Additive Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

Question 10

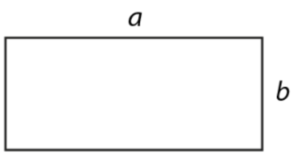
How confident are you that you understand and can support children to develop strategies for solving problems with two unknowns?

1 ☐ 2 ☐ 3 ☐ 4 ☑

How would you respond …?

a. How many solutions can you find to this question? How could you check you have found all the solutions?

'A rectangle with sides a and b has a perimeter of 30 cm. a is a two-digit whole number and b is a one-digit whole number. What are the possible values of a and b?'



b. Year 6 have earnt 200 stars; the stars are either gold or silver. They have 30 more gold stars than silver. How many are gold?

What error in approach has the child taken in answering the problem? How could a bar model be used to expose the structure?

$\frac{1}{2}$ of 200 = 100

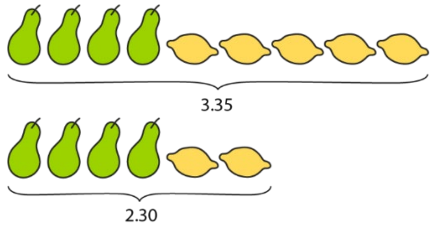
100 + 30 = 130

130 stars are gold

c. Four pears and five lemons cost £3.35, and four pears and two lemons cost £2.30. Find the cost of one lemon.

What approaches could be used here?

Use a representation to discuss the steps children would need to take to solve this problem.



Responses

Note your responses to the questions here before you engage with the rest of this section:

Did you notice that…?

a. Before starting to find the solution, encourage children to draw a diagram to deduce that a + b = 15 cm. Then, work systematically to ensure all possible solutions are found. For example, beginning with 'a' as the smallest possible two-digit number (10).

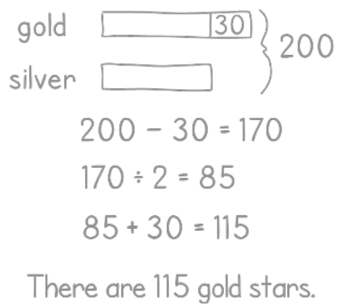
You may wish to ask children to reason why a = 15 and b = 0 isn't a solution. These values satisfy the conditions that a + b = 15, 'a' is a two-digit number and 'b' is a one-digit number, but they don't make sense in the context of the problem (because the rectangle would become a line). Some children might incorrectly include this solution if they get 'carried away' filling in the table, forgetting the context of the problem, which imposes its own implicit criterion (namely, that both unknowns must be greater than zero).

a (cm) (two-digit)	b (cm) (one-digit)	a + b (cm)
10	5	15
11	4	15
12	3	15
13	2	15
14	1	15

b. In part b, the child has split the 200 into two parts: the number of gold stars and the number of silver stars. That gives them 100 of each. Then they've added on 30, because we know that there are 30 more gold stars; this has given an incorrect answer.

Child A has taken a common, but incorrect, approach. Adding 30 on to the 100 to give 130 gold stars, means there are only 70 silver stars left (out of the 200), so the difference between the number of gold and silver stars is now 60, rather than the desired 30.

This is one way of representing the problem with bars:



c. When deciding on a representation, some children may choose to draw two separate diagrams – one for each piece of information given – while others may draw one diagram combining both pieces of information. Based on their previous experience, some children may choose to draw bars, whilst others may choose to draw circles (or similar shapes) as they think of the individual pieces of fruit.

Whatever the type of diagram, the key feature is a layout that clearly exposes the difference between the two scenarios since we are looking to relate one piece of information to the other to find a solution.

In this example, the diagram must clearly expose that the difference between the two purchasing situations is the cost of three lemons.

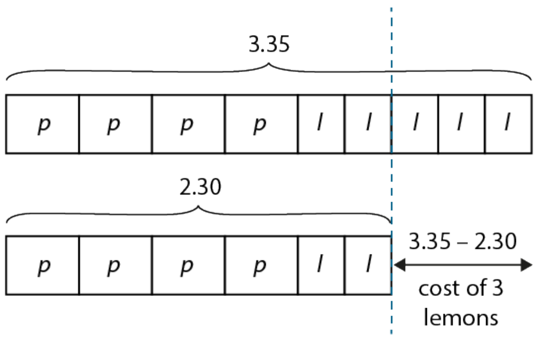
To draw attention to this, look again at the two pieces of information provided and ask children:

- *'What's the same?' (the number of pears)*
- *'What's different?' (the number of lemons and the total cost)*

The cost of 3 lemons = 3.35 – 2.30 = 1.05

So, the cost of 1 lemon = 1.05 ÷ 3 = 0.35

'One lemon costs 35 p.'



