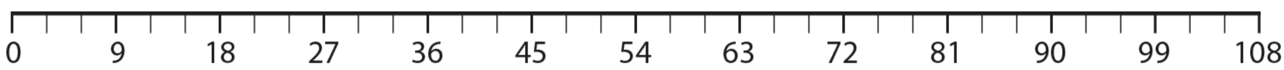
Steps in learning

	Guidance	Representations
4:1	Now, the same teaching sequence as described in *Teaching points 1* and *2* can be repeated for the nine times table. Since the steps here are the same as those described earlier, please refer to *Teaching point 1* for more detailed guidance. It is recommended that you ensure that children are fluent with the three and six times tables before beginning work on the nine times table. Begin by enumerating objects in groups of nine by skip counting in nines and writing the associated multiplication equations. Use contextual examples of groups of nine alongside nine-value unitising counters and a number line. Work through a variety of numbers of groups of nine.	*'How many apples are there? Count in groups of nine.'*  • *'Nine, eighteen, twenty-seven, thirty-six. There are thirty-six apples.'* • *'There are four groups of nine; there are thirty-six altogether.'* • *'There are nine, four times; there are thirty-six altogether.'* $4 \times 9 = 36$ $9 \times 4 = 36$ • *'Four is a factor.'* • *'Nine is a factor.'* • *'The product of four and nine is thirty-six.'* • *'Thirty-six is the product of four and nine.'*

4:2 Include writing the equations $0 \times 9 = 0$ and $9 \times 0 = 0$. By this point, children will know that when one of the factors is zero, the product will be zero.

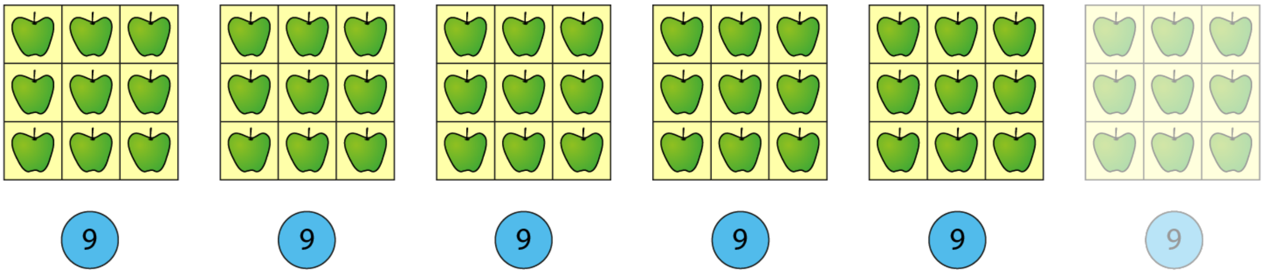
4:3 Practise skip counting, forwards and backwards in nines between 0 and 108, regularly outside the main maths lesson, so that children begin to develop fluency with this counting sequence before moving onto the next step. Use familiar representations such as a number line and the Gattegno chart.

Number line:



4:4 Now, using a familiar context, work systematically to construct the nine times table, beginning with zero nines and working up to twelve nines. Use a ratio chart to record the number of groups and the product as you go, and also write the multiplication equations (two equations for each times-table fact). Use the same form of language as described in step *1:4* and exemplified below.

Building up the nine times table:



$0 \times 9 = 0$	$9 \times 0 = 0$
$1 \times 9 = 9$	$9 \times 1 = 9$
$2 \times 9 = 18$	$9 \times 2 = 18$
$3 \times 9 = 27$	$9 \times 3 = 27$
$4 \times 9 = 36$	$9 \times 4 = 36$
$5 \times 9 = 45$	$9 \times 5 = 45$
$\mathbf{6 \times 9 = 54}$	$\mathbf{9 \times 6 = 54}$

Number of boxes of 9 apples	Total number of apples
0	0
1	9
2	18
3	27
4	36
5	45
6	**54**

- *'Six groups of nine is equal to fifty-four.'*
- *'Six times nine is equal to fifty-four.'*

- *'Nine, six times is equal to fifty-four.'*
- *'Nine times six is equal to fifty-four.'*

4:5	Once the ratio chart and full set of equations are complete, ask children questions, encouraging them to use the chart/equations for support, for example: • *'How many apples are there if there are seven boxes of apples?'* • *'If I had seventy-two apples, how many boxes of nine would that be?'* • *'If the product is eighty-one, what are the factors?'* • *'Why are eight times nine and nine times eight both equal to seventy-two?'* • Dòng nǎo jīn: *'Jon says that he subtract one from all the products in the ten times table to create the nine times table because there is one fewer in each group. Is he right?'*	Complete ratio chart and nine times table:

Number of boxes of 9 apples	Total number of apples
0	0
1	9
2	18
3	27
4	36
5	45
6	54
7	63
8	72
9	81
10	90
11	99
12	108

$0 \times 9 = 0$	$9 \times 0 = 0$
$1 \times 9 = 9$	$9 \times 1 = 9$
$2 \times 9 = 18$	$9 \times 2 = 18$
$3 \times 9 = 27$	$9 \times 3 = 27$
$4 \times 9 = 36$	$9 \times 4 = 36$
$5 \times 9 = 45$	$9 \times 5 = 45$
$6 \times 9 = 54$	$9 \times 6 = 54$
$7 \times 9 = 63$	$9 \times 7 = 63$
$8 \times 9 = 72$	$9 \times 8 = 72$
$9 \times 9 = 81$	$9 \times 9 = 81$
$10 \times 9 = 90$	$9 \times 10 = 90$
$11 \times 9 = 99$	$9 \times 11 = 99$
$12 \times 9 = 108$	$9 \times 12 = 108$

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Now practise chanting the nine times table, with the written times table for support, using a variety of representations, including:

- stacked number lines (as shown opposite)
- the Gattegno chart
- concrete representations
- pictorial representations.

Use the following language:

- *'One group of nine is equal to nine. Two groups of nine is equal to eighteen…'*
- *'One times nine is equal to nine. Two times nine is equal to eighteen…'* then shortening to *'One nine is nine, two nines are eighteen…'*

and

- *'Nine, one time is equal to nine…'* *'Nine, two times is equal to eighteen…'*
- *'Nine times one is equal to nine…'* *'Nine times two is equal to eighteen…'*

Regular practice should be undertaken, including outside the main maths lesson, until children are fluent.

