# Core concept 2.2: Solving linear equations

This document is part of a set that forms the subject knowledge content audit for Key Stage 3 maths. The audit is based on the NCETM Secondary Professional Development materials and there is one document for each of the 17 core concepts. Each document contains audit questions with check boxes you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications and explanations, and further support links. At the end of each document there is space to type reflections, targets and notes. The document can then be saved for your records.

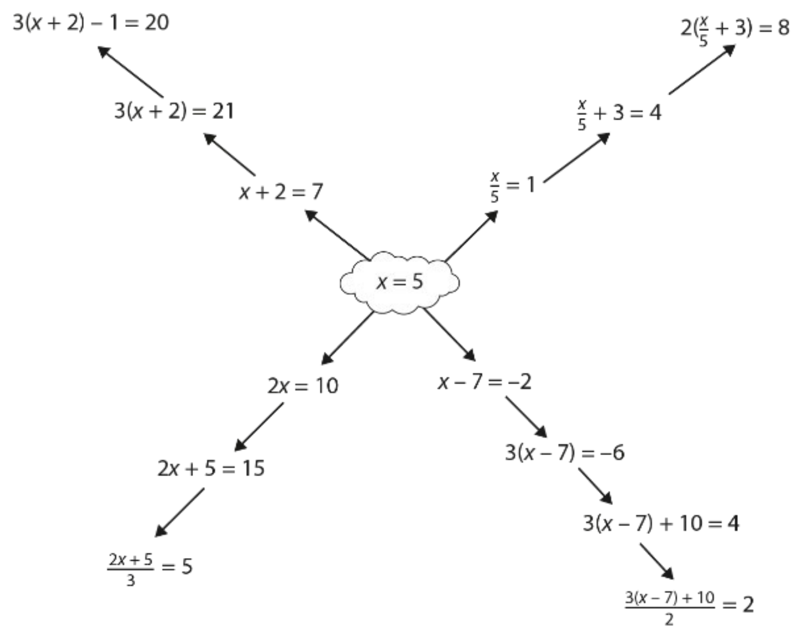
## 2.2.1 Understand what is meant by finding a solution to a linear equation with one unknown

How confident are you that you understand and can explain what a linear equation is, and what is meant by finding its solution?

1 ☐ 2 ☐ 3 ☐ 4 ☐

At Key Stage 3, algebra is considered as generalised arithmetic and built on a secure understanding of equivalence, simple algebraic manipulation and of finding unknown values.

A useful way of supporting students' appreciation of what makes a solution is to offer them the opportunity to create their own equations starting from a given value, as in this 'spider diagram'.




Students should appreciate that $x = 5$ is a linear equation (to which the solution is obvious) and that all other linear equations that are a transformation of this have the same solution. This also links to the awareness that linear equations have only one solution.

Students are also expected to be able to:

- Reason whether certain values are or are not solutions to particular equations. *'Samira says that $x = 3$ is a solution to the equation $7 - 5x = 8$. Is she right? If so, explain why; if not, explain why not and correct her.'*
- Interrogate equations that do not have a solution and explain why. *'What happens when you try to solve $4x + 6 = 4x$? What does this mean? Why is there no solution?'*

### Further support links

- NCETM Secondary Professional Development materials: 2.2 Solving Linear Equations, pages 9–17

## 2.2.2 Solve a linear equation with a single unknown on one side where obtaining the solution requires one step

How confident are you that you can explain how to find the solution of a linear equation requiring a single additive or multiplicative step?

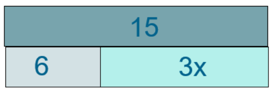
1 ☐ 2 ☐ 3 ☐ 4 ☐

Solving simple linear equations requiring a single step is built on understanding that these are the formulation of one operation on an unknown number, and how they can be solved by undoing the operation to find the value of the unknown.

Students should be able to recognise alternative versions of the family of four within that structure, e.g., $5 + 3 = 8$, so $8 – 3 = 5$, $8 – 5 = 3$ and $3 + 5 = 8$. This understanding will enable them to construct the other three rearrangements of the equation $x + 3 = 10$: $3 + x = 10$, $10 – 3 = x$, $10 – x = 3$.

A simple linear equation can be represented using a bar model as follows:

**$6 + 3x = 15$**




A similar process can be followed for equations of the form $x – a = b$, $ax = b$ and $\frac{x}{a} = b$. In each case, one of the rearrangements will result in a form from which the solution can be calculated.

An important awareness is that, if $a = b$, then $a + c = b + c$ and $a \times c = b \times c$.

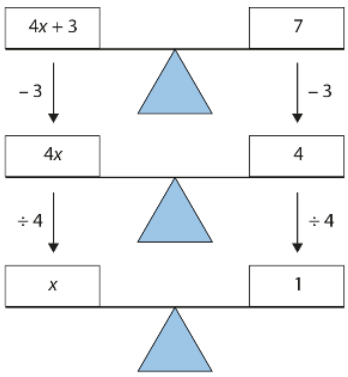
## 2.2.3 Solve a linear equation with a single unknown where obtaining the solution requires two or more steps (no brackets)

How confident are you that you can model how to solve a linear equation requiring two or more steps, including linear equations involving reciprocals?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Solving linear equations with two or more steps involves the undoing of these operations in the correct order to find the value of the unknown.

When using the balance method, one operates in the same way on both sides of an equation to maintain equality, for example:




It is also useful to consider what constitutes the most efficient solution. For example, in the equation $5x – 14 = 6$, it is important to:

- understand that any operation applied to both sides of the equation will result in equality being maintained
- reason why some operations lead to a solution more quickly than others.

Students will benefit from exploring these ideas with a wide variety of linear equations with unknowns on both sides and, through these experiences, become aware that all equations of the type $ax + b = cx + d$ can be reduced to the form $Ax + B = C$.

Exploring equations such as $6 - 2x = x + 9$ will usefully give rise to a discussion about whether to subtract $x$ from both sides or to add $2x$ to both sides. Such discussions of efficiency and ease of calculation will support the development of approaches to solving equations of the form $a - x = b$.

Similarly, consideration of equations of the form $\frac{a}{x} = b$ and $\frac{a}{x} + c = b$ will help students see that these can be transformed into $a = bx$ and $a + cx = bx$.

### Further support links

- NCETM Secondary Professional Development materials: 2.2 Solving Linear Equations, pages 18–24
- For information on using algebra tiles to solve linear equations see NCETM: Using Mathematical Representations at Key Stage 3: Algebra tiles, pages 11–17

## 2.2.4 Solve efficiently a linear equation with a single unknown involving brackets

How confident are you that you can model how to solve a linear equation involving brackets?

1 ☐ 2 ☐ 3 ☐ 4 ☐

By considering a range of linear equations involving brackets, students should explore looking at the structure of an equation to help decide the most efficient method for solving it. For example, $3(x - 2) = 27$ can be simplified directly to $x - 2 = 9$ rather than multiplying out the brackets first.

Through discussion, students can secure and deepen their understanding of solving linear equations and reflect on the efficiency and elegance of the solutions. The following examples suggest that it can sometimes be useful to use common factors to simplify, rather than multiplying out the brackets:

- $2(x + 1) + 3(x + 2) = 10$
- $2(x + 1) + 3(x + 1) = 10$
- $2(x + 1) + 2(x + 2) = 10$

Attention may also be given to the way in which different representations of the same equation may suggest different methods. For example, in the equation $\frac{1}{3}(x + 3) = 5$, students may want to expand the brackets, while the same equation represented as $\frac{x+3}{3} = 5$ may lead students to multiplying by three as a first step. Students should be made aware of these different representations in order to make informed and flexible decisions about the most efficient route to a solution.

## Notes