Problems with two unknowns

This section will explore problems with two unknowns, considering how bar model representations can be used to expose the structure of the problem. There may be one solution or several.

When children are working on problems that have more than one solution, it is important to focus on how they work rather than the solution. Encourage them to:

- ask what is the same and what is different about the information given
- work in small, logical steps
- look for connections and patterns within the solutions
- check their solutions
- reason about whether they have found all possible solutions.

Before children work on problems with two unknowns, they will already have had experience understanding different structures and should be able to discuss these without reference to specific numerical values. Time should be given to explore the structures of problems and the appropriate representations of these, such as with Cuisenaire® rods, before applying this understanding to drawings to expose the structure.

Opportunities to discuss different strategies and approaches allow children to consider ways to approach problem solving. The example below shows five different ways that a problem may be approached, with the emphasis being on analysing approaches, as opposed to children finding a solution to the problem. Some of these are incomplete and some have errors, allowing children to carefully examine and compare them, which is invaluable to developing their reasoning and understanding.

Year 6 have earnt 200 stars; the stars are either gold or silver. They have 30 more gold stars than silver. How many are gold?'

$\frac{1}{2}$ of 200 = 100

100 + 30 = 130

130 stars are gold

g = 130, s = 70 x

g = 100, s = 100 x

g = 105, s = 95 x

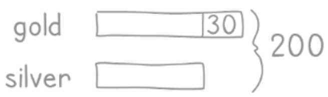
g = 110, s = 90 x

g = 111, s = 89 x

g = 120, s = 75 x

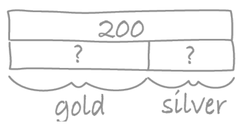
gold	silver	difference
150	50	100
140	60	80
130	70	60
120	80	40
110	90	20
115	85	30

115 stars are gold



200 − 30 = 170

170 ÷ 2 = 85



You don't have enough information to solve it

The aim of this type of activity is for children to reflect on their choice of strategy and consider why approaches may have resulted in incorrect solutions. Also, to consider that even when solutions are incorrect, some steps may have been correct.

Have a look at the examples. Consider the strategy each child has used and why it may or may not have resulted in the correct final solution. Are you able to use a bar model to represent the problem?

Bar models are powerful for solving problems with two unknowns, but it is important that children do not see them as a standalone entity, rather something that can be used alongside other diagrams or drawings. For some mathematical problems, it is very useful to draw a model; for others, it isn't relevant and strategies such as trial and improvement or reasoning may be a better approach.

I spent £29.90 on fish and chips. One fish cost £3.20 and one portion of chips cost £1.50. How many portions of each did I buy?

This problem is not suited to a bar model but would lend itself to a trial and improvement approach. Encourage children to work systematically to solve this problem, making a list of multiples of £3.20 and multiples of £1.50, before looking for a combination of multiples that make a total of £29.90.

Number of portions	Fish	Chips
1	£3.20	£1.50
2	£6.40	£3.00
3	£9.60	£4.50
4	£12.80	£6.00
5	£16.00	£7.50
6	£19.20	£9.00
7	£22.40	£10.50
8	£25.60	£12.00
9	£28.80	£13.50
10	£32.00	£15.00

Children need to think about how many multiples they might need in their table; they don't need to go over £29.90 in either column. When the have found nine or ten multiples of £1.50, they should check if they can find a solution before working out more multiples. Once children have found one solution (seven fish and five portions of chips), ask them if there are any more solutions and how they might check (by completing the table up to £29.90 in the 'chips' column).

Children will need to apply their systematic approach to problems that involve more than one solution, to ensure they find all possibilities. In the practice question, there are several different solutions for the perimeter of the rectangle.

Common errors in this area may include:

- children not knowing where to begin or feeling they do not have enough mathematical information to complete the task
- children only finding the value of one of the unknowns
- not understanding the representation they are using as it is being used by rote
- being unable to find all the solutions as a systematic approach has not been used.

What to look for

Can a child:

- use a flexible approach when representing problems?
- discuss the structure of the problem?
- justify why different strategies are more suited to certain problems?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 1: Number, Addition and Subtraction

- Topic 1.31: Problems with two unknowns

Notes:
Key learning from support material and self-study:
What I will focus on developing in my classroom practice: