MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.2 Introducing 'whole' and 'parts': part–part–whole

Teacher guide | Year 1

Teaching point 1:

A 'whole' can be represented by one object; if some of the whole object is missing, it is not the 'whole'.

Teaching point 2:

A whole object can be split into two or more parts in many different ways. The parts might look different; each part will be smaller than the whole, and the parts can be combined to make the whole.

Teaching point 3:

A 'whole' can be represented by a group of discrete objects. If some of the objects in the group are missing, it is not the whole group – it is part of the whole group.

Teaching point 4:

A whole group of objects can be composed of two or more parts and this can be represented using a part–part–whole 'cherry' diagram. The group can be split in many different ways. The parts might look different; each part will be smaller than the whole group and the parts can be combined to make the whole group.

Overview of learning

In this segment children will:

- be introduced to the concept of 'whole' and develop an understanding of this word as used in mathematics to mean an entire single object (e.g. an apple) or a group of objects (e.g. five pencils)
- develop an understanding of the word 'part' as used in mathematics to mean a portion of a whole object or of a group of objects
- be introduced to the structure of part–part–whole relationships and explore these within real-world contexts
- explore the ways in which part–part–whole relationships can be represented using concrete apparatus, leading to pictorial and symbolic representations, which become increasingly abstract.

The purpose of this segment is to introduce the concepts of 'whole' and 'part'. Examples should be carefully chosen throughout to ensure variation, so that children are exposed to the concepts in a range of contexts. Through use of this variation, together with exposure to 'non-concept' examples ('not whole'), the children will gain a depth of understanding.

Teaching will begin with the context of a single object, before moving on to examples where the whole is made up of several discrete objects. Children will explore how a whole can be split into parts and how these parts can be combined again to make the whole. They will develop an understanding that a part cannot be greater than the whole.

The focus of this segment is not on calculation but, through exploration, on the development of an understanding of the part–part–whole structure. Children will gain experience of identifying whole groups, for example, *'I have five pencils'*, and identifying parts within a group, for example, *'One pencil is red and four pencils are blue.'* They will not generate abstract number sentences, but will explore different ways of representing their understanding. Number sentences and calculations using the part–part–whole structure will be introduced in segment *1.5 Additive structures: introduction to aggregation and partitioning*.

Children will become fluent in drawing part–part–whole cherry models to represent a variety of contexts. The alternative bar model representation of part–part–whole will be introduced in segment *1.3 Composition of numbers: 0–5*.

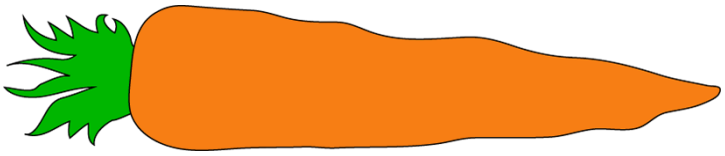
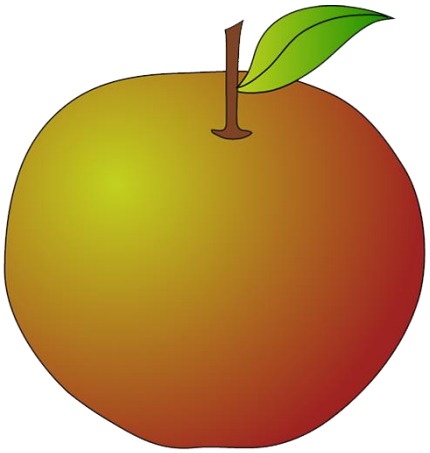
At this stage, whole groups should be restricted to ten items or fewer, ensuring variation in terms of the size of the whole and parts within this set. The larger the set, the more ways there are to partition it. Systematically partitioning numbers to ten in different ways will be explored in more detail in segments *1.3 Composition of numbers: 0–5* and *1.4 Composition of numbers: 6–10*. The focus here is for children to understand that quantities can be partitioned in a variety of ways, rather than on learning what those ways are.

