Secondary Magazine
Issue 82

National Centre for Excellence in the Teaching of Mathematics

Contents

Contributors to this issue include: Laurinda Brown, Mary Pardoe, Richard Perring and Peter Ransom.

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for Excellence in the
Teaching of Mathematics

From the editor

Welcome to Issue 82 of the Secondary Magazine.

The Interview is with Laurinda Brown. In 2006, an NCETM grant was awarded to the University of Bristol's Graduate School of Education, in which Laurinda is Senior Lecturer in Mathematics Education. In the Final Report of the grant project, The Economy of Teaching Mathematics, of which Laurinda was the leader, we are reminded of the principle of 'spurious purposes' in teaching mathematics.

The principle is that if you want students to learn some mathematics you get them interested in doing something that is not ostensibly learning that mathematics – you get them committed with motivation to a spurious purpose – and, in engaging in this 'spurious' activity, they draw on and develop understanding of the mathematics that you want them to learn.

Many of the topics on which we focus in issues of this magazine can provide 'spurious purposes': the subject of the Focus on… in this issue, strategies that ensure that you will be the winner in games of chance played with non-transitive dice, is a good example. While students are busy being surprised, and trying to work out how to beat their opponents, they are necessarily developing and applying ideas involved in probability theory, such as the concept of equally likely outcomes.

In our focus on non-transitive dice we lean heavily on material presented appealingly to learners by Dr James Grime. In What is the point?, another recent video by Singingbanana, James draws attention, with passion, to an aspect of doing mathematics that also frequently features in thoughts and discussions about learning mathematics:

> *One of the students put up their hand and asked 'What is the point of that?' …*
>
> *You are mistaken if you think that mathematics is only for practical purposes. Mathematics is the intersection of many subjects, but particularly science, philosophy and art. The practical use of mathematics might be science and physics and mechanics, and things like that. But if you only study it for that purpose you are going to miss out on a lot of discoveries – discoveries that might find applications after the fact ...*
>
> *Studying mathematics for its own sake is equally as valid and equally as worthwhile even when there is no immediate practical purpose. Mathematics is a very creative subject …*
>
> *The pleasure comes from creating something – something new, something original, something that has never been done before…*
>
> *What we're searching for is mathematical truth, but to reduce it to practical purposes means that you're going to miss out on a lot of that truth, and a lot of that beauty, and a lot of that discovery.*

The Subject Leadership Diary provides links to some unusual classroom resources that ***are*** related (loosely) to 'practical purposes', such as space exploration, but which might also provide some excellent 'spurious purposes'!

To what extent in our teaching do we bear in mind the principle of 'spurious purposes'? And, if we do apply that principle, is the 'What is the point of that?' question as likely to be asked?

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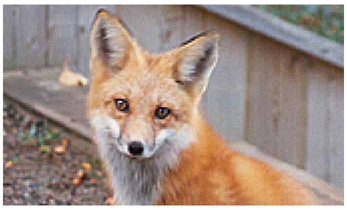


National Centre for Excellence in the Teaching of Mathematics

It’s in the News!
Fox scales Shard

The fortnightly *It’s in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but a framework which you can personalise to fit your classroom and your learners.

A fox has been found living off scraps of food left by builders at the top of the UK’s highest building!

When completed, The Shard, near London Bridge, will be 1 016 feet high. The fox cub, named Romeo by workers, is thought to have climbed to the peak of the building using the central stairway.

Fortunately, the story has a happy ending. Romeo was given an all-clear by the vet, a good feed, and was released back onto the streets of Bermondsey.

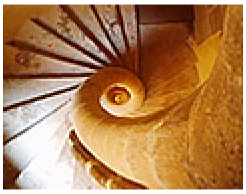
This resource uses a graphic from *The Sun* newspaper to illustrate this story as a context to explore proportion and scale drawing.

Download this *It's in the News!* resource - in PowerPoint format

Image Credits
Page header - Urban fox photograph by digitalprimate some rights reserved

The Interview

Name: Laurinda Brown



About you: I taught mathematics for 14 years before working as a curriculum developer and then as a university lecturer, primarily as a mathematics PGCE tutor. During this time I have developed my skills as a teacher and as a researcher, enjoying, particularly, editing journals. I edited Mathematics Teaching, a professional journal, for five years (1988-1992) and For the Learning of Mathematics, an international research journal, for four years (2003-2007). I have also recently completed a guest editorship of the Journal of Mathematics Teacher Education (October and December 2010, Volume 13, issues 5 and 6).

I like to work collaboratively and develop structures in anything I do so that there is a team approach. It seems to me that in my work with different groups of people, whether they are mathematics educators from across Europe, PGCE student mathematics teachers, or students in a classroom learning mathematics, these structures are similar – as I subordinate my teaching skills in the support of learning. My passion, however, is the learning and teaching of mathematics, and I continue to enjoy doing mathematics.

The most recent use of mathematics in your job is...
I am currently grappling with the uses of modelling in financial systems that led to the recent collapse, and it entails, at the moment, using mathematics as a learner, getting to grips with someone else's mathematics. But I hope to transform all this into questions and structures to support teachers in enabling their students to explore and use effectively the models and different interpretations that do not begin with the assumption of growth.

