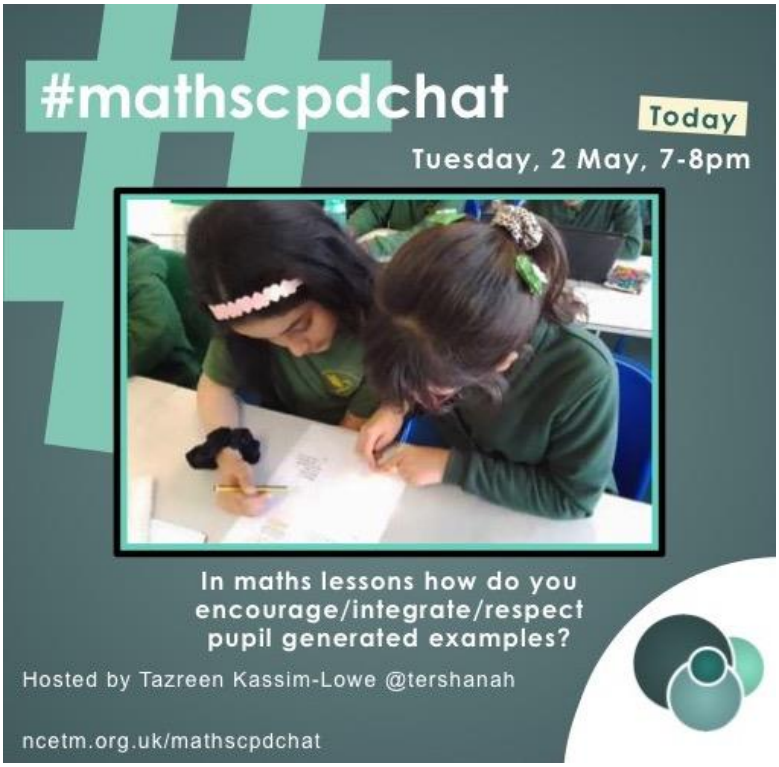
#mathscpdchat 2 May 2023

In maths lessons how do you encourage/integrate/respect pupil-generated examples?
Hosted by Tazreen Kassim-Lowe

This is a summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



The links shared during this discussion were:

Algebraic Thinking which are materials from the NCETM. They are designed for teachers to explore ideas about algebra with (KS3) students, and to encourage the students to think in algebraic terms about a range of problems and questions. It was shared by Tazreen Kassim-Lowe

Thinkers which is a book from the ATM written by Chris Bills, Liz Bills, John Mason and Anne Watson. It promotes a classroom culture in which learner-generated examples are provoked so that they enhance and deepen students' mathematical thinking and learning. It was shared by Tazreen Kassim-Lowe

The power of student-generated examples in mathematics which is a blog by Robert Talbert. He provides reasons and examples to support the belief that there is great value in placing the main work of example generation in maths into the hands of the students. It includes a link to the article, *Student-generated examples in the learning of mathematics* by Anne Watson and John Mason. It was shared by Tazreen Kassim-Lowe

Submit A Solution which is a page on the NRICH website which provides guidance about submitting, for publication on/by NRICH, pupils' solutions/responses to NRICH tasks. It includes notes about aspects of solutions that are greatly valued. It was shared by Tazreen Kassim-Lowe

Maths Carrolls which is a webpage where you will find a template in which pupils/students can write their own examples, and a variety of example 'examples' provided by Nathan Day. It was shared by Peter Williams

Focus on... Quadrigon which is an article in the archived NCETM Secondary Magazine, Issue 68. It was shared by Mary Pardoe

An idea for the classroom - regions created by reflections which is an article in the archived NCETM Secondary Magazine, Issue 71. It was shared by Mary Pardoe

Focus on... spirolaterals which is an article in the archived NCETM Secondary Magazine, Issue 78. It was shared by Mary Pardoe

An illustrated summary of the discussions in this #mathsCPDchat follows.