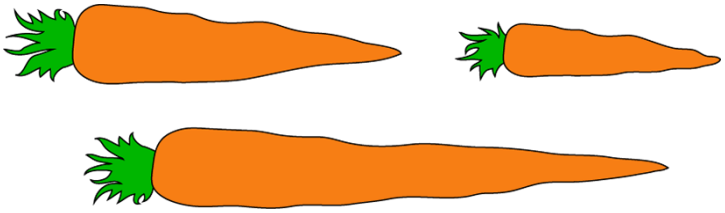
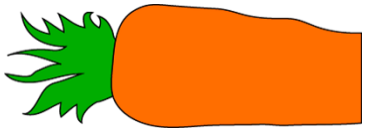
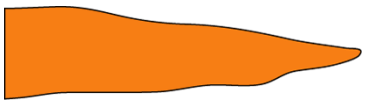
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

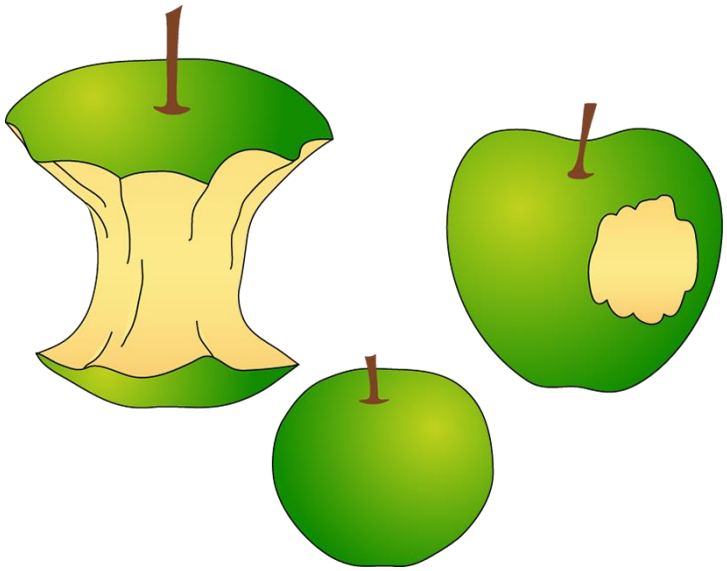
Teaching point 1:

A 'whole' can be represented by one object; if some of the whole object is missing, it is not the 'whole'.

Steps in learning

	Guidance	Representations
1:1	Note that 'whole' and 'hole' are homophones so ensure that children aren't thinking about a 'hole' when you are discussing a 'whole'. Begin to introduce the idea of a 'whole' using familiar objects. As a starting point, it is important to use wholes that will remain recognisable once split into parts – for example, avoid using a length of ribbon, as it can be more difficult for children to recognise it as a whole object, once it has been cut up. The objects you select now will be used in later steps, in which you will remove part of the whole – so for consistency in representations, consider using wholes here that can easily be cut or broken up. An appropriate object, at this stage, would be a whole carrot. Introduce the term 'whole', asking *'What makes this a whole?'* Children should be able to explain that 'whole' means that they have all of the carrot. Show a variety of whole objects, such as a whole apple or a whole banana. Make sure that you vary the orientation of each object to draw attention to what is important. Other useful questions you can ask to promote exploration and a depth of understanding include: • *'Is this a whole apple?'* • *'Why is it a whole apple?'* • *'Can you see anything else that is a whole in the classroom?'* The nature of the objects may lead to interesting discussions, which will	*'This is a whole carrot, because I have all of it.'*  *'Is the leaf part of the whole apple?'* *'What about the stalk?'* 