Why mathematics?
I came top of the class all the way up school. Mathematics was a subject that I did not need to revise before examinations. My being able to do it was not linked to memorisation. The crunch time was when I was choosing my A-level subjects. I wanted to do double mathematics and English for my three academic subjects, but Further Mathematics had, in my northern girls' grammar school, always been against English on the option lines, and there was not about to be a change. I was aware in a vague way that I would always read but that if I stopped doing mathematics that would be it. My inspirational English teacher, whilst disappointed, gave her sort of blessing with, *'Well, if you're going to be a mathematician, be a literate mathematician'*, and I suppose that is what I've become, enjoying, during my career, editing professional and academic journals related to mathematics education. From this A-level combination, I was on a path to read mathematics at university – all three of us in that Further Mathematics group did.

Some mathematics that amazed you is...
Before A-level, mathematics was mostly exercising routines. I felt that I was not understanding what was going on, and then sometime later something would click and the new routines became connected, fitting in with the mathematics I had already mastered. I became aware that the other girls in my group did not see connections, and quickly forgot any routine they were presented with, often not being prepared to do what seemed to them to be meaningless repetition. There were times in my mathematics lessons when I had finished and others had hardly started. My mathematics teacher at the time gave me a book called Flatland and started a continuing journey through, and fascination with, dimensions – that continues today. My motivation to invent the first time-machine was let go of at some stage, but I continue to be absorbed by many-dimensional spaces including fractal dimension!

A significant mathematics-related incident in your life was…

Watching the first episode of Doctor Who, attracted by the music, getting to grips with Time and Relative Dimensions in Space, the TARDIS and wanting to invent the first real time machine. I still read science fiction, anything with time in the title, and also philosophical and scientific tracts – but I am not now actively pursuing my childhood ambition.

Who inspired you?

There has to be room here for a few people. I am from a northern working class background and, reflecting back, I must have posed quite a challenge for my primary teachers. From an early age I loved reading, having been taught to read before going to school, by my sister. I could be disruptive to others in class because of finishing quickly and not being allowed to do anything else. In my last year of primary school, I had a teacher called **Mr Bates**, and learning was different, far more challenging. He told my mother that I got bored easily and I know he worried about whether I would cope in the grammar school with a formal teaching style. He began setting me different tasks, such as write down all the words beginning with 'a'. Initially I remember not engaging, writing down a few random words, but he came past me and looked disappointed. Did I want to please him? Was I fired by the realisation that I was missing something? I started to write down the dictionary!! – and when I looked at the dictionary sometime later, was fascinated with words starting with double 'a', (aardvark, …) – I did not have any double 'a' letters in my list!

This sort of logical organisation leads directly in my mind to going to sleep seeing how far I could get doubling numbers, being fascinated by when this is easy and when it is hard, or saying the alphabet backwards, literally. He had inspired through the 'dictionary' task a thirst for infinite order under my own control.

Dr Wilkinson taught me English from year 9 to year 11 and it was difficult to turn away from being taught by her. As head of the sixth form, she gave us all a reading list so comprehensive that I still refer to it today. The list came with the injunction, *'read the first four books on the list and never lament your lot again'*. What would be the four books you would put on such a list? Dr Edith Bone, Seven Years Solitary; Alan Burgess's book about Gladys Aylward, The Small Woman; To Sir with Love by E R Braithwaite, and the Diary of Anne Frank. She also invited us to read a book by an author and, if we liked it, obviously go on and read the rest of their books. If we did not like it, then it might be the only book of that author that we did not like so we needed to read more.

I did my PGCE year with Dick Tahta, learning to teach mathematics. Given the reactions of my classmates at school to the subject, I knew that there had to be a different way to teach mathematics that was more accessible to more people – and in the first session of the course we were invited to work at a problem called 'frogs'. There were lots of resources available and the mathematics was visual and tangible. We used pegboards and colours, getting up close to the structure of the problem, extending our ideas and discussing the many different ways of approaching the problem. I knew immediately that here was a place where I would be able to work at teaching mathematics in a different way. Dick became a life-long friend and through him I eventually engaged with the work of Caleb Gattegno supporting my work in subordinating teaching to learning.

The best book you have ever read is…

Not possible for me to say this – for the reasons above (there might be four) – and that I've spent a lifetime reading. I like books that make me see differently – so I would recommend Coxeter's Regular Polytopes as a follow up to *Flatland* if I am thinking books on mathematics.

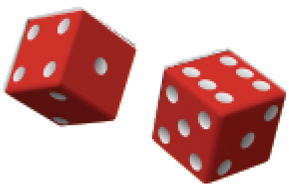
If you weren't doing this job you would…

Well, that might happen sooner than we think. I can't imagine a job that I would enjoy doing more. Challenging and being challenged by supporting the next generation of mathematics teachers is a great job to which I'm well suited. However, that job might not exist soon and I will become more the academic, perhaps, or simply retire…. In my retirement I will continue to sing in an *a cappella* choir, and spend a lot more time playing my piano.

