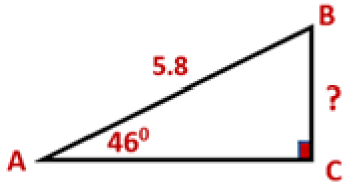
Secondary Magazine 148

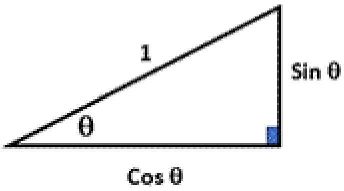


National Centre for Excellence in the Teaching of Mathematics

Welcome to the first Secondary Magazine of the summer term. With the exam season upon us, and gained time looming, we take the opportunity to ask if your school is thinking about teaching for mastery for 2018-2019, and talk to one school that has adopted the approach. And if revising trigonometry with Y11s has exposed a lack of real understanding, then we'd like to invite you to eavesdrop on a discussion about a longer-term approach to the topic.

Don't forget that all previous issues are available in the Archive.

This issue's featured articles



Trigonometry: new at Foundation tier. Two teachers discuss approaches to support deep understanding for all students

Do you have an approach to introducing trigonometry that really exposes the structure of the concept and relationships? At this time of year, it often becomes evident that students are simply trying to remember SOH CAH TOA, and a series of tricks for choosing the correct formula and applying it. In this article, two teachers discuss how a continuous approach to multiplicative reasoning in Key Stages 2 and 3 could support a deep understanding of trig in KS4. Why not use this as pre-reading for department curriculum development work?

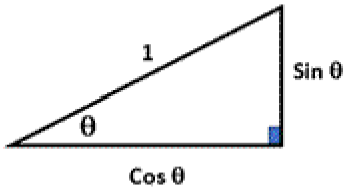
Teaching for mastery at secondary: what about our 'top' and 'bottom' sets?

The national teaching for mastery programme in secondary is now well underway (see bullets below for more information), with a second cohort of Mastery Specialists in training, and a third being recruited for the Autumn. Departments often wonder how adopting such an approach would still allow them to cater for their full spread of attainment. In this article, we visit a school in Surrey that has adopted a teaching for mastery approach and found it able to meet the needs of students in all sets.

And here are some other things for your attention:

- Teaching for mastery in secondary phase: opportunities to find out more or get involved:
  - train to be a Mastery Specialist – recruiting now
  - apply for your department to supported by a Mastery Specialist in adopting teaching for mastery
  - listen to the experiences of two teachers involved in the programme on our most recent podcast
  - read about one school's experience in this issue's second feature article
- Keep an eye on BBC Breakfast this summer, where the presenters are studying for and taking GCSE maths. Their experiences may be encouraging for your students.
- Thinking about professional development? Grants and funding opportunities are available from various bodies:
  - travelling fellowships for teachers travelling abroad to see good, innovative practice
  - grants to encourage study of the history of mathematics
  - bursaries to attend STEM Learning's four-day summer school, New to teaching A level mathematics.

- The new Advanced Maths Support Programme (AMSP) began work on 1 May to support teachers of all level 3 mathematics (AS/A level Maths and Further Maths, and Core Maths). Led by the same team at MEI, it will build on the work of the Further Maths Support Programme (FMSP) and expand to support Core Maths and to provide additional support in areas of low participation. More information about plans for the programme can be found in MEI's April newsletter.
- Although funding for Underground Mathematics (the free resource for teachers of A level maths) has come to an end, the website lives on, and if you haven't used it yet, is well worth a visit.



Trigonometry: new at Foundation tier. Two teachers discuss approaches to support deep understanding for all students

In this article we have fashioned a conversation between two imaginary maths teachers, Wendy and Meera (who is also a Head of Maths). This is based on some of the very real and constructive conversations that participants in the 2013/14 NCETM KS3 Multiplicative Reasoning Project became engaged in. Wendy and Meera are here considering Meera's department's approach to teaching trigonometry in the light of the increased demands of the new GCSE, particularly at the Foundation tier. In the discussion Wendy draws on her experience as a participant in the Multiplicative Reasoning project to offer some thoughts on an alternative approach to developing trigonometry in the curriculum.