The host tweeted this invitation to the chat an hour before it was due to begin:

Click on the image above to go to Esther's tweet in order to see/listen-to the video clip.

Later, during the chat, the following tweets were posted:

With the next tweet the host opened the chat …

Tazreen Tershanah @tershanah · May 2

Thanks for joining this #mathscpdchat: In maths lessons how do you encourage/integrate/respect pupil-generated examples?

Don't forget to use the hashtag: #mathscpdchat

I will do my best to respond to posts and comments thoughtfully. @NCETM

… and her first main question, following swiftly …

Tazreen Tershanah @tershanah · May 2

Here is a starter question:

Question 1.

What does the term 'pupil-generated examples' mean to you?

… generated the next four conversations. First, a comment from Mr Hawes prompted a conversation consisting of two threads, this next one, which includes a reminder of the five reasons that Anne Watson and John Mason have identified for encouraging pupils to think of their own examples …

MrHawesMaths @HawesMaths · May 2

As it says on the tin. Examples generated by students like you might see in Frayer models. In the other hand, it could be examples that have been modelled and worked on by a student in the case of I go we go you go. #mathscpdchat

Tazreen Tershanah @tershanah · May 2

#mathscpdchat Thanks @HawesMaths . So scaffolded examples in the physical sense (i.e. with a frayer model or graphic organise) and in the thinking sense (getting pupils to model their way of thinking to others based on a particular method)?

MrHawesMaths @HawesMaths · May 2

Exactly that. #mathscpdchat a good question to fire out sometimes is getting students to generate examples using some form of context. Gets them to think about where and how the maths could be used.

Tazreen Tershanah @tershanah · May 2

#mathscpdchat Like the idea of attaching real life contexts and experiences. Reminds me of Watson and Mason's work on using pupil generated examples....

Tazreen Tershanah @tershanah · May 2

#mathscpdchat
Watson and Mason (2005)
1. Experiencing structure
2. Experiencing and extending the range of Variation
3. Experiencing Generality
4. Experiencing the constraints and meanings of conventions
5. Extending example-spaces and exploring boundaries

… and this, in which we have repeated Mr Hawes' tweet that generated both threads, and which includes ways of encouraging pupils to think of their own examples …

MrHawesMaths @HawesMaths · May 2

Exactly that. #mathscpdchat a good question to fire out sometimes is getting students to generate examples using some form of context. Gets them to think about where and how the maths could be used.

Mary Pardoe @PardoeMary · May 2

And sometimes pupils own examples just HAVE to materialise ... from the way a task is presented. For example this is from here:
ncetm.org.uk/media/kvabsy1u.
#mathscpdchat

Focus on…spirolaterals

Many British mathematics teachers first came across spirolaterals, as activity 62, Worms, in Points of Departure 1, which was published by the ATM in 1986. The example is Frank Odds' first school-boy doodle – this 90°, 1, 2, 3, 'worm track'…

Students usually very much enjoy creating their own spirolaterals, asking their own questions about them, and trying to reach some general conclusions. Mathematical symmetry is likely to feature in at least some of their questions.