Focus on…non-transitive dice

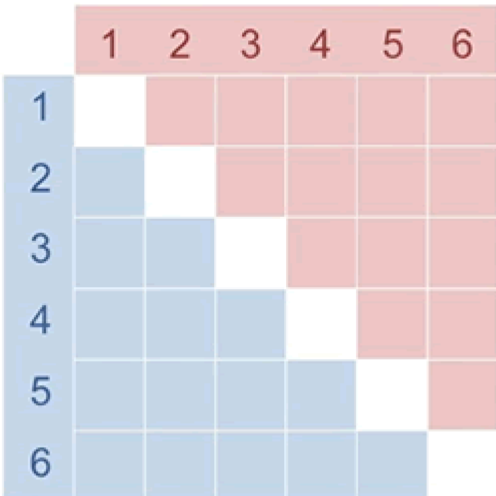
Many students enjoy exploring interesting phenomena that can occur when two people each throw a die and the 'winner' is the person with the highest score.

If the dice are normal, unbiased, dice…



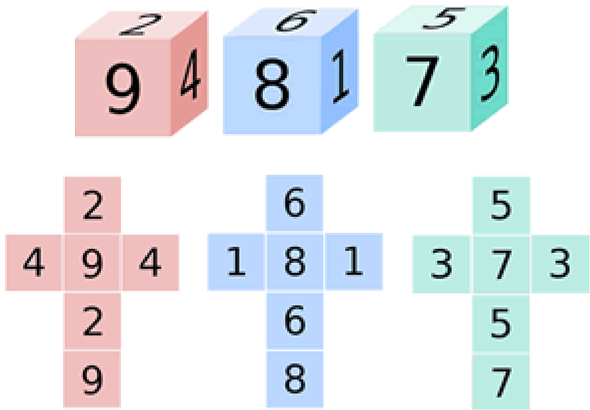
...the game is fair.

Each player is as likely to win as the other player…



…so that, as the competition is repeated more and more times, the accumulated result approaches a draw.

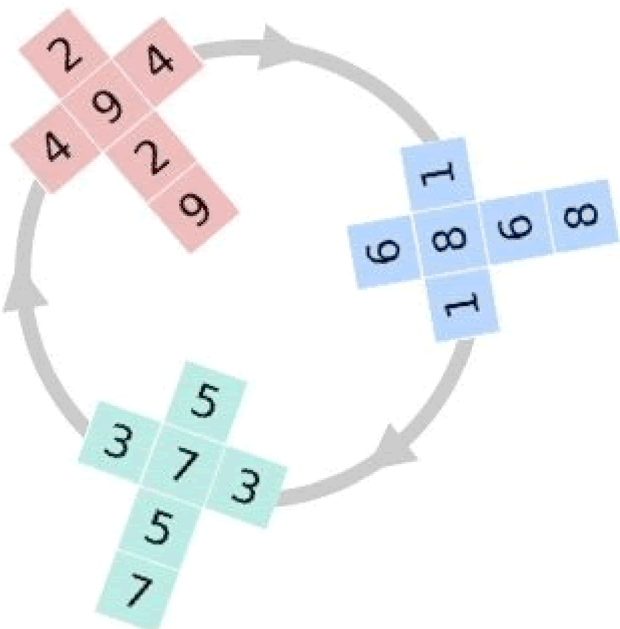
But, what if the dice are unbiased, but not normal? For example, what happens if the two players each choose a die from this set of three dice?



Students could make this set of dice, and play against each other in pairs – with a game consisting of, say, 20 throws, and the final winner being the player who has 'clocked up' the highest number of wins by the end of the game. Can students discover a winning strategy?

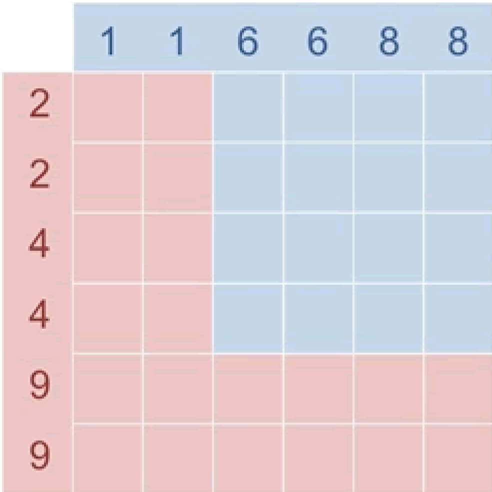
Eventually they may discover that the second player to choose a die can ***always win*** by adopting this very simple strategy:

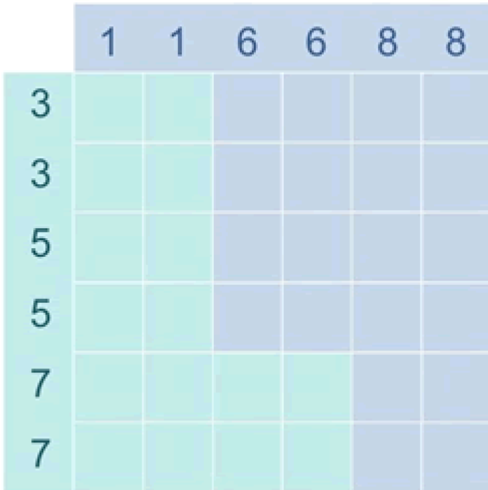
– pick the die that immediately precedes, in this cycle, the die chosen by your opponent…



You could challenge your students to try to shed light on ***why*** this is a winning strategy!