W How's it going? I hear you're busy implementing changes to your scheme of work given the changes in the new curriculum and GCSE - ratio and proportion even has its own strand!

M Yes, but it's trigonometry we are wrestling with at the moment - it's now examined in the foundation tier GCSE. It's the algebraic skills we have doubts on.

W I thought it was a geometric idea?

M Sure, we have always introduced trig as ratios of sides of right angled triangles, but for pupils it often comes down to choosing and manipulating the right trig formula. However, if I'm honest many pupils don't really get what's going on, as essentially, we probably move to procedures involving the formulas rather too quickly.

W A few years back I participated in the NCETM KS3 Multiplicative Reasoning Project and I remember we had some teachers get very excited about how multiplicative reasoning could link together so many topics across the curriculum, including providing a potential alternative approach to trigonometry.

M Oh yes, multiplicative reasoning? What was that project about?

W Well briefly; ideas of ratio and proportion underpin many maths topics across KS1 to KS4. A key skill here is knowing how many times bigger one number is than another (multiplicative reasoning) rather than how much bigger it is (additive reasoning). Without getting into too much detail the project focussed on a small number of representations that support pupils model these problems and reason multiplicatively.

M Representations – now that's something we're interested in. We may be joining a local Maths Hub Workgroup that uses multiplicative reasoning as the basis of developing continuity in approaches across KS2 into KS3.

What particular representations would you be referring to here?

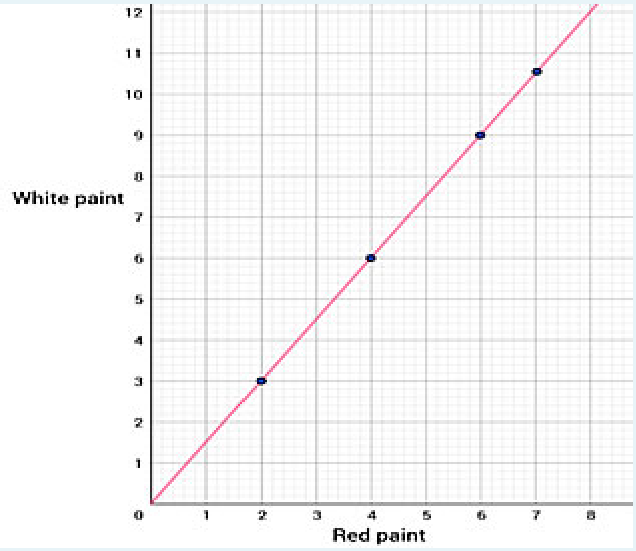
W Principally – bar model, double number line and ratio table ie; moving from concrete to more abstract representations.

M Mm interesting – anyway what has this got to do with trigonometry?

W

I remember we had been working on a graphical representation of a problem as the basis of exploring equivalent ratios:

Red and white paint are mixed in ratio 2:3 giving a pink shade.
i) Explore different quantities of red and white paint that make the shade of pink
ii) How much white paint is needed to be mixed with 7 litres of red paint?



Yes - certainly a nice context for exploring the algebraic properties of direct proportion.

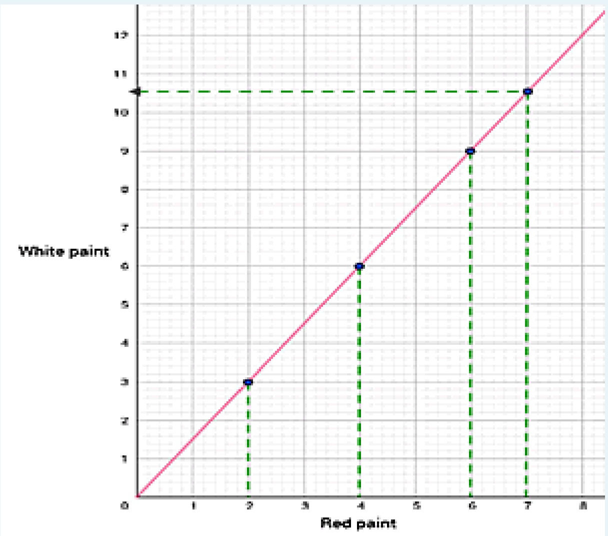
Looking at a problem from different mathematical perspectives can often help make connections and deepen insight. This is also something we are keen to look into as part of developing our pedagogy

- but again, what has this got to do with trig?

M

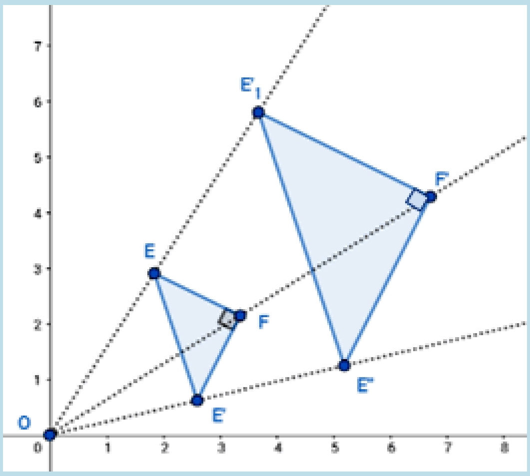
W

When we started annotating the graph to see how the equivalent ratios might appear - visually they formed a nest of right angle triangles which for many of us clearly linked to a familiar image we use for teaching enlargement.



Yes I can see the similarities *(oops how appropriate!)*.

M



It's a familiar image for our pupils. I'd say that certainly by the end of KS3 the majority of our pupils can align corresponding sides, determine the scale factor of enlargement from appropriate known pairs and use this to work out any missing sides – *I sound confident don't I!*

OK then - no arguments as an introduction, but wouldn't we fairly soon still be in formula territory?

W

Hold on for a bit longer - in the project pupils often moved towards extracting information into a ratio table in order to make sense of the problem and apply the maths. So in the problem given:

Red paint *(litres)*	White paint *(litres)*
2	3
× 7/2 ↓	↓ × 7/2
7	?

Yes this gives a rich layout to explore ratio relationships. I'm getting a feeling for where you might be going with this.

M

W

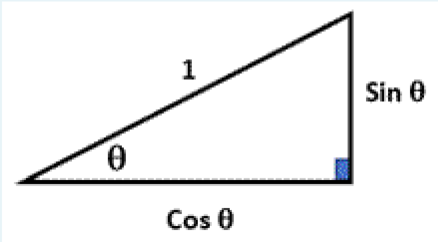
Well it was the scale factor enlargement image you refer to that prompted an interesting insight from the group – *for any right-angled triangle containing an angle Θ, our calculator gives us all the sides of an 'enlargement' (similar) triangle with that angle in it.*

Oh - I suppose you mean because your calculator can give you Cos Θ and Sin Θ they will be the sides of a right-angle triangle of unit hypotenuse?

M

W

Yes, the thing is to draw it – look, this is what you know for any angle Θ:



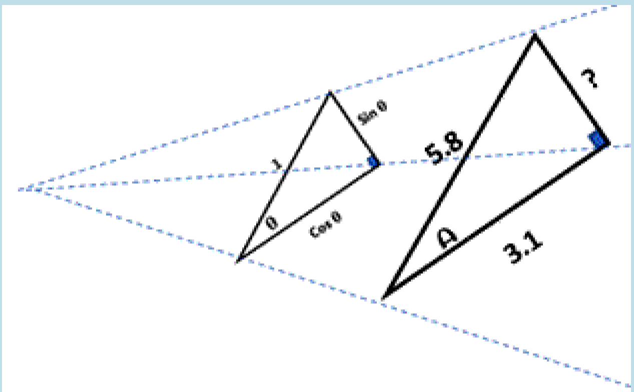
A powerful image indeed if you understand its significance?

M

W

Well this is why it follows on naturally from the work you mention on enlargement. If you can align it correctly with say the right-angled triangle given in a trig question then as you said before – 'if you can determine the scale factor of enlargement then you can work out any missing sides'.

M



Mm perhaps – how would the working be set out?

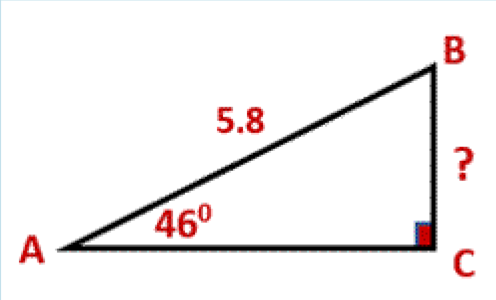
W

As with the paint example earlier – a typical tool in the MR project, for setting out working in ratio and proportion problems, is the ratio table.

M

Ok then - here is a typical non-contextual trig problem:

Given a right-angled triangle ABC with AB = 5.8 cm and angle BAC = 46°. Find the length BC



W

There was always some debate about the best way to represent information from a problem in a ratio table and there was no general consensus, or even agreement on whether a standard way should be proscribed.

M

Well I guess to some degree it might be down to pupil's previous experience using ratio tables to model problems. For a pupil to represent the problem in a way that allows them to make sense of it is often 90% of the way to solving it?

W

Yes - good point. However, in response to your question, I can show you one layout offered and discussed by the teachers which I have applied to your example problem:

	Hyp	Opp	Adj
Unit rt triangle with $\theta = 46^0$	1	sin 46^0	cos 46^0
	× 5.8 ↓	× 5.8 ↓	
Target triangle with $\theta = 46^0$	5.8	?	

What do you think?

M

It'd be good to see other ways of structuring working out for a problem like this.

However, referring to the point I just made: in our department we have begun to realise that all too often in a situation like this, we would proscribe the setting out - you know going all procedure and drill straight away rather than allow pupils to experiment a bit with layouts that make sense to them, then justifying and comparing approaches as part of a class discussion of what might be an optimal approach.

Anyway, back to your setting out. One thing I note for this particular problem, is that the hypotenuse is one of the known sides, hence aligns with the unit hypotenuse giving the scale factor of enlargement rather easily. It may not be so easy for many, if the hypotenuse is one of the unknowns?

W

Well, in the situation we are envisioning, by the end of KS3 pupils will have had some experience at setting out ratio and proportion problems in such a table, and would be used to working out multipliers between any numbers and in any direction.

M

Mm hopeful - but I can certainly see there might be advantages to such approaches offering different insights into the nature of trig.

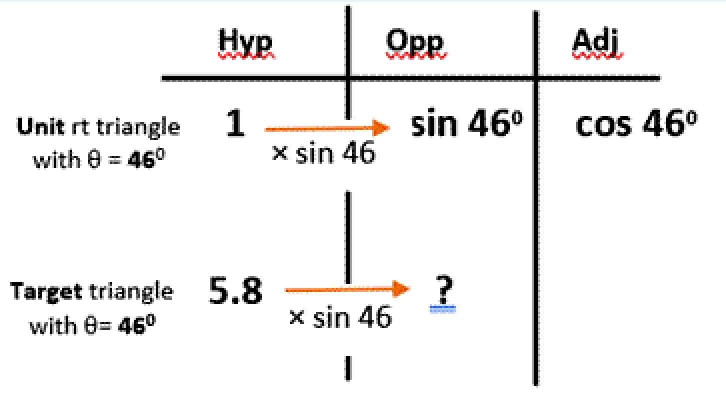
However, I still feel the use of formula is desirable, particularly as key trig ratios now need to be known, and for higher pupils an understanding of sine, cosine and tangent as functions, is required.

W

With regard to your point on the trig ratios as formulae, one of the advantages of the ratio table is how it allows you to explore the relationships in a number of ways.