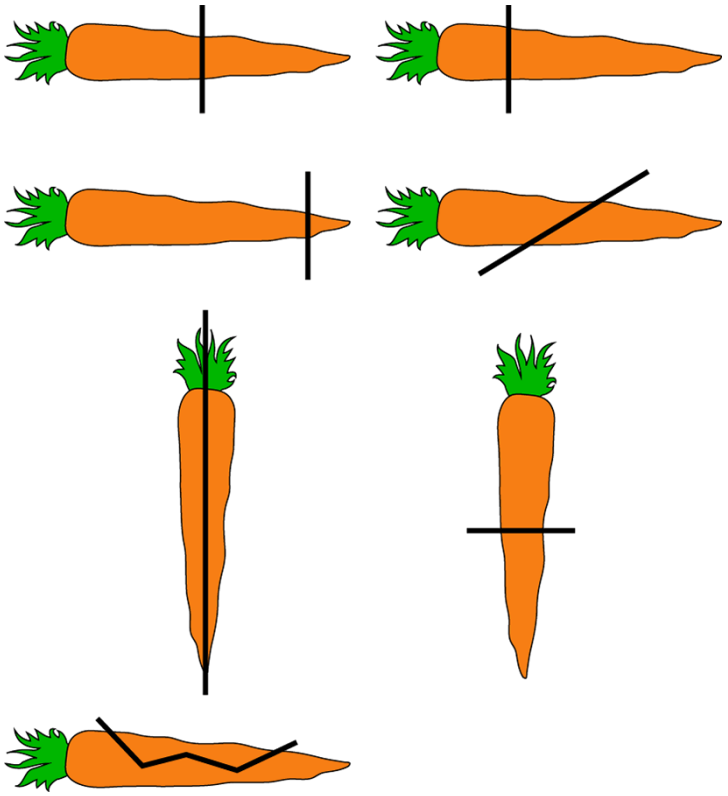
	develop children's reasoning skills. With an apple, for example, you might ask: • *'Is the leaf part of the whole apple?'* • *'What about the stalk?'* Build towards use of the stem sentence: ***'This is a whole ___, because I have all of it.'***	
1:2	Once children are able to identify whole objects, introduce the concept that wholes can be different sizes. Use different-sized examples of a given object to draw attention to the essential feature of a whole as being a complete object, regardless of its size. Ask questions such as: • *'What do you notice?'* • *'What's the same?'* • *'What's different?'* • *'Is the small carrot a whole carrot? How do we know?'* Include a range of familiar objects of different shapes and sizes.	• *'What do you notice?'* • *'What's the same?'* • *'What's different?'* • *'Is the small carrot a whole carrot? How do we know?'* 
1:3	To provide variation, now move on to the concept of 'not whole' (non-concept) by showing part of a whole object. This will help to clarify the essential feature of a whole: that it represents the entire object. For example, show half of a carrot, asking children *'Is this a whole carrot?'* and eliciting an explanation for their answer. Children should be able to explain that *'It is not a whole carrot because I don't have all of it.'* This leads to the stem sentence: ***'This is not a whole ___ because I don't have all of it.'*** An alternative stem, that begins to introduce the word 'part' in this context, would be: ***'This is not a whole ___ because I only have part of it.'***	• *'Is this a whole carrot?'* • *'Why / why not?'*  • *'Is this a whole carrot?'* • *'Why / why not?'* 

	To promote depth, ask children to identify: • 'not wholes' in a group of whole objects and 'not wholes' • 'wholes' in a group of whole objects and 'not wholes'. Continue to use a variety of shapes and sizes of given objects.	*'Which image shows the whole apple?'*  *'Does it matter that the whole apple is the smallest?'*