They may 'see' that, as shown in these diagrams, …



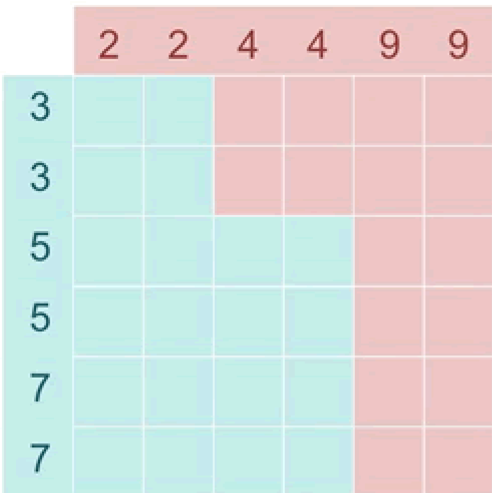


… in the long run …

> a player throwing the red die will win a game against a player throwing the blue die because, when both dice are thrown, in more than half (in 20 out of 36) of the equally likely possible outcomes, the score on the red die is greater than the score on the blue die,
>
> and
>
> for exactly similar reasons, a player throwing the blue die will win a game against a player throwing the green die.

So red beats blue, and blue beats green. Students might, therefore, expect that red would beat green – as it would if 'beats' was a *transitive* relation!

But, as this diagram shows, …



… in fact green beats red!

So, with this set of dice, 'beats' is NOT a transitive relation. ***R beats B*** and ***B beats G*** does NOT imply that ***R beats G***. Instead, ***R beats B***, ***B beats G*** and ***G beats R***. So these three dice together form a set of *non-transitive* dice.

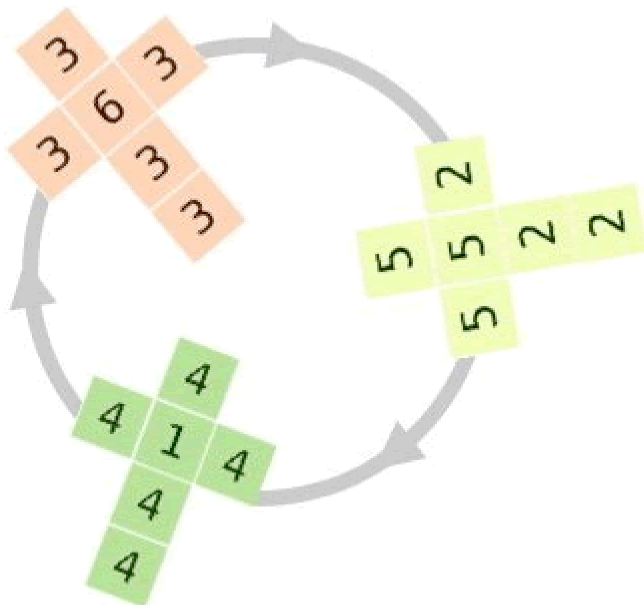
This particular set of non-transitive dice has another unusual property. With any pair of dice that are adjacent in the 'priority' cycle, the probability of the player with the 'best' die winning in one 'go' is the same – it is 5/9 for all three pairs.

Another surprising phenomenon

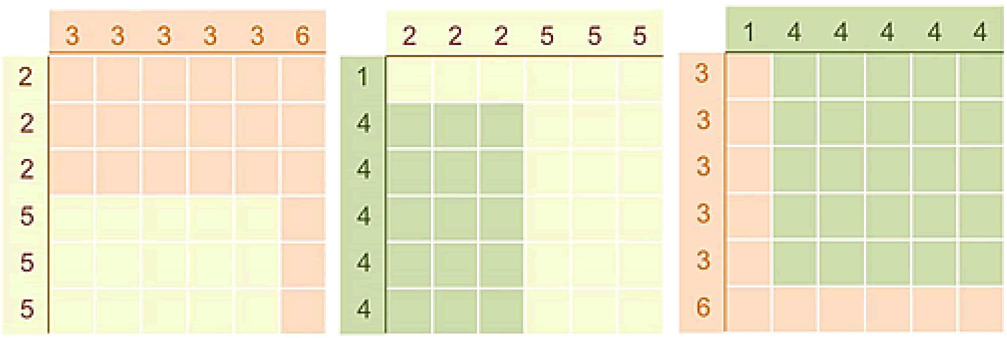
What do students think will happen if, as before, each player chooses a die from a set of non-transitive dice, but throws TWO of that die, instead of just one, thus scoring the total shown on the two dice?