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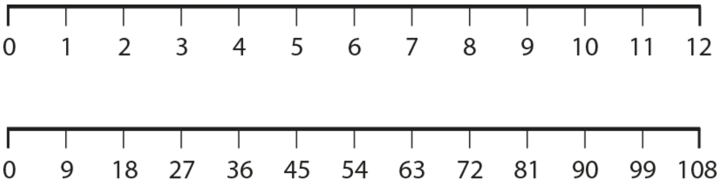

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Provide practice, similar to that in steps *1:7* and *2:7*, in the context of groups of nine.

Example word problems:

- *'What is the product of "5" and "9"?'*
- *'There are nine batteries in a packet. How many batteries are there in eight packets?'*
- *'A fire extinguisher contains nine litres of water. How much water is there in four fire extinguishers?'*

Children should write a multiplication equation for each problem, rather than simply writing the product.

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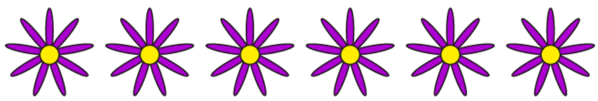
Completing multiplication equations:

'For each picture, complete the equations to show how many petals there are altogether.'

$5 \times 9 = \square$ $9 \times 5 = \square$



$\square \times 9 = \square$ $9 \times \square = \square$

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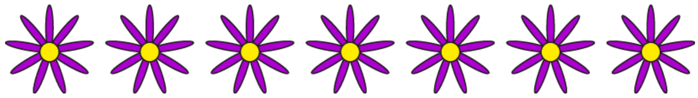
At this stage, children can recite the nine times table up to the number they need to find the answers or use the multiplication chart for reference. Plenty of practice will be needed over an extended period until children are fluent in the isolated multiplication facts (for example, just knowing that seven times nine is sixty-three, rather than having to recite the times table up to seven nines).

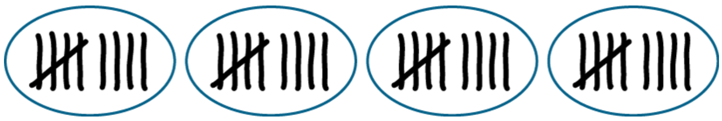


$\square \times \square = \square$ $\quad$ $\square \times \square = \square$

Representing multiplication facts:

'The tally marks represent:'

$4 \times 9 = 36$



'Draw tally marks to represent:'

$7 \times 9 = 63$

Missing-number sequences/problems:
'Fill in the missing numbers.'

0	9	18	27	36							

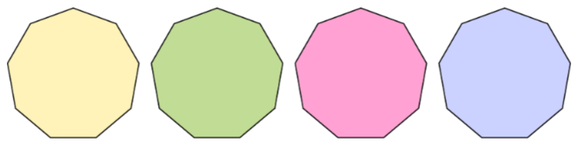
108	99	90									

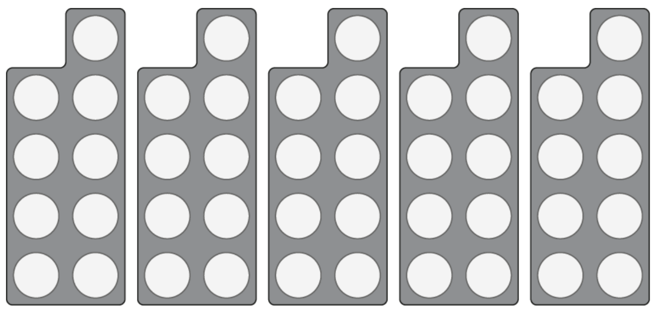
	1		
	3		
	5		
$9 \times$	7	$=$	
	9		
	11		

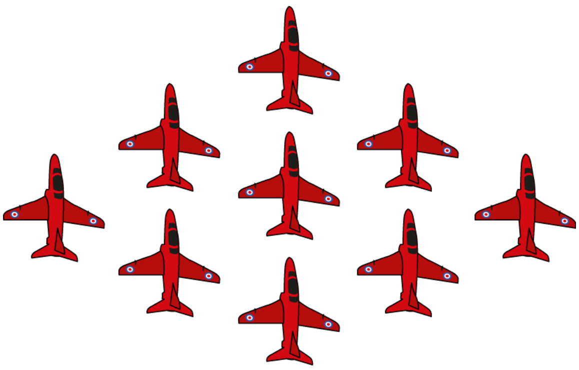
0		
2		
4		
6	$\times 9 =$	
8		
10		
12		

Dòng nǎo jīn

'Which of these pictures could represent groups of nine? Write two multiplication equations for each picture that represents groups of nine.'







4:8

Similarly to steps *1:8* and *2:8*, ask children what patterns they can see in the nine times table, prompting for the following:

- The products alternate between odd and even.
- Working down the list, the product increases by nine each time.

Focus on the fact that adjacent multiples of nine have a difference of nine, and that this knowledge can be used to find the next or previous multiple of nine from a given multiple.

Remind children that:

- nine can be added to a number by adding ten and then subtracting one
- nine can be subtracted from a number by subtracting ten and then adding one.

Show how this strategy can be used to help us recite the nine times table/count in multiples of nine, and to find the next/previous multiple of nine from a given multiple.

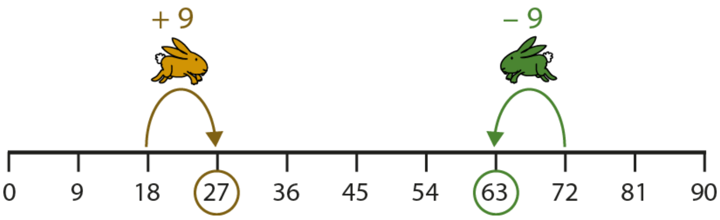
Then challenge children to build the nine times table from facts that they already know, using the rule about adjacent multiples.

Finding adjacent multiples – ratio chart and number line:

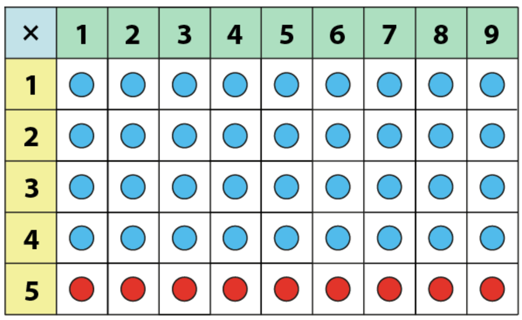
	× 9
0	0
1	9
2	18
3	
4	36
5	45
6	54
7	
8	72
9	81
10	90
11	99
12	108

↓ + 9 $3 \times 9 = 2 \times 9 + 9$

↑ − 9 $7 \times 9 = 8 \times 9 - 9$



Finding adjacent multiples – array chart:



$4 \times 9 = 5 \times 9 - 9$

		Building the nine times table from known facts: *'Build up the nine times table from facts we already know.'*

$0 \times 9 =$	0
$1 \times 9 =$	9
$2 \times 9 =$	18
$3 \times 9 =$	27
$4 \times 9 =$	36
$5 \times 9 =$	45
$6 \times 9 =$	54
$7 \times 9 =$	
$8 \times 9 =$	72
$9 \times 9 =$	
$10 \times 9 =$	90
$11 \times 9 =$	
$12 \times 9 =$	

4:9	Provide children with varied practice based on their knowledge that adjacent multiples of nine have a difference of nine. Example word problems: • *'Jordan is making nine-sided shapes with sticks. He's already made ten shapes from ninety sticks. If he adds another nine-sided shape, how many sticks will there be?'* • *'There were eight flowers in a flower bed, each with nine petals. If one of the flowers is picked, how many petals are left in the flower bed?'*	Missing- symbol problems: *'Fill in the missing symbols (+ or –).'* $7 \times 9 = 8 \times 9 \bigcirc 9$ $11 \times 9 = 10 \times 9 \bigcirc 9$ $9 \times 9 \bigcirc 9 = 8 \times 9$ $9 \times 8 = 9 \times 9 \bigcirc 9$ True/false questions: *'True or false?'* $9 \times 11 - 9 = 9 \times 12$ $7 \times 9 + 9 = 6 \times 9 + 9 + 9$ Dòng nǎo jīn • *'Fill in the missing numbers.'* $30 \times 9 = 270$ so $31 \times 9 = \square$ $9 \times 18 = 162$ so $9 \times 19 = \square$ $21 \times 9 = 189$ so $20 \times 6 = \square$ $9 \times 17 = 153$ so $9 \times 16 = \square$

		• *'Some children are trying to do this calculation:'* $15 \times 9 = ?$ *'Their teacher writes down this equation to help them:'* $16 \times 9 = 144$ *'Mark each child's answer as correct (✓) or incorrect (✗). Explain.'*

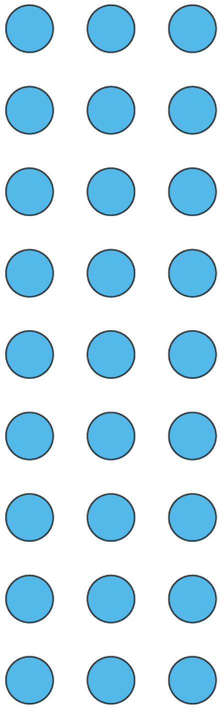
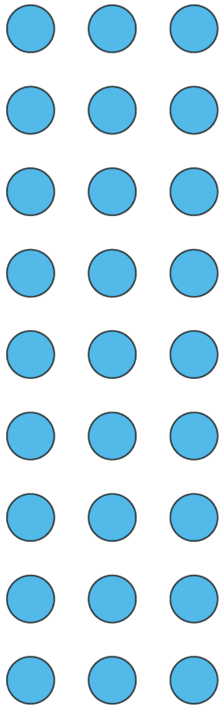
	✓ or ✗	**Why?**
Cara writes: 15 × 9 = 144 + 10 – 1 = 153		
Isabelle writes: 15 × 9 = 144 – 9 = 136		
Bo writes: 15 × 9 = 144 – 9 = 144 – 10 + 1 = 134 + 1 so 15 × 9 = 135		
Bryony writes: 15 × 9 = 144 – 10 – 1 = 134 – 1 so 15 × 9 = 133		

4:10

Provide children with general practice that includes extending nine times table facts to:

- multiplication problems about nine equal groups (as opposed to those about groups of nine), as shown opposite and here: *'Six children each have nine stickers. How many stickers do they have altogether?'*
- non-contextual division problems (writing equations; linking multiplication and division equations; missing-number problems); use intelligent practice as shown opposite
- contextual division problems, for example:
  - *'A grocer has sixty-three apples. If nine apples will fit in each box; how many boxes does the grocer need?'* (quotitive division)
  - *'The school cook has forty-five kilograms of potatoes shared equally between nine bags. What mass of potatoes is in each bag?'* (partitive division)
  - *'Tariq is swimming in a pool that is nine metres wide. How many widths would Tariq have to do to swim fifty-four metres?'* (quotitive division)
- multi-step contextual problems, for example:
  - *'Muffins come in boxes of nine. Tariq buys five full boxes for his party but eats two muffins on the way home. How many muffins does Tariq have left for his party?'*
  - *'Tariq's sister, Rania, has six boxes of nine muffins; Tariq gives her four more muffins. How many muffins does Rania have now?'*

Multiplication problems about nine equal groups:

'Circle the groups of nine and complete the sentence.'	*'Circle the nine equal groups and complete the sentence.'*
	
There are ___ groups of nine.	There are nine groups of ___.

$9 \times \boxed{} = 27$

$\boxed{} \times 9 = 27$

- *'Does this represent a fact in the nine times table?'*

- *'Eight children baked seventy-two samosas together at school, and shared them equally. Each child ate one samosa at school. How many did each child have left to take home?'*
- *'Lily baked six rows of nine cookies. She took one cookie away from each row to give to her friends. How many cookies does Lily have left?'*
- Dòng nǎo jīn: *'If apples are stored in boxes of nine, how many boxes are needed for one hundred apples?'*

Non-contextual division problems:

- *'Fill in the missing numbers.'*

  $9 \times 2 = \square$ $\qquad \square = 12 \times 9$

  $2 \times 9 = \square$ $\qquad \square = 9 \times 12$

  $18 \div 9 = \square$ $\qquad \square = 108 \div 9$

- *'True or false?'*

  $81 \div 9 - 9 = 9 \times 0$

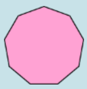
- *'What multiplication fact can be used to solve this division calculation?'*

  $72 \div 9 = ?$

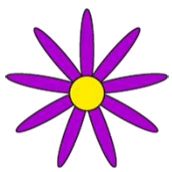
  I can use this multiplication fact: ___ × ___ = ____

Contextual problems:

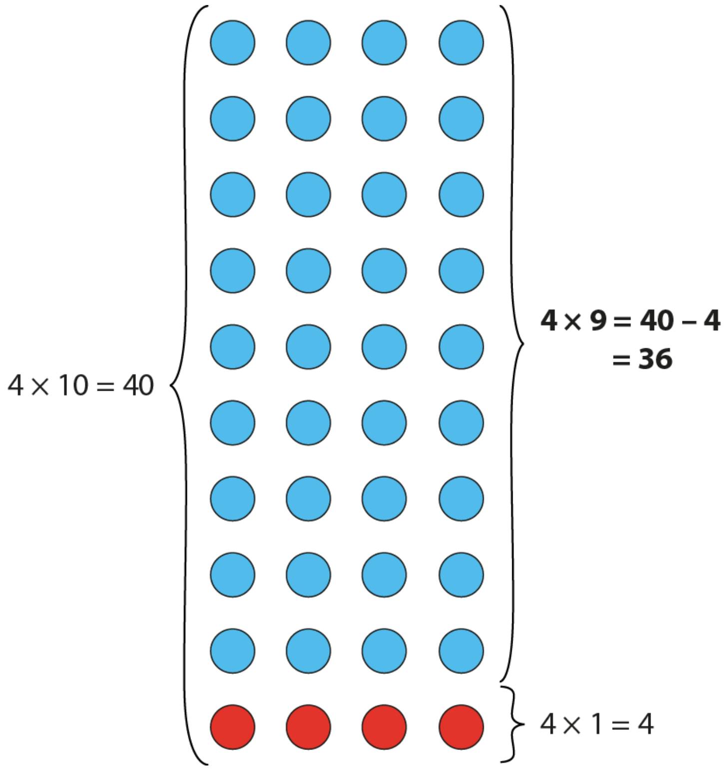
- *'Fill in the missing numbers to complete the table.'*

Number of nonagons 	0	1		3	4		6	
Total number of sides		9	18		36	45		63

- *'One flower has nine petals.'*

  'Draw some of these flowers to show forty-five petals.'

4:11	To complete this teaching point, spend a little time exploring how facts in the nine times table can be found using known facts in the ten times table. Note that, as with step *2:11*, this strategy uses the distributive law, which will be covered in detail in segment *2:10 Connecting multiplication and division, and the distributive law*; for now, use arrays, as exemplified opposite, to make the link between the multiplication facts, rather than writing out full mixed-operation equations such as: $4 \times 9 = 4 \times 10 - 4 \times 1$	

Teaching point 5:

Products in the nine times table are triple the products in the three times table. Products that are in the three, six and nine times tables share the same factors.

Steps in learning

5:1

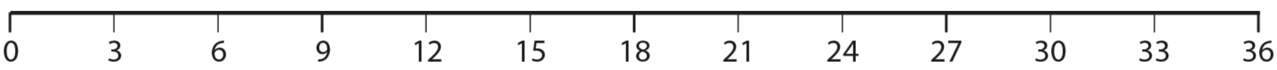
This teaching point explores the relationship between the three and nine times tables. Until now, all explored links between times tables have involved doubling/halving (comparing the five and ten, the two and four, the four and eight, and the three and six times tables). This teaching sequence follows a similar approach to *Teaching point 3* (relationship between the three and six times table), but now the relationship is one of triples/thirds. Unlike previous times table relationships, where children had known doubling/halving strategies to draw upon, children do not have known tripling/'thirding' strategies; as such this teaching point explores the relationship only in terms of tripling (from three times table facts to nine times table facts), since children *do* have strategies for adding three numbers (*Spine 1: Number, Addition and Subtraction*, segment *1.11*). The focus here is on exploration of the relationship between the three and nine times tables, but we also begin to lay foundations for the scaling structure of multiplication (segment *2.17 Structures: using measures and comparison to understand scaling*), supporting children to begin to think beyond doubling in terms of scaling up.

First, practise counting forwards from zero in multiples of three and then in multiples of nine. Then split the class in half and 'double skip count' in threes and nines up to 36, in a similar way to that described in step *3:1*. Use representations such as:

- a number line with both multiples of three and multiples of nine labelled
- the Gattegno chart.

Discuss and record the pattern in a table as shown below.

Number line:

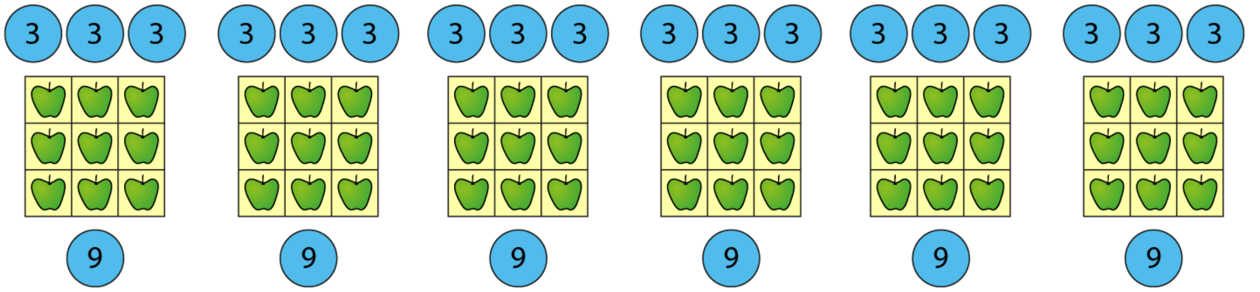


	Comparing counting in multiples of three and nine:

Number	Counting in 3s	Counting in 9s
0	✓	✓
1		
2		
3	✓	
4		
5		
6	✓	
7		
8		
9	✓	✓
10		
11		
12	✓	
13		
14		
15	✓	
16		
17		
18	✓	✓
19		
20		
21	✓	
22		
23		
24	✓	
25		
26		
27	✓	✓
28		
29		
30	✓	

5:2	Now discuss the relationship between the number of threes and the number of nines in terms of *groups* of three or *groups* of nine, using a familiar pictorial representation, and three- and nine-value counters. Work upwards from three threes/one nine in a similar way to that described in step *3:2*. Since the tripling relationship is new, if children are familiar with Numberblocks these can be used to support the connection between one nine and three threes. You can also use a bar model to summarise the relationships. Work towards the generalisation: ***'For every one group of nine, there are three groups of three.'***

Example 1:



<table>
<tr><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr>
<tr><td colspan="3">9</td><td colspan="3">9</td><td colspan="3">9</td><td colspan="3">9</td><td colspan="3">9</td><td colspan="3">9</td></tr>
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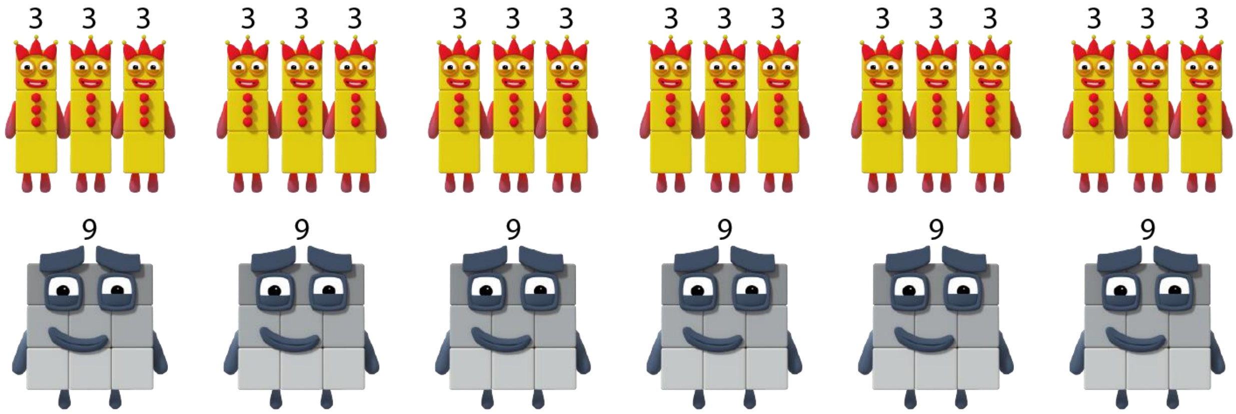
Example 2:



<table>
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<tr><td colspan="3">9</td><td colspan="3">9</td><td colspan="3">9</td><td colspan="3">9</td><td colspan="3">9</td><td colspan="3">9</td></tr>
</table>

5:3

Now show the three and nines times tables and ask children:

- *'What's the same?*
- *'What's different?'*

Prompt for the following:

- Products in the nine times table are also in the three times table.
- Every third multiple of three is in the nine times table.

As a class, sort some numbers into a Venn diagram, as shown opposite, and discuss the patterns and connections.

Dòng nǎo jīn: *'Ted says that all multiples of three are also multiples of nine and all multiples of nine are also multiples of three. Is he correct?'*

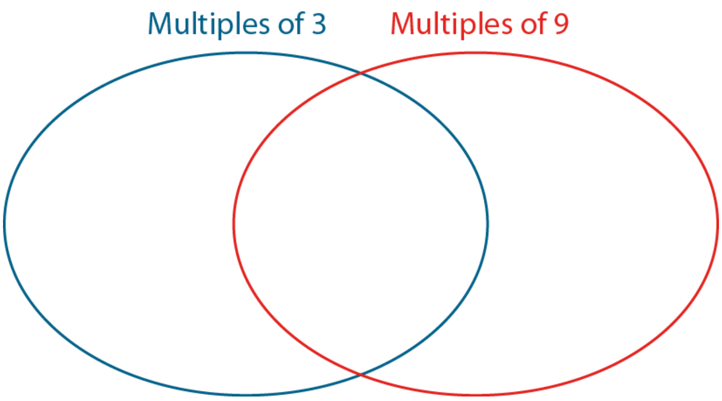
Comparing the three and nine times tables:

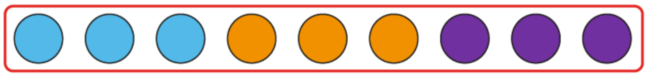
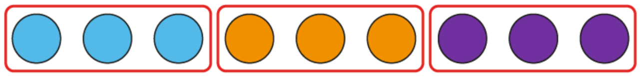
$3 \times 0 = 0$	$9 \times 0 = 0$
$3 \times 1 = 3$	$9 \times 1 = 9$
$3 \times 2 = 6$	$9 \times 2 = 18$
$3 \times 3 = 9$	$9 \times 3 = 27$
$3 \times 4 = 12$	$9 \times 4 = 36$
$3 \times 5 = 15$	$9 \times 5 = 45$
$3 \times 6 = 18$	$9 \times 6 = 54$
$3 \times 7 = 21$	$9 \times 7 = 63$
$3 \times 8 = 24$	$9 \times 8 = 72$
$3 \times 9 = 27$	$9 \times 9 = 81$
$3 \times 10 = 30$	$9 \times 10 = 90$
$3 \times 11 = 33$	$9 \times 11 = 99$
$3 \times 12 = 36$	$9 \times 12 = 108$

Sorting numbers:

'Place these numbers in the diagram. In the overlapping section, you should place the numbers that are both multiples of three and nine. Numbers that are neither multiples of three nor nine should go outside the circles.'

0	3	9	32	15	31	36	63	30



5:4	Now compare pairs of equations with the same product (e.g. $6 \times 3 = 18$ and $2 \times 9 = 18$). Since a triple relationship is less familiar to children than a double relationship, begin by looking at one nine/three threes, using an array with different coloured counters to draw attention to the three groups of three in nine. Use the language exemplified opposite to support children in making links, including introducing the term 'triple' to mean 'three times' (compare with the connection between the word 'double' and 'two times'). Ensure that children are clear on the meaning of 'triple' before adding another group of nine counters; then repeat, adding more rows of counters until children are confident with the pattern.	One nine / three threes: • *'There are nine counters.'*  • *'How many groups of nine are there?'*  $1 \times 9 = 9$ $9 \times 1 = 9$ • *'There is one group of nine. One times nine is equal to nine.'* • *'How many groups of three are there?'*  $3 \times 3 = 9$ • *'There are three times as many threes as there are nines.'* • *'There are three groups of three.'* *'Three times three is equal to nine.'* • *'We can say triple three is equal to nine.'* • *'Triple means "three times".'* $1 \times 9 = 9$ $\downarrow \times 3$ $3 \times 3 = 9$ • *'<u>One</u> times nine is equal to nine, so <u>triple-one</u> times three is equal to nine.'*

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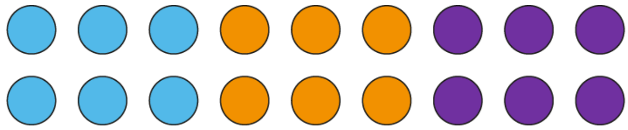
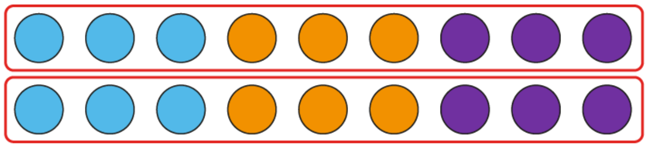
Two nines / six threes:

- *'There are eighteen counters.'*



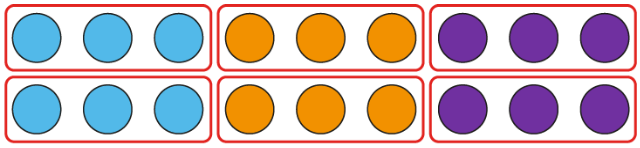
- *'How many groups of nine are there?'*



$2 \times 9 = 18$ $9 \times 2 = 18$

- *'There are two groups of nine. Two times nine is equal to eighteen.'*

- *'How many groups of three are there?'*



$6 \times 3 = 18$ $3 \times 6 = 18$

- *'There are six groups of three.'*
- *'Six times three is equal to eighteen.'*

$$\begin{array}{ccccccc} 2 & \times & 9 & = & 18 \\ \downarrow{\times 3} & & & & \\ 6 & \times & 3 & = & 18 \end{array}$$

- *'Two times nine is equal to eighteen, so triple-two times three is equal to eighteen.'*
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<td>5:5</td>
<td colspan="2">
Apply the idea of same product to a contextual example; you can use the apples and counters from step 5:2, along with a number line.

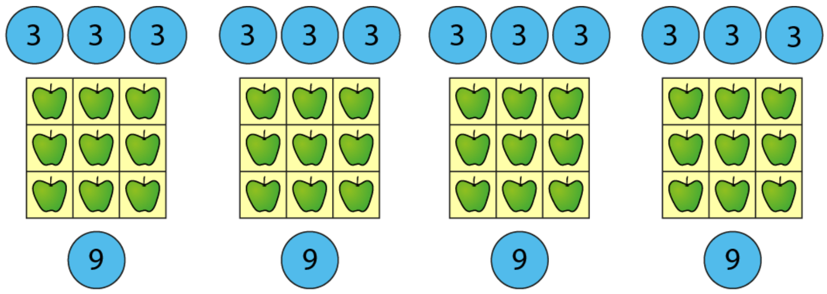
For a given number of boxes of apples (for example, four):

- ask how many groups of nine there are (four)
- represent the groups of nine with nine-value counters
- represent the jumps of nine on a number line
- write a multiplication equation.
</td>
</tr>
</table>

Then:

- ask how many groups of three there are (12)
- represent the groups of three with three-value counters
- represent the jumps of three on the same number line as the jumps of nine
- write a multiplication equation.

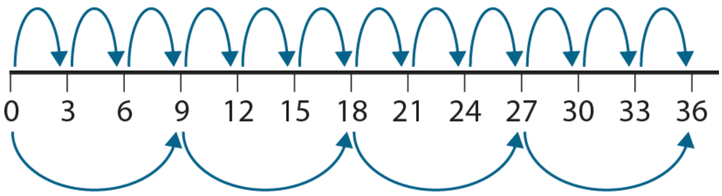
Use a bar model, and the language introduced in step *5:4*, to summarise the relationships.



$12 \times 3 = 36$

$4 \times 9 = 36$

12 threes



4 nines

36											
3	3	3	3	3	3	3	3	3	3	3	3
9			9			9			9		

5:6

Provide children with some intelligent practice and contextual problems related to pairs of three and nine times-table facts that have the same product. Include related pairs of division facts.

Example word problems:

- *'If I can cut a piece of ribbon equally into four nine-centimetre lengths, how many three-centimetre lengths would I be able to cut?'*
- *'A baker makes ninety buns.'*
  - *'If he puts them in boxes of nine, how many boxes will he need?'*
  - *'If he puts them in boxes of three, how many boxes will he need?'*

Missing-number problems:

'Fill in the missing numbers.'

$1 \times 9 = 3 \times 3$

$2 \times 9 = \boxed{} \times 3$

$3 \times 9 = \boxed{} \times 3$

$4 \times 9 = 12 \times \boxed{}$

$\boxed{} \times 9 = 15 \times 3$

$9 \div 3 = 3$ $\quad$ $9 \div 9 = \square$

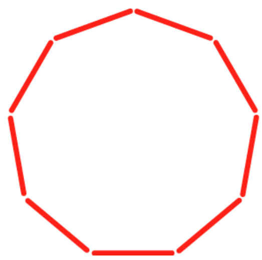
$18 \div 3 = \square$ $\quad$ $18 \div 9 = \square$

$27 \div 3 = \square$ $\quad$ $27 \div 9 = \square$

Contextual problem:

'Jake and Misha are making triangles and nonagons using sticks.'

One triangle uses three sticks.



One nonagon uses nine sticks.

- *'Jake makes nine triangles. How many nonagons can Misha make using the same number of sticks?'*
- *'Now, Misha makes four nonagons. How many triangles can Jake make using the same number of sticks?'*

5:7 Now shift the focus onto pairs of facts where one factor is the same and the other is either three or nine, e.g.:

$5 \times 3 = 15$

$5 \times 9 = 45$

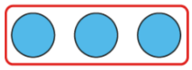
Begin by comparing one three and one nine, using arrays. Draw attention to the fact that one row of nine can be made up of three rows of three (make duplicates of the original array and arrange them end-to end), so the product in the nine times table fact ($1 \times 9 = 9$) is three times the product in the three times table fact ($1 \times 3 = 3$). Use the following stem sentences to describe the relationships: '***Nine is triple three, so ___ nines is triple ___ threes.'***

Then add another row to each array, to compare 2×3 and 2×9; repeating until the pattern is clear.

Comparing one three and one nine:

- *'How many threes are there?'*

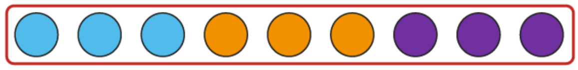
- *'There is one group of three.'*



- $1 \times 3 = 3$ *'One three is three.'*
- $3 \times 1 = 3$ *'Three, one time is three.'*

- *'How many nines are there?'*

- *'There is one group of nine.'*

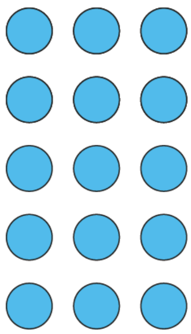


- $1 \times 9 = 9$ *'One nine is nine.'*
- $9 \times 1 = 9$ *'Nine, one time is nine.'*

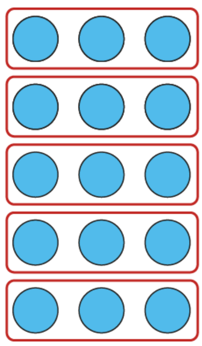
- *'Nine is <u>triple</u> three, so one nine is <u>triple</u> one-three.'*

Comparing five threes and five nines:

- *'How many threes are there?'*

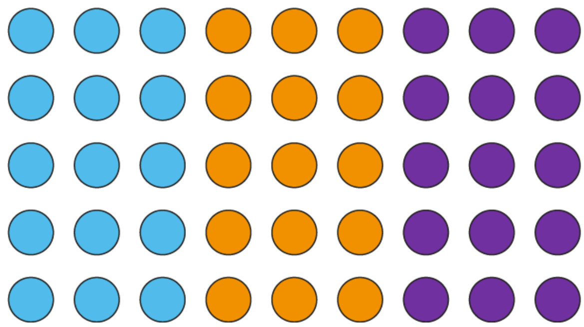


- *'There are five groups of three.'*

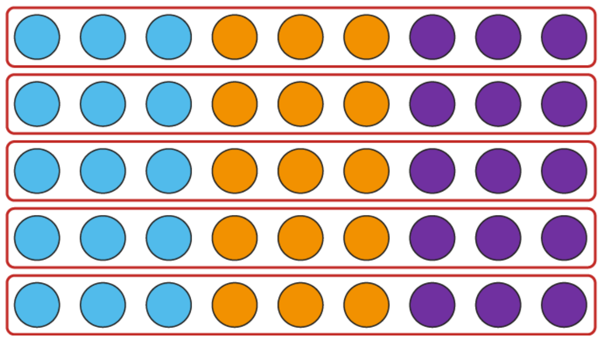


- $5 \times 3 = 15$ *'Five threes are fifteen.'*
- $3 \times 5 = 15$ *'Three, five times is fifteen.'*

- *'How many nines are there?'*

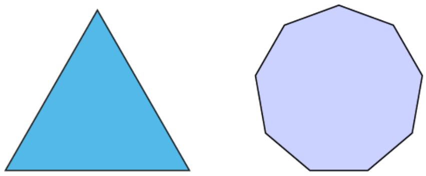


- *'There are five groups of nine.'*



- $5 \times 9 = 45$ *'Five nines are forty-five.'*
- $9 \times 5 = 45$ *'Nine, five times is forty-five.'*

	• *'Nine is triple three, so five nines is triple five-threes.'* $5 \times 3 = 15$ $\downarrow$ triple $5 \times 9 = 45$	
5:8	Provide children with some intelligent practice and contextual problems related to pairs of facts where one factor is the same and the other is either three or nine.	Missing-number/symbol problems. • *'Fill in the missing numbers.'* $2 \times 3 = \square$ $3 \times 4 = \square$ $2 \times 9 = \square$ $9 \times 4 = \square$ $6 \times 3 = \square$ $3 \times 8 = \square$ $6 \times 9 = \square$ $9 \times 8 = \square$ • *'Fill in the missing symbols (<, >, or =).'* $4 \times 3 \bigcirc 1 \times 9$ $2 \times 9 \bigcirc 3 \times 3$ $2 \times 9 \bigcirc 6 \times 3$ True/false questions: *'True or false?'* $6 \times 3 + 9 = 7 \times 3$ $6 \times 9 + 3 = 7 \times 9$

		Contextual problem: *'Felicity and Hamid have some shapes.'*  triangle nonagon • *'Felicity has four triangles. How many sides does she have altogether?'* • *'Hamid has the same number of nonagons. How many sides does he have altogether?'*

Teaching point 6:

Divisibility rules can be used to find out whether a given number is divisible (to give a whole number) by three, six or nine.

Steps in learning

	Guidance	Representations
6:1	Knowing if a dividend is divisible exactly by a divisor (to give a whole number) without having to do a full calculation, is a useful skill. Children have already learnt the divisibility rules for divisors of two, five and ten (segment 2.6 *Structures: quotitive and partitive division*) and for divisors of four and eight (segment 2.7 *Times tables: 2, 4 and 8, and the relationship between them*). In this teaching point, children explore and apply the divisibility rules for divisors of three, six and nine. Note that, since children are working within the context of integers, throughout this teaching point the statement 'can be divided by' implies 'gives a whole number when it is divided by'. Begin by exploring the divisibility rule for three. Since this divisibility requires children to find the sum of the digits in multi-digit numbers, it is important to make a clear distinction between the terms 'number' and 'digit'. Display and discuss some two-digit numbers, modelling the correct use of the terms, and then find the sum of the digits, as exemplified opposite. Practise as a class until children are confident with the language and summing the digits in two- and three-digit numbers, for example: • *'What is the sum of the digits in "35"?'* • *'Can you think of a three-digit number where the digit in the hundreds column is "2" and the sum of all the digits is no more than "4"?'*	Modelling the language of 'number' and 'digit': 17 • *'This is the number seventeen.'* • *'One and seven are the digits in seventeen.'* • *'The sum of the digits is eight.'* $1 + 7 = 8$

6:2

Once children are confident with the language, briefly practise skip counting in threes and chanting the three times table to remind children of the multiples of three up to thirty-six. Then present numbers up to and including 18 in a table and, as a class, find the sum of the digits of each number, as shown opposite.

Ask children what they notice about the resulting table, working towards the generalisation: ***'For a number to be divisible by three, the sum of the digits of the number must be divisible by three.'***

Continue the table beyond 18, in multiples of three, to verify that the rule continues to work.

Then extend the rule beyond known multiples of three, beginning with two-digit numbers (e.g. 42 and 72) and then exploring some three-digit numbers (e.g. 102 and 423). Once you have established that the rule is valid beyond 36, practise as a class identifying whether particular numbers are divisible by three, ensuring you use a mixture of numbers that both are and aren't multiples of three.

Revealing the divisibility rule for three:

Number	Sum of the digits
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	1
11	2
12	3
13	4
14	5
15	6
16	7
17	8
18	9
	'For a number to be divisible by three, the sum of the digits of the number must be divisible by three.'