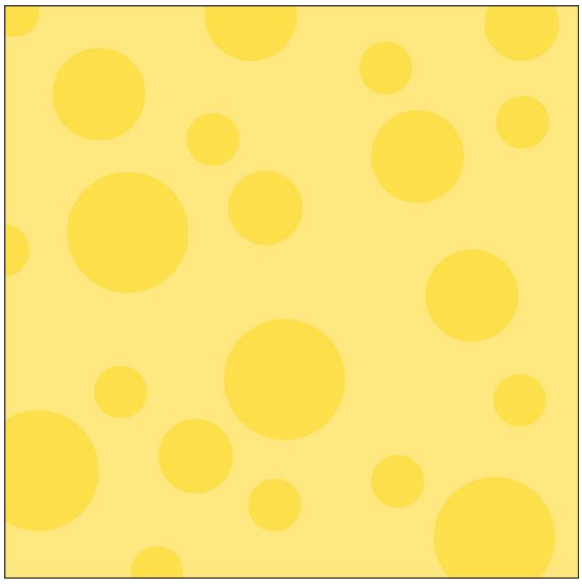
Teaching point 2:

A whole object can be split into two or more parts in many different ways. The parts might look different; each part will be smaller than the whole, and the parts can be combined to make the whole.

Steps in learning

	Guidance	Representations
2:1	Once children have mastered identifying and describing 'wholes' and 'not wholes', introduce the concept that a whole can be split into parts. For practical work, you could use bananas, since they are easy for the children to cut up themselves. Alternatively you could use pictures of a fruit/vegetable, which can be cut in different ways using scissors. Use tasks and questions such as these to develop the idea that parts can be different in size and shape, and to embed concepts of equality/inequality: • *'Can you cut your banana into two parts that don't look the same?'* • *'If you eat both parts, will you be eating the whole banana?'* (This begins to hint at the concept that the parts can be combined to make the whole.) • *'Can you cut your banana into two parts that are about the same size?'* • *'Can you cut your banana into two parts so that one part is a lot bigger than the other part?'* • *'How could you cut your banana to give you the biggest possible part? What do you notice about the other part?'* (This draws attention to the fact that if one part is large, the other part will be small.) • *'Can you cut your banana so that one part is bigger than the whole? Why not?'* (This draws attention to the important concept that no matter how large a part is, it cannot be larger than the whole.)	Dòng nǎo jīn: *'Which two pictures show the whole carrot being split in the same way?'* 

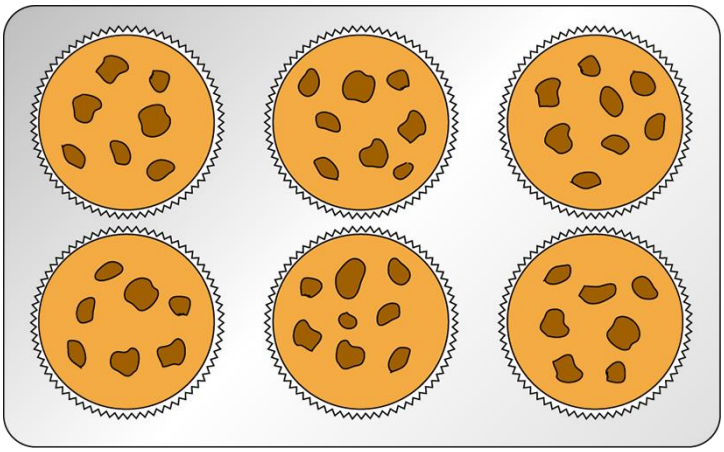
	Throughout this step, encourage children to use the following generalised statements: • ***'A whole can be split into two parts in lots of different ways.'*** • ***'A whole is always bigger than a part of the whole.'*** • ***'A part is always smaller than its whole.'*** To promote depth, use a dòng nǎo jīn question: show pictures of the same object in a variety of orientations, split into two parts in a variety of ways; ask children to identify a pair of images which show the whole being split in the same way.	
2:2	Once children are secure in their understanding that a whole can be split into two parts, explore splitting wholes into more than two parts, continuing to reinforce the key concepts covered in step *2:1*. You could begin by providing a banana and asking *'Can you cut your banana into more than two parts?'* Discuss the different ways that children respond to this task, asking: • *'Has anyone cut their banana into three parts?'* • *'How many parts do other children have?'* • *'What is the largest number of parts that anyone has split their banana into?'* Use the following question to reinforce the concept that the parts, no matter how many there are, will combine to make the whole: *'If you ate all of your parts, how much of the banana will you have eaten?'* Make children aware of the generalised statement:	

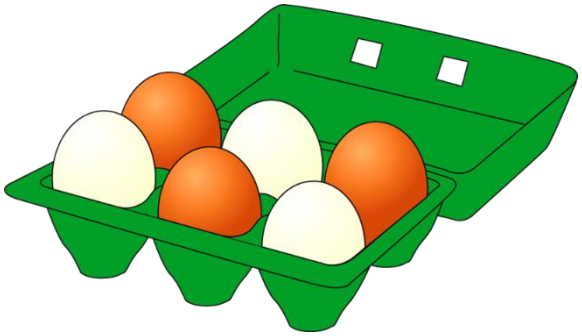
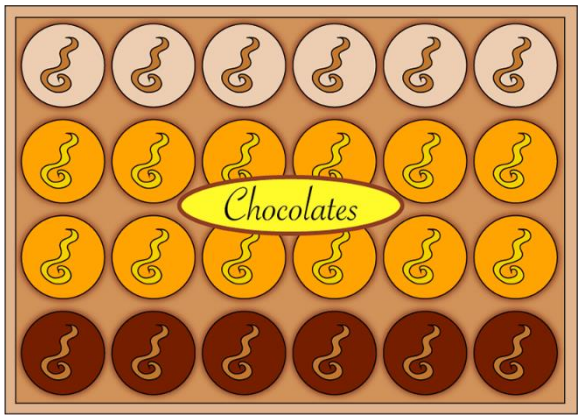
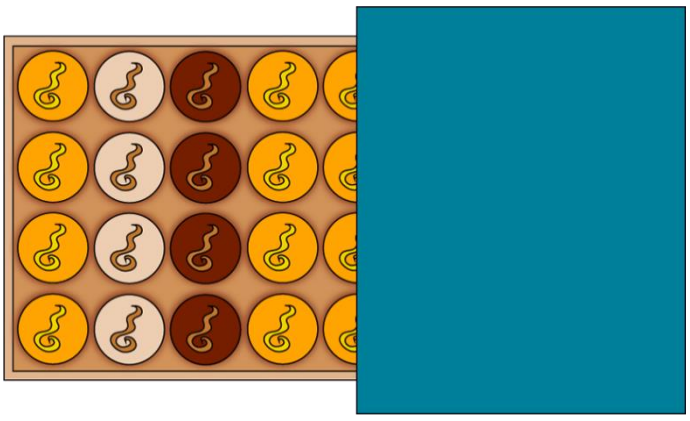
	'A whole can be split into more than two parts in lots of different ways.' Continue to include discussions about whether it matters that the parts are different sizes or shapes to secure children's understanding of the essential features of parts and wholes.	
2:3	Provide further challenge by setting a dòng nǎo jīn task of the form: *'Can you cut this picture of a slice of cheese into four parts so that one part is bigger than the other three? Can you do this in more than one way?'* Other similar tasks include cutting the slice into four parts such that: • one part is a lot bigger than the other three • there are two large parts and two small parts • the parts are about the same size but look different • the parts all look the same. Provide more than one picture per child, and always ask *'Can you do this in more than one way?'* By setting parameters in this way, children have to show understanding of the comparative vocabulary used. Questions with more than one solution, such as these, encourage children to reason mathematically and creatively, and provide opportunities for them to reflect upon what's the same and what's different.	*'Can you cut this picture of a slice of cheese into four parts so that one part is bigger than the other three? Can you do this in more than one way?'* 