In the previous ratio table the vertical multipliers represented the scale factor of enlargement but below I indicate the horizontal multipliers which represent the internal (trig) ratios of the triangles, ie sin 46 or cos 46 and this can allow an exploration that links directly with an approach involving the traditional trig 'formulas' for ratios

e.g. hyp × sin 46 = opp



M

OK, I can see how this allows some nice connections to be made.
It all appears, on the surface, to be rather neat - building on ratio and proportion approaches from KS2 and continuing to apply the same representations to wider suitable topics as we move through KS3 into KS4, including a possible *progression in geometry towards trig that we have talked about here*.

However, we have staff who have strong views and compelling arguments on how trig should be taught. But come to think of it, putting this into the mix alongside their approaches might generate a richer professional development discussion?

In the end the question for us is not only how accessible this is, in reality, for our foundation pupils, but also the flexibility it offers to develop understanding for the needs of all our pupils.
I guess the only way to find out is to try stuff out with pupils?

W

Well yes it would be good to share any thoughts on this and examples of trying out approaches with pupils.

A point the group of teachers we worked with in the project felt strongly about was the importance of *'a progression that developed these approaches over time allowing connections that built understanding and competence'*. Hence, this was more of a scheme of work development than just a few lessons added in to a unit on trig.

M

Mm that takes a bit of a departmental commitment – collaborative department CPD and planning stuff!

Anyway – as you say: be good to get a bit of discussion and share thoughts on this – that might help kick start things for us.

W

Anyway, please do let us know how you get on.

M

Yes, many thanks. I hope we can speak again soon!

The Multiplicative Reasoning Project (2013/14) is now complete, but much of the work done informs the ongoing Maths Hub Y5-8 Continuity project. Anyone wishing to get involved in this, or any other Maths Hub Work Group in 2018/19, should contact their local Maths Hub. Materials developed by the original Multiplicative Reasoning Project, to support KS3, are still available on the project's webpage.

Teaching for mastery at secondary: what about our 'top' and 'bottom' sets?

Two Y7 maths lessons, on the same day, in the same school – one with Set 1 of 6, the other with Set 3. What would you expect to see in your school? What would be the similarities? The differences?

When we visited two such lessons at Tomlinscote School in Frimley, Surrey, we noticed how similar they were. All were learning about finding the nth term of a linear sequence, all were accessing the same starting point, all tackled the same questions and had access to the same prompts for thinking and exploring at greater depth.

What was most striking was the participation – both classes were expected to follow and contribute to the learning as a class, sharing their insights and explaining the mathematics to one another.

The teacher of both classes, Teresa Booth, is one of the first cohort of Secondary Mastery Specialists, trained by the Maths Hubs network. There is no blueprint for introducing teaching for mastery into secondary schools in England, and Mastery Specialists are exploring ways in which to make teaching for mastery work in their own contexts.

In Teresa's school, after a year of learning about the principles of teaching for mastery, they settled on two main changes:

- The new scheme of work is linear rather than spiral, so a topic is studied for a substantial period of time (5-6 weeks) rather than disparate weeks scattered over the year. This allows a steady pace so that teachers can introduce topics in small steps and all can keep up. Differentiation is achieved through depth rather than by speed or topic.

- All sets study the same material. The department is resourcing the scheme of work with a selection of problems, exercises, visual representations etc. in the form of PowerPoint slides, for each topic. Teachers then pick the ones most suitable for their class.

So what does this look like in the classroom?

Most notably, the lesson feels like a class working collaboratively to understand together and to help one another understand. Teresa checks regularly, with a show of 1-5 fingers, how well students feel they understand. And when they get it, students are encouraged to verbalise their understanding regularly. Teresa asks searching questions that get at the structure of linear sequences, using - and expecting students to use - precise language and terminology such as 'arithmetic sequence', 'geometric sequence', 'ascending', 'descending', 'nth term', and 'position'.

At one point she asks a boy for an explanation of how he got his answer. He says,

> "I know, but I can't explain it."
>
> A little while later she comes back to him and asks again, and he is able to explain what he has done.
>
> "You see, you can explain it", she encourages.
>
> "I know", he says, "because I heard what they said". There is a strong recognition in the class that this is a valid way to learn from one another, rather than cheating or copying.

There is a sense of enthusiasm evident in both classes that we see, but particularly Set 3. Students are pleased when they gain an understanding or see a connection. And Teresa is seeking, all the time, to draw out those connections: between the difference and the related times table, between the difference and the nth term, between the 0th term and the nth term, and how a descending sequence affects the nth term.

Teaching for mastery embraces much of what has long been widely considered good classroom practice.

So, what looks different to good teaching in any other context…?

The pupils complete less written work than might be expected in other schools – the lesson is based around an exercise that involves finding the nth term for only eight sequences:

> 7, 12, 17, 22, 27...
>
> 2, 7, 12, 17, 22...
>
> 3, 0, 15, 21, 27...
>
> 10, 12, 14, 16, 18...

-4, -1, 2, 5, 8...

20, 30, 40, 50, 60...

½, 1½, 2½, 3½, 4½...

(Pupils in all sets work on these sequences, with those in lower sets provided with additional time and scaffolding to get to the nth term. Previously, those in higher sets would have covered sequences including the nth term, in a single lesson in Y7, a pace that allows little time to embed knowledge. Those in lower sets would have covered 'number patterns' with no expectation of encountering the concept of generalisation).

Teresa explains:

> "We now undertake 'intelligent practice' in class through our use of Variation Theory, rather than long lists of very similar questions that the students used to do almost on auto-pilot without deepening their understanding of the concept at all".

The learning largely takes place in the discussion around the exercise. What is different about the nth terms? What is the same? How do they relate to other features we can spot in the sequences?

To cater for pupils moving at different paces, Teresa sets a 'challenge question' alongside the exercise:

> "What is the 0th term for each sequence? What do you notice?"

There is a palpable sense of excitement as pupils around the room make the connection between the 0th term and the constant term in the nth term. And then Teresa pushes further with:

> "Can you explain why?"

We're intrigued to know what the pupils make of this kind of interactive lesson style. One girl tells us:

> "I like maths this year more than at primary school. We work more as a team rather than individuals".

A boy says:

> “At primary school, the lessons usually involved a couple of minutes of explanations, then lots of questions”.

And Teresa says:

> “We are seeing some really positive results in Year 7, not only in their attainment and depth of understanding, but in their enjoyment of maths. A student survey we ran earlier in the year showed an enthusiasm for maths that we hadn’t seen before. Our students also seem much more confident in their maths – they can articulate their understanding, and are willing to share ideas in class and make mistakes because they want to understand. Even for the lower prior-attaining students, we make sure that every lesson starts with a review of the prior learning and pre-requisite knowledge so that the learning is accessible to all from the very start of the lesson, and then this foundation is built upon in small steps throughout each lesson ensuring that students are not left behind early on”.

Is there any discernible difference between teaching the top set and the third? Teresa says:

> "It has been really eye-opening to me to have two classes with different prior attainment and on different ‘flight paths’, but to be teaching them the same material. Students of all attainment levels can understand the same concepts, provided the starting point is accessible and they can progress in small-enough steps to keep up. I find students in both classes display a willingness to have a go, to explore ideas through discussion, and a confidence and enjoyment in their own learning.
>
> "However, in this lesson the top set group were quicker to articulate their thinking, and made the connection with the algebra much more quickly. It was interesting to see many in the top set used substitution of zero into the nth term to find the 0th term, whereas in Set 3 they subtracted from the 1st term, and only made the connection to the nth term afterwards through questioning. I was able to slot in another more challenging problem at the end with the top set, because of their speed.
>
> "Also, I think the higher prior attaining students are also much more likely to remember their learning, and as you descend the sets then retention of learning becomes much more of an issue. I think the issues around retention and making connections explains a lot about differences in attainment: you can have two students who both understand a concept really well, but one of them gets great marks in an exam due to good retention, and the other doesn’t".

We are currently recruiting for a new cohort of Mastery Specialists: more details and application form are here. We are also inviting schools interested in implementing teaching for mastery to join a teaching for mastery Work Group, collaborating with other schools and with the bespoke support of a local Mastery Specialist - find out more.