		Summary up to twelve threes: <table><tr><th>Number</th><th>Sum of the digits</th></tr><tr><td>3</td><td>3</td></tr><tr><td>6</td><td>6</td></tr><tr><td>9</td><td>9</td></tr><tr><td>12</td><td>3</td></tr><tr><td>15</td><td>6</td></tr><tr><td>18</td><td>9</td></tr><tr><td>21</td><td>3</td></tr><tr><td>24</td><td>6</td></tr><tr><td>27</td><td>9</td></tr><tr><td>30</td><td>3</td></tr><tr><td>33</td><td>6</td></tr><tr><td>36</td><td>9</td></tr><tr><td></td><td>'For a number to be divisible by three, the sum of the digits of the number must be divisible by three.'</td></tr></table>
6:3	Provide children with some practice relating to divisibility by three.	Abstract problem: *'Circle the numbers that are divisible by three.'* 3 103 96 13 113 106 23 123 116 33 133 126 Contextual problems: *'For each example below, circle the numbers that are possible.'* • *'Easter eggs are sold in packs of three. Which number of Easter eggs can be arranged so that there are only full boxes?'* 86 87 88

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- *'A factory produces cartons of juice in packs of three. They can make the production line go slightly faster or slightly slower. If they want to make full packs of three each hour, how many cartons of juice should they make each hour?'*

740 741 742
</td>
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<td>6:4</td>
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Now move on to the divisibility rule for six. Since this rule relies on the divisibility rules for two and three, and on the connection between the two and six times table, briefly review:

- the divisibility rule for two (even numbers are divisible by two)
- skip counting in sixes/chanting the six times table, and reminding children of the fact that all multiples of six are even numbers
- 'double skip counting' in threes and sixes, and the relationship between groups of three and groups of six, including the following generalisations:
  - ***'For every one group of six, there are two groups of three.'***
  - ***'The product of an even number and three is a product in the six times table.'***

You can use familiar representations from earlier in this segment, such as dice showing six dots (two columns of three), six legs on a bug (three legs on each side) and stacked number lines.

Then, as a class, sort some numbers into a Venn diagram according to whether they are divisible by two, by three, or by *both* two *and* three. With the multiplication chart for the six times table visible, ask children what they notice, working towards the generalisation: '***For a number to be divisible by six, the number must be divisible by both two and three.'***

You can use tables similar to those in step *6.2*, working systematically to
</td>
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Revealing the divisibility rule for six:

'Place these numbers in the diagram. In the overlapping section, you should place the numbers that are divisible by both two and three. Numbers that are neither divisible by two nor three should go outside the circles.'

<table>
<tr><td>2</td><td>4</td><td>6</td><td>9</td><td>12</td><td>14</td><td>15</td><td>18</td></tr>
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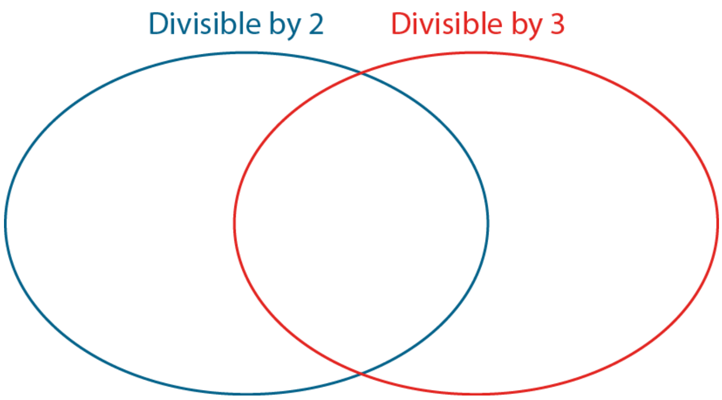

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<tr><td></td><td>verify the rule, and then extend beyond known multiples of six, i.e. beyond 72, and into the three-digit numbers.</td><td></td></tr>
<tr><td>6:5</td><td>Provide practice similar to that in step 6.3, related to divisibility by six.
Dòng nǎo jīn: 'Think of a number bigger than 400 and smaller than 410 that:
• is divisible by six
• is divisible by three, but not divisible by six.'</td><td>'Put a tick in the correct boxes to show which numbers are divisible by two, three and six.'
<table>
<tr><th></th><th>Divisible by 2</th><th>Divisible by 3</th><th>Divisible by 6</th></tr>
<tr><td>24</td><td></td><td></td><td></td></tr>
<tr><td>48</td><td></td><td></td><td></td></tr>
<tr><td>63</td><td></td><td></td><td></td></tr>
<tr><td>336</td><td></td><td></td><td></td></tr>
<tr><td>588</td><td></td><td></td><td></td></tr>
<tr><td>693</td><td></td><td></td><td></td></tr>
</table></td></tr>
<tr><td>6:6</td><td>Finally, explore the divisibility rule for nine. Briefly practise skip counting in nines/chanting the nine times table. Then, in the same way as in step 6.2, examine the sum of the digits of numbers, and ask children to spot the pattern for numbers that are multiples of nine, working towards the generalisation: 'For a number to be divisible by nine, the sum of the digits of the number must be divisible by nine.'
Then extend the rule beyond known multiples of nine.</td><td>Summary up to twelve nines:
<table>
<tr><th>Number</th><th>Sum of the digits</th></tr>
<tr><td>9</td><td>9</td></tr>
<tr><td>18</td><td>9</td></tr>
<tr><td>27</td><td>9</td></tr>
<tr><td>36</td><td>9</td></tr>
<tr><td>45</td><td>9</td></tr>
<tr><td>54</td><td>9</td></tr>
<tr><td>63</td><td>9</td></tr>
<tr><td>72</td><td>9</td></tr>
<tr><td>81</td><td>9</td></tr>
<tr><td>90</td><td>9</td></tr>
<tr><td>99</td><td>9</td></tr>
<tr><td>108</td><td>9</td></tr>
<tr><td></td><td>'For a number to be divisible by nine, the sum of the digits of the number must be divisible by nine.'</td></tr>
</table></td></tr>
</table>

<table>
<tr>
<td>6:7</td>
<td>Again, provide practice, now related to divisibility by nine.
Example word problems:
• 'A grocer has 209 apples.'
  • 'Can he split them into full packs of nine?'
  • 'The grocer finds that two of the apples are rotten. Can he split the remaining apples into packs of nine?'
• Dòng nǎo jīn: 'Mr Whitehouse tells his class that 504 is divisible by nine.'
  • 'Rishi says that 504 must also be divisible by three.'
  • 'Anna says that 504 must also be divisible by six.'
'Are they right? Why/why not?'</td>
<td>Circle the numbers that are divisible by nine.

9   108   63
19   118   263
29   168   563
99   198   963</td>
</tr>
</table>