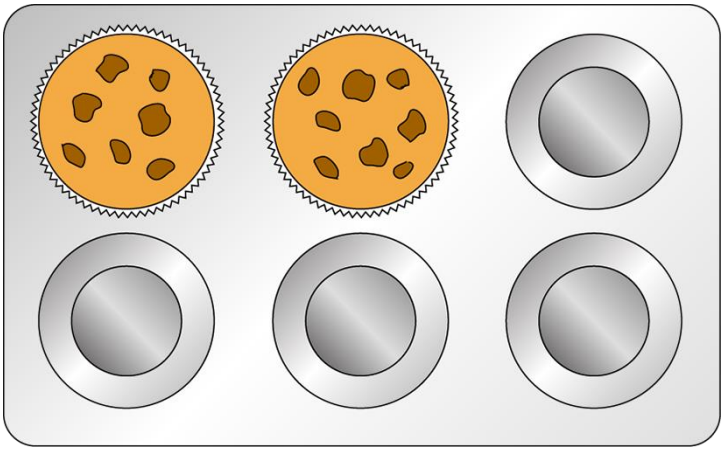
Teaching point 3:

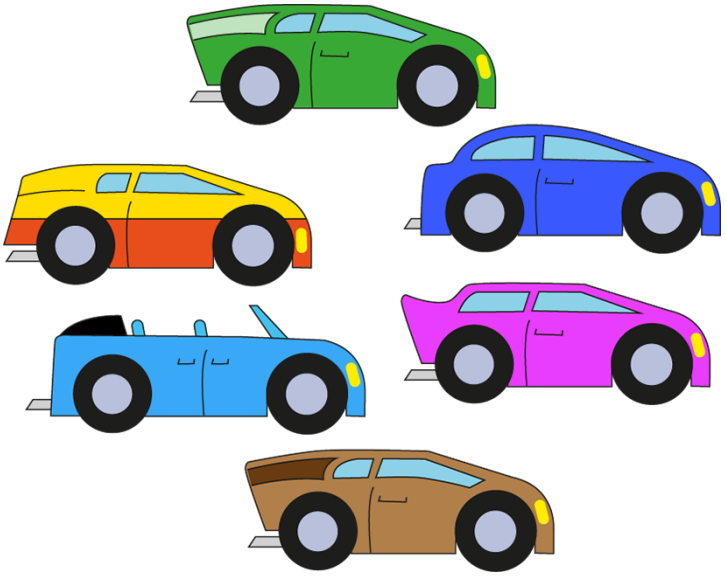
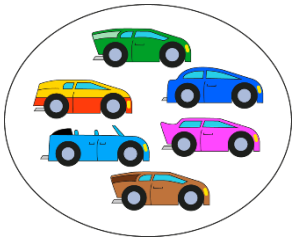
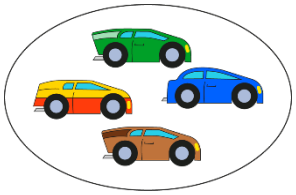
A 'whole' can be represented by a group of discrete objects. If some of the objects in the group are missing, it is not the whole group – it is part of the whole group.

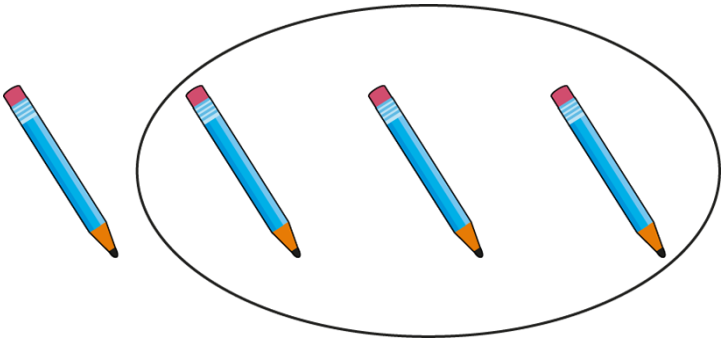