Tazreen Tershanah @tershanah · May 2

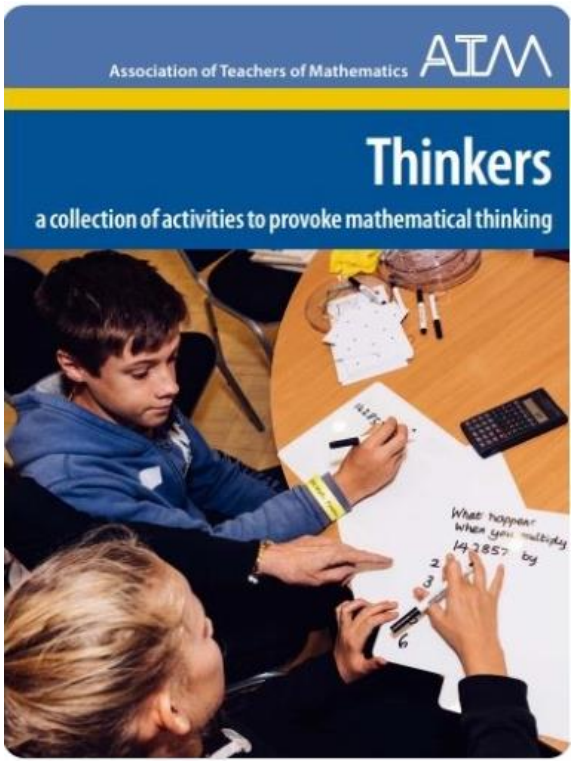
#mathscpdchat Really great example of 'Experiencing Generality' and, in fact, seeking it out by creating their own examples. Thanks @PardoeMary

Tazreen Tershanah @tershanah · May 2

#mathscpdchat This reminds me of a time where I asked my year 3s to submit solutions to @nrich. Generating their own examples for submission really motivated them to be communicate their thinking through their examples: nrich.maths.org/6274/submitsol...

Tazreen Tershanah @tershanah · May 2

#mathscpdchat



Another direct reply, this time from Peter Williams, to Tazreen's first question …

What does the term 'pupil-generated examples' mean to you?

… prompted the following discussion:

Peter Williams @MathsImpact · May 2

A great tool for gathering current understanding of a topic, especially on mini whiteboards so you can see a range of different responses quickly.

Typically they can be phrased as "show me a..."

#mathscpdchat

Peter Williams @MathsImpact · May 2

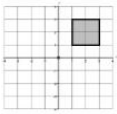
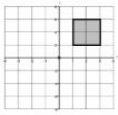
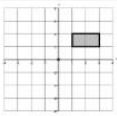
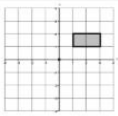
For a slightly more structured approach, @nathanday314 had some maths carroll diagrams which are designed to generate student examples, but with specific constraints.

#mathscpdchat

Primes	Is prime	Is not prime
Feels prime	139	119
Does not feel prime	149	129

interwovenmaths.com
Maths Carrolls - Interwoven Maths
Template Here! PPT Template Examples

Ratios	Simplifies	Does not simplify
Feels simplifiable	35 : 10 : 15	12 : 10 : 15
Does not feel simplifiable	351 : 104 : 156	121 : 104 : 156

Squares	A square	Not a square
Looks like a square		
Doesn't look like a square		

Percentages by @studymaths	Easy to calculate	Hard to calculate
Looks easy to calculate	25% of 64	25% of 25
Looks hard to calculate	64% of 25	64% of 64