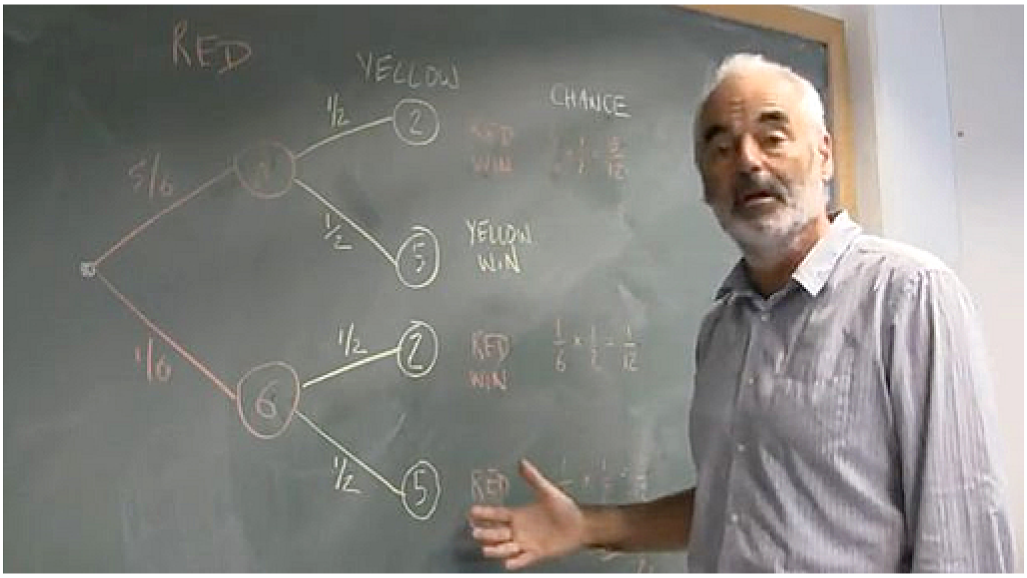
James Grime and David Spiegelhalter demonstrate and explain in this engaging and illuminating article and video a further extraordinary phenomenon that occurs with some sets of non-transitive dice when the 'which die beats which die' game is played in this way.

If challenged to confirm that, when two players each threw just one of the non-transitive dice used in the first part of the video, this was the cycle of priority …



… students might have sketched diagrams such as these …





In order to try to understand why the cycle REVERSES when each player throws two dice instead of one die, students might now show themselves all the equally likely outcomes for each ‘go’ in this new version of the game by creating images such as …

	3	3	3	3	3	6
3	6	6	6	6	6	9
3	6	6	6	6	6	9
3	6	6	6	6	6	9
3	6	6	6	6	6	9
3	6	6	6	6	6	9
6	9	9	9	9	9	12

	2	2	2	5	5	5
2	4	4	4	7	7	7
2	4	4	4	7	7	7
2	4	4	4	7	7	7
5	7	7	7	10	10	10
5	7	7	7	10	10	10
5	7	7	7	10	10	10

	1	4	4	4	4	4
1	2	5	5	5	5	5
4	5	8	8	8	8	8
4	5	8	8	8	8	8
4	5	8	8	8	8	8
4	5	8	8	8	8	8
4	5	8	8	8	8	8

… and doing some calculations. For example, a student might reason that…

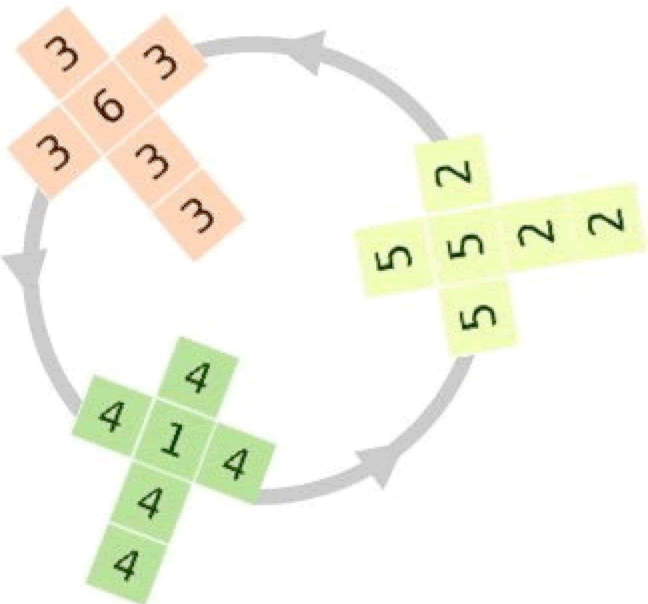
> *When two six-sided dice are thrown there are 6^2 (36) equally likely outcomes.*
> *So when two players each throw two six-sided dice there are 36^2 (1 296) equally likely outcomes.*
>
> *Orange can win against yellow in 9 x 36 + 18 x 11 + 9 x 1 = 324 + 198 + 9 = 531 ways.*
> *Yellow can win against orange in 18 x 25 + 9 x 35 = 450 + 315 = 765 ways.*
> *(531 + 765 = 1 296)*

Therefore yellow beats orange.

Yellow can win against green in 1 x 9 + 11 x 27 + 25 x 9 = 9 + 297 + 225 = 531 ways.
Green can win against yellow in 10 x 9 + 25 x 27 = 90 + 675 = 765 ways.
(531 + 765 = 1 296)
Therefore green beats yellow.

Green can win against orange in 25 x 25 = 625 ways.
Orange can win against green in 11 x 36 + 25 x 11 = 396 + 275 = 671 ways.
(625 + 765 = 1 296)
Therefore orange beats green.

So, when two players each throw TWO of these dice, orange beats green, green beats yellow and yellow beats orange – the cycle is reversed!

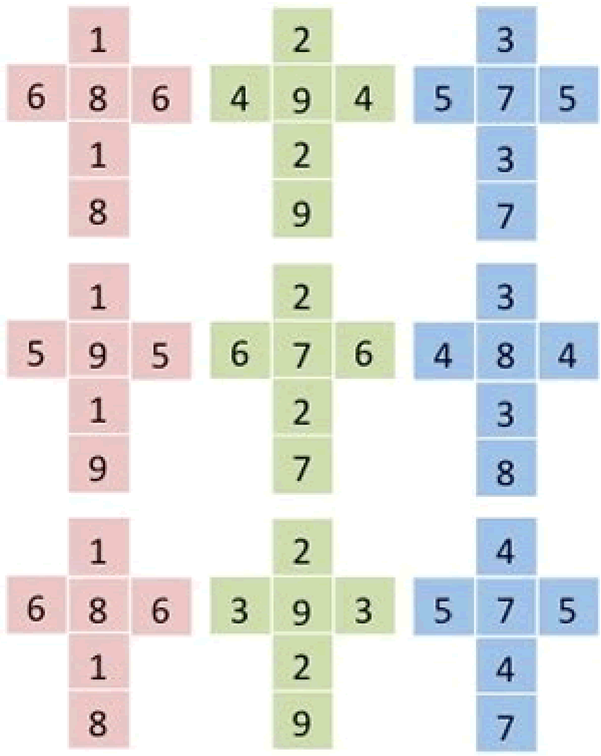


An introductory puzzle

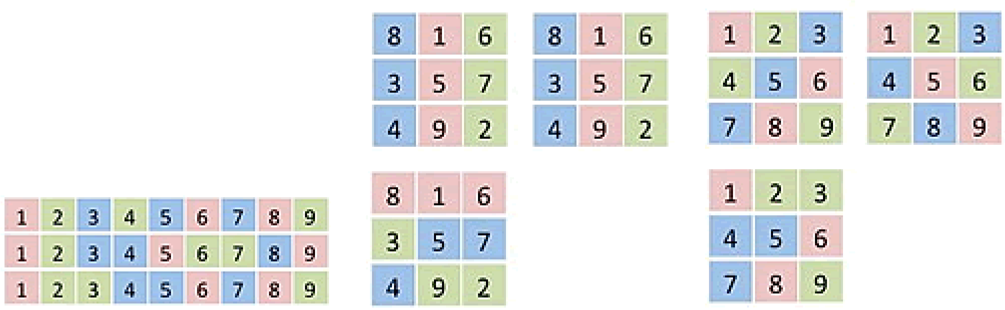
Before your students watch James Grimes' video, or read his article in *Plus* magazine, Curious Dice, you might challenge them to solve the 'easy' *Non-transitive Dice* puzzle presented by Gurmeet Singh Manku on the *Delightful Puzzles* page of his website.

When presenting the puzzle be careful not to reveal inadvertently the solutions that Gurmeet Manku provides! Challenge students to design a set of three dice so that, when using those dice, they can always win a dice-throwing game in which an opponent chooses one of the dice, then they choose another, and then several times they simultaneously each throw their die. They must design the dice so that whichever die their opponent chooses they can pick another die that will give them, on average, a higher score than their opponent. But there is a further condition which simplifies the task by eliminating all but a few possibilities – each die must have faces that show three different whole numbers between 1 and 9, with pairs of opposite faces being identical.

Three solutions (at one of which we have already looked) are given …



The arrangements of numbers on the dice display some intriguing patterns!
These diagrams show how in each set the numbers from 1 to 9 are distributed on the different dice …



The number-squares with 816 in the top row are magic squares!

Students could explore number patterns associated with other sets of non-transitive dice.

Two excellent puzzles from NRICH, which, alternatively or additionally, you might pose in order to get students started, are A Dicey Paradox and Dicey Dice.

Many sets of non-transitive dice

Although many sets of various numbers of non-transitive dice, which you and your students may enjoy creating, using and investigating, have already been discovered, it is possible that your students might find some sets that are as yet unknown – what a great challenge is that!

Some more of the many sources of information and inspiration for anyone taking up that challenge are suggested below.

In his article Dr James Grime reminds us that, as long ago as December 1970, Martin Gardner wrote about The Paradox of the Nontransitive Dice and the Elusive Principle of Indifference in his Mathematical Games column of Scientific American. You could read Nontransitive Dice and Other Paradoxes, which is Chapter 22 in The Colossal Book of Mathematics by Martin Gardner.

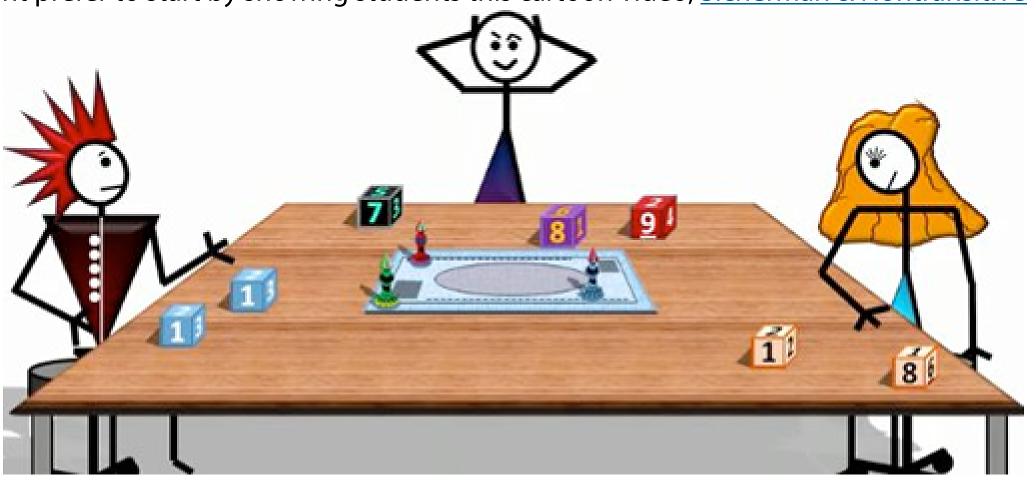
At Ivars Peterson's Math Trek the Tricky Dice and Tricky Dice Revisited pages are useful, as is this article at NRICH by Toni Beardon.

Bradley Efron, who is Max H. Stein Professor of Statistics and Biostatistics at Stanford University, is recognised as having been the first person to appreciate the special properties of non-transitive dice. The set of four dice with which the exploration started are called Efron dice.

Dr James Grime, in his articles already cited, discusses the non-transitive dice discovered by Oskar van Deventer, with which one player can beat two opponents in three-player games. Students could make a set of the seven Oskar dice.

A third possible way of introducing students to non-transitive dice – an alternative to approaching the ideas via the Singingbanana video or by posing a puzzle – is through the well-known game Rock, Paper, Scissors. Simon Singh takes this approach in this article.

Or you might prefer to start by showing students this cartoon video, Sicherman & Nontransitive Dice!



If you are teaching adults you might consider using this 13-minute video Use Non-Transitive Dice to cheat your friends!, but beware – although this film may draw students to the mathematics, considerable quantities of alcohol are in evidence!

Image Credits
Screenshots from the the Singing Banana Youtube video, used with permission; Red dice by Stephen Silver, in the public domain

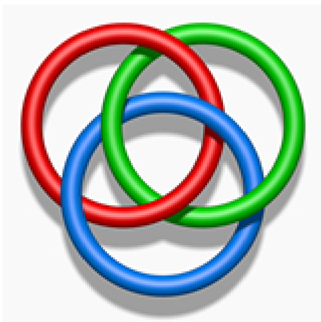
5 things to do this fortnight

- Professor Marcus du Sautoy is focusing on our cities: *"The cities we live in are bubbling with mathematics that has helped to shape the urban environment. Over the next few months I'm going to be working with a group of mathematicians here in Oxford to devise some walking tours of the city to reveal these hidden mathematical stories."* If you or your students accept Marcus's invitation to tell a story about maths in your city you will become part of a virtual mathscape of cities around the world – and you might win a prize! To find out more visit the Maths in the City website.

- To celebrate the **2012 Olympic and Paralympic Games**, the Millennium Mathematics Project, at Maths and Sport: Countdown to the Games is offering a wide audience – from school students and their teachers to members of the public – special **free online mathematical articles and activities**. New content will be published throughout the run-up to the Games.

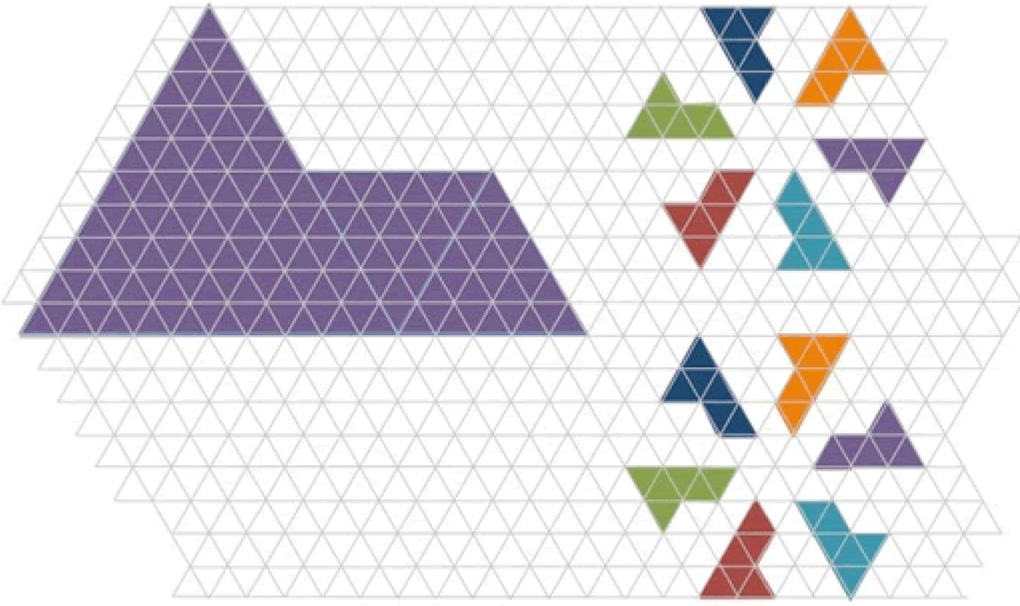
- The first European Girls' Mathematical Olympiad will take place at Murray Edwards College, Cambridge, in April 2012. Competitors will try to solve eight very hard problems during nine hours of examinations over two days! The organisers intend that this competition will become an annual event, moving around Europe every year. A **nationwide talent search** to find the girls to form the UK team at EGMO 2012 will begin with a *Competition Challenge* in June 2011, by means of which students will be selected to train for the competition.

- Professor Kokichi Sugihara from the Meiji Institute near Tokyo has made a career of creating optical illusions. You and your students may enjoy seeing some of them in this video. The focus was on illusions in Issue 74 of this magazine, which was published at about the same time as Visual curiosities and mathematical paradoxes in *Plus* magazine.



Borromean rings by Jim Belk

- Have you explored the Mathematics Centre that is managed by Black Douglas? This world of **alternatives to text-based learning** is intended to support you in enabling your students to learn to work like mathematicians. You will find many unusual tasks and ideas for lessons – brought richly to life with beautiful photographs and fascinating writing by teachers and students.



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Borromean rings image by Jim Belk, in the public domain

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Issue 82

National Centre for Excellence in the Teaching of Mathematics

Subject Leadership Diary

Easter is in sight now and that is always a time for some interesting cross-curricular mathematics with science and religious studies. Easter is a moveable feast, meaning it is not fixed in relation to the civil calendar. In AD 325 the date of Easter was determined as the first Sunday after the full moon following the vernal equinox. Now, we can do some astronomical links with the phases of the moon and the fact that at the equinoxes there are equal hours of light and dark, since that is when the sun crosses the equatorial plane. There is a great opportunity here for some 3D geometrical work in visualising the Earth and its place in our solar system. The date of Easter varies between 22 March and 25 April – we can look at these dates and do some statistical work (if we so wish). It is interesting to note that in 2008 we had a very early date for Easter (March 23) and this year we have a very late date (April 24). How likely is it that if you didn't live in this period (2008-2011) you would experience Easter dates that are over a month apart?

Now why this sudden interest in astronomy I hear you ask? In the last diary there was mention of searching for exoplanets (planets outside our solar system) and now I'm working with the moon! Well, it all started with the Texas Instruments International Conference, which I attended over half-term in San Antonio. I was fortunate to be invited to give a workshop and exhibit some work I have done in school on various topics, and NASA had a presence at the conference. I attended one of their sessions and was amazed at what they have available to do in the mathematics classroom. There is a big move in the USA to incorporate space exploration into math and science, and NASA have free downloadable materials for both teachers (educators) and pupils (students). We owe it to our students to provide challenging materials to keep them focussed on their studies and to provide insight into science, technology, engineering and mathematics (STEM). For example, to link in some biology I searched the NASA site on the word 'bones' and a variety of materials emerged which includes video clips and worksheets – such as Next Generation Spacecraft – Orion which deals with decomposing figures and calculating areas, and Space Shuttle Ascent, which involves linear and quadratic equations. There is a universe of useful material 'to boldly go' and explore!

Of course, the conference did not consist of just NASA sessions. I learned about the UK's Bloodhound SSC project – the Project Director, Richard Noble, writes '*We are pushing the limits and inspiring our young engineers and scientists with our incredible car capable of 1 000 mph*'. Take a look, and then inspire your students! This is just the thing to energise everyone near the end of a long term.

The number of students at our after-school revision sessions has started to increase slightly now the examination season is approaching. For GCSE, we work with the same topics at both Foundation and Higher levels each week in two separate classes. It is interesting that the number of males is increasing faster than that of females, but there are still more females who attend. This pattern seems to repeat itself every year. The males seem confident on the outside, perhaps because they focus on what they do know (which is not always very much), whereas the females seem worried on the outside because they focus on what they do not know (which, once again, is not always very much)! I am forever grateful to my faculty who give their time so willingly to help out at these voluntary sessions.

We are gearing up to our annual Easter Egg challenge. In Key Stage 3 we use one lesson and homework in which students design a box for a creme egg. Students get to measure the samples we have and they can work in pairs or on their own to produce an attractive box – they have to consider things from the producer's point of view as well as the consumer's! This links in well with some 3D work on solids and surface area. A second lesson is used to judge the entries and award the prizes – a nice activity that combines mathematics and art.

I hope you all have an excellent break over Easter and that you don't eat too much chocolate!