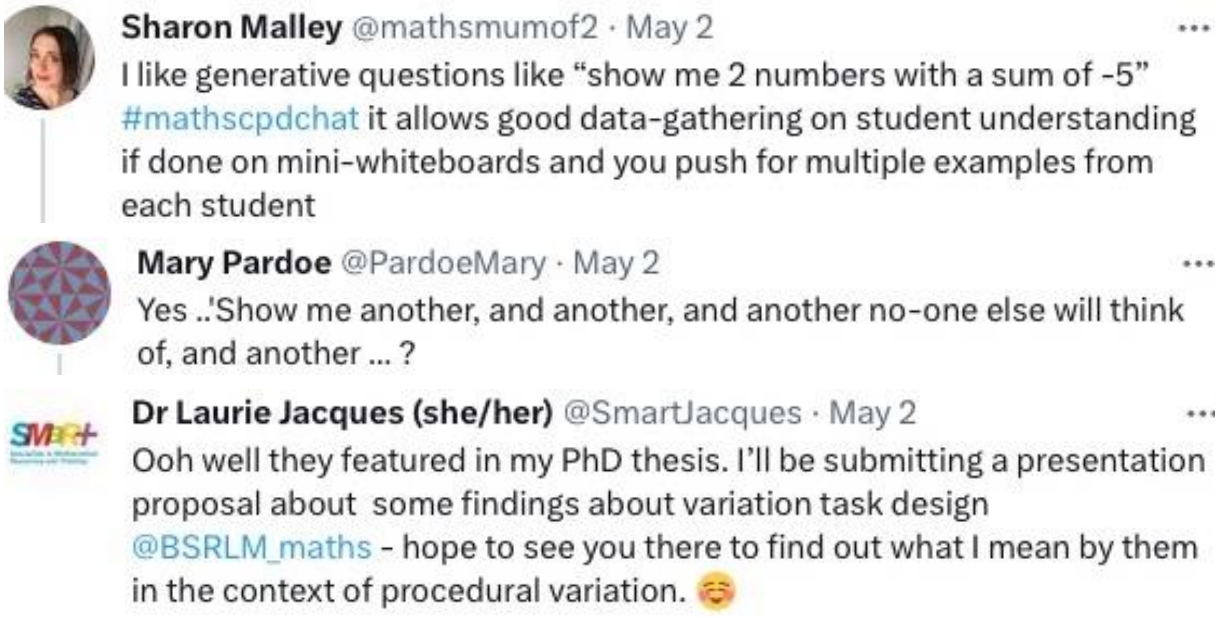
Powers	Easy to calculate	Hard to calculate
Looks easy to calculate	100^2	2^{100}
Looks hard to calculate	2.01^0	$0.2^{0.1}$

Integrals	Easy to calculate	(Very) Hard to calculate
Looks easy to calculate	$\int e^{-2x}\,dx$	$\int e^{-x^2}\,dx$
Looks hard to calculate	$\int 2^{-ex}\,dx$	$\int e^{-2^x}\,dx$

The third conversation resulting from a reply to this question …

What does the term 'pupil-generated examples' mean to you?

… was started by Sharon Malley's reminder that the use of mini-whiteboards by pupils is often useful when they are generating and sharing their own examples, another and another and another … … …



Tazreen Tershanah @tershanah · May 2

#mathscpdchat show me 2 numbers with a sum of -5.....and another and another. Mini-whiteboards can be powerful as pupils have ownership over that space. I recall using the visualiser to highlight particularly thought provoking examples from students to discuss.

Tazreen Tershanah @tershanah · May 2

#mathscpdchat or even a general example, a peculiar example and a non-example in order to experience 'the constraints and meanings of conventions' Watson and Mason (2002)

Tazreen Tershanah @tershanah · May 2

#Mathscpdchat Particular, Peculiar, General
Write down a particular/peculiar/general number that leaves a remainder of 1 when divided by 7. Write down a number that leaves a remainder of 1 when divided by 7 which is peculiar in some way.

Tazreen Tershanah @tershanah · May 2

#mathscpdchat Can highly recommend @ATMMathematics 'Thinkers' e-book for more generative questions and activities🙌🏽 @mathsmumof2

Tazreen Tershanah @tershanah · May 2

@MathsImpact This thread might interest you, since you mentioned the power of mini-whiteboards. #mathscpdchat

The fourth conversation in response to this first main question …

What does the term 'pupil-generated examples' mean to you?

… provides more suggestions of helpful resources and strategies that teachers have used:

MrHawesMaths @HawesMaths · May 2

Another situation is to put an answer on the board say '12' and students have to come up with lots of questions that have that solution. Ranging from equations to ratio to fraction to area and perimeter. The possibilities are vast and responses so interesting #mathscpdchat

Neil "not Santa" Eley @neileley · May 2

Used to do this with y7 and Y8 Only allowed one question on a topic.

Tazreen Tershanah @tershanah · May 2

#mathscpdchat I recently watched a lesson of a secondary colleague who selected examples which highlighted particular algebraic structured, lettered them (A-E) and asked the whole class which was more appropriate for the given situation. Very powerful.

Tazreen Tershanah @tershanah · May 2

These resources were used as a stimulus for that particular lesson:

ncetm.org.uk
Algebraic Thinking
Materials for exploring algebra with KS3 students

Tazreen Tershanah @tershanah · May 2

#mathscpdchat. Just to note that the above resource is a great way to encourage 'experiencing structure'. Had to mention it, as it was the only principle from Watson and Mason (2002) not mentioned so far!

Tazreen Tershanah @tershanah · May 2

#mathscpdchat *Watson and Mason (2005)

Tazreen's second main question …

Tazreen Tershanah @tershanah · May 2

Feel free to keep commenting on Question 1. What does 'pupil-generated example' mean to you?

Here is another question to get us thinking and reflecting:

Question 2. Do you integrate pupil generated examples in your maths lessons? If so, how? When? #mathscpdchat

… prompted the sharing of an example:

Mary Pardoe @PardoeMary · May 2

I have done when looking at reflection. This (that I wrote ages ago) is from here: ncetm.org.uk/media/sn4dyohr... #mathscpdchat

An idea for the classroom – regions created by reflections

This is a task in which students create their own examples, and that they could tackle in pairs on

Prompt students:

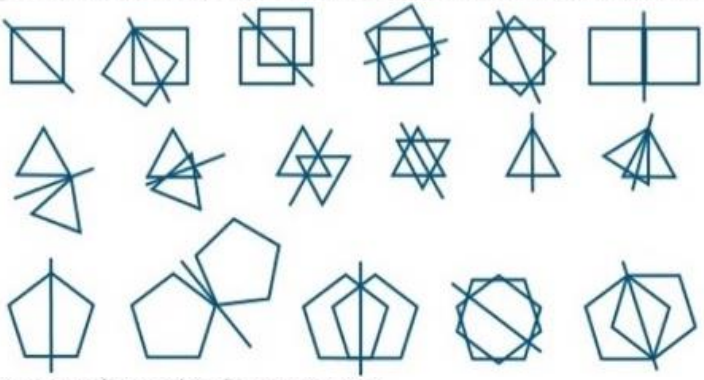
- *think about what you can change*
- *ask your own questions, and try to answer them.*

Encourage students to explore their own examples by changing aspects of the situations that they create. For example, they may change the position of the mirror line but keep the same polygon. Or they might change the number of sides of the polygon while retaining the regularity.

From time to time, invite particular students to link their computer to the large screen, show their diagrams to the whole class, and talk about what they are finding. These brief whole-class episodes should generate discussion and prompt students to ask their own questions, such as:

- *what are the maximum and minimum possible numbers of regions when the shape that is reflected is a square?*
- *what numbers of regions is it impossible to make?*
- *when you reflect a square, are there any numbers of regions that you can only make in one way?*
- *are the maximum and minimum possible numbers of regions the same for all regular polygons?*
- *is the maximum number of regions related to the number of sides of the polygon?*
- *how is the maximum number of regions related to the number of sides of the polygon?*

In one lesson, the diagrams that students created and printed out to support their findings included these:



Encourage students to generalise and make conjectures.

In one lesson students conjectured that:

- *when any regular polygon is reflected the minimum number of regions is always 2*
- *you cannot make three regions when you reflect a square*
- *when an equilateral triangle is reflected the number of regions is never more than 8*
- *you cannot make more than ten regions when you reflect a square*
- *you cannot make three regions with any regular polygon*
- *you cannot make an odd number of regions with any regular polygon*
- *whatever regular polygon is reflected the maximum number of regions is twice one more than the number of sides of the polygon.*

A few students were able to express this last generalisation algebraically:

- *the maximum number of regions is 2(n + 1), where n is the number of sides of the polygon that is reflected.*

Tazreen Tershanah @tershanah · May 2

Thanks for sharing @PardoeMary #Mathscpdchat Wonderful resources. Especially like how it addresses '5. Extending example-spaces and exploring boundaries' with questions like 'Is it possible to make...?'

In response to Tazreen's third main question …

Tazreen Tershanah @tershanah · May 2

Feel free to respond to questions 1 and 2. A few comments in the last minute have lead nicely on to the 3rd question of the evening:

Question 3. Do you plan to integrate pupil generated examples in maths lessons or is it spontaneous? Please give examples #mathscpdchat

… there were three threads/discussions. This one was started by a reply from the host herself:

Tazreen Tershanah @tershanah · May 2

#mathscpdchat This blog talks about planning for meaningful pupil generated learning in maths lessons

rtalbert.org

The power of student-generated examples in mathematics

Why don't we have students generating their own examples more often? And how might such a strategy of student-generated exampl...

Tazreen Tershanah @tershanah · May 2

#mathscpdchat Although after a secondary colleagues wonderful lesson, they said that the pupil generated examples were not part of the original lesson plan. So perhaps it is possible to make pupil-generated examples meaningful, even if it is spontaneous?

Mary Pardoe @PardoeMary · May 2

See your retweet just now @AllThingsMaths, (about #changingperceptions) might this not change some perceptions of what can be done effectively in maths lessons? **#mathscpdchat**

> **Tazreen Tershanah** @tershanah · May 2
>
> #mathscpdchat This blog talks about planning for meaningful pupil generated learning in maths lessonsrtalbert.org/the-power-of-s...
>
> Show this thread

Maryse @AllThingsMaths · May 2

I've briefly skimmed it and from what I've seen, yes. I like students writing questions too, approaching it from elsewhere. I've done a 1 to 1 on circle theorems with a visually impaired student and it was really interesting to observe how she approached >

Maryse @AllThingsMaths · May 2

problems. We started by just having her feel the question and then generated everything she could explore. Then angles in semicircle, then she explored some more. Not quite what the #mathscpdchat question was, but still approaching it from student led.

Maryse @AllThingsMaths · May 2

I always say it's like 2 Islands. What we know. What we want to know. Then we build a bridge between. It's the bridge building that is the problem solving. We touched on isosceles triangles, congruency, similarity, Pythag and trig. All student generated. #mathscpdchat

This next thread includes another suggested strategy to generate pupils' own examples …

Maryse @AllThingsMaths · May 2

Forgot it was Tuesday 🤦‍♀️

I often plan but then sometimes a particular response takes us slightly off track. Sometimes I'll "put a pin it" to return to later.

#mathscpdchat

Tazreen Tershanah @tershanah · May 2

No worries @AllThingsMaths #Mathscpdchat Yes. It opens up more avenues to explore. Watson and Mason (2002) talk about pupil generated examples as a larder (metaphorically): when you go looking for one thing you automatically find and recategorize another thing!

Maryse @AllThingsMaths · May 2

A particular example has probably been mentioned, but it's the "the answer is ... What was the question?"

Directed number, factors, primes are examples where this strategy has been used.

Tazreen Tershanah @tershanah · May 2

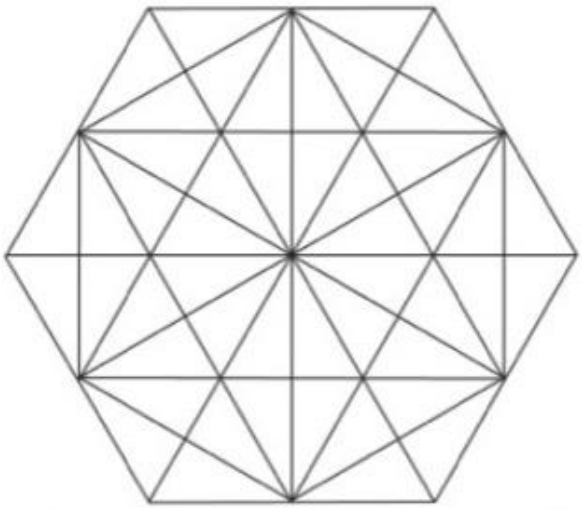
#mathscpdchat This has not already been mentioned. Brilliant. Thank you. A lovely example of 'Experiencing and extending the range of Variation'.

… as does the third thread created in response to Tazreen's third question:

Mary Pardoe @PardoeMary · May 2

This is an image I have used (planned to use) time and time again to get pupils generating examples ... wrote re it here: ncetm.org.uk/media/1o5fq3xm. #mathscpdchat

Focus on...Quadrigon



The great richness of this situation is in the opportunities that the board itself provides for students to explore their own examples of mathematical objects and relationships between them, ask their own questions, and consolidate what they know in creative and aesthetically satisfying ways.

I have observed students seeing in the Quadrigon board, and investigating, many different objects and relationships, such as those suggested by the following questions – which are just a few of the many questions that students might ask themselves.

Tazreen Tershanah @tershanah · May 2

Thanks @PardoeMary I love that this particular task encourage pupils to generate, not only their own examples, but their own questions! #mathscpdchat

The host's fourth, and last, main question …

Tazreen Tershanah @tershanah · May 2

A final question to think about before we finish our discussion in the next few minutes. Thank you to all who have contributed so far! Keep using the hashtag!

Question 4. How can pupil generated examples work with teacher generated examples in the maths classroom?#mathscpdchat

… again generated three threads. This short thread was about another strategy …

Mary Pardoe @PardoeMary · May 2

Teacher can show two 'things', maybe images (their example) and ask 'What is the same, what is different about them?' the pupils give THEIR examples of what is same/different. I'll post an example later when I've found it! #mathscpdchat

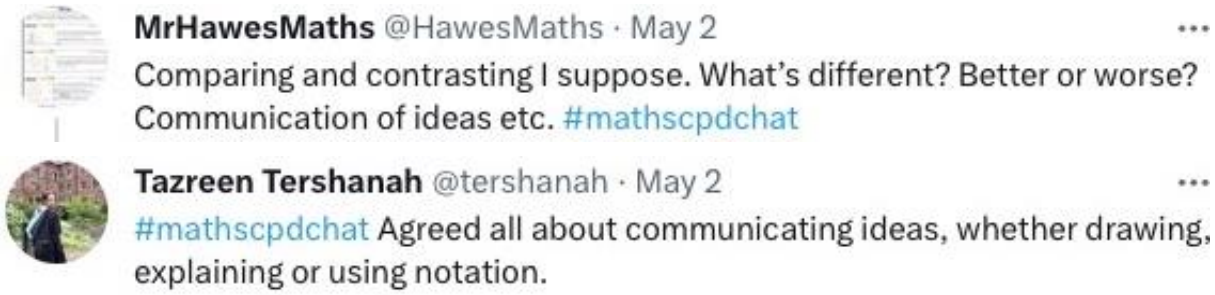
Maryse @AllThingsMaths · May 2

Yes... this.

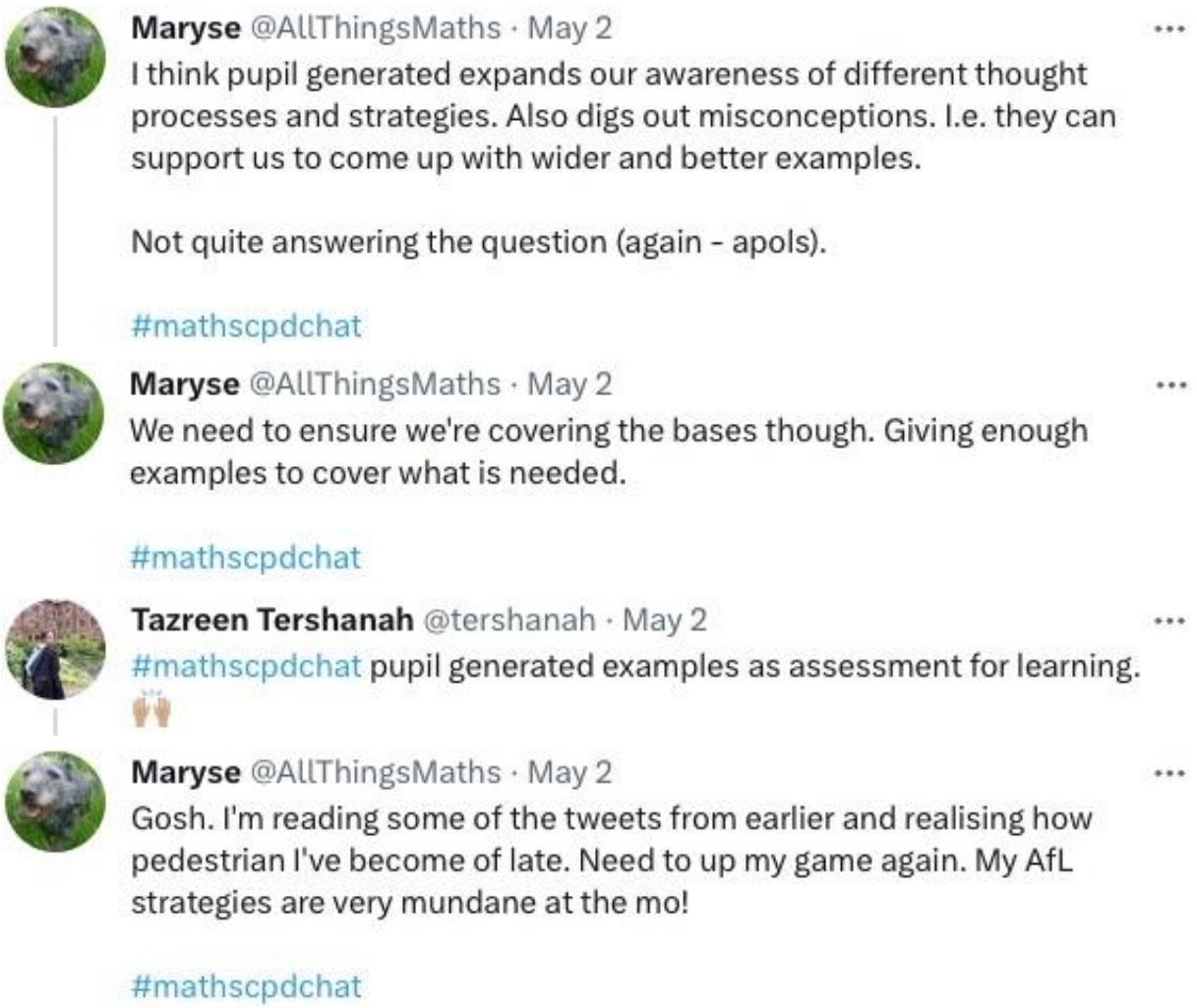
Tazreen Tershanah @tershanah · May 2

#mathscpdchat I have given students a number or term to exemplify with drawing e.g. a year 3 class 'There are 3 groups with 3 in each group'. Some beautiful and whimsical examples (e.g. 3 bowls with 3 fish in each) compared with my array or bar model.

… this thread conveyed another important thought …



… and the last thread was a reminder that, when pupils share examples they have thought of themselves, they reveal aspects of what they know and can do in/with mathematics:



Tazreen summed up the content of this #mathsCPDchat, the topic of which she had suggested, in her closing message:

Tazreen Tershanah @tershanah · May 2

Time's up on tonight's #mathscpdchat. Feel free to keep commenting.
Today we have discussed:
1. what pupil generated examples are
2. When, how and why we use them
3. Whether they are planned or spontaneous
4. The balance between teacher and pupil generated examples

Maryse @AllThingsMaths · May 2

👏👏👏 on hosting. Apols for being late to the party.

Tazreen Tershanah @tershanah · May 2

#mathscpdchat No worries. Always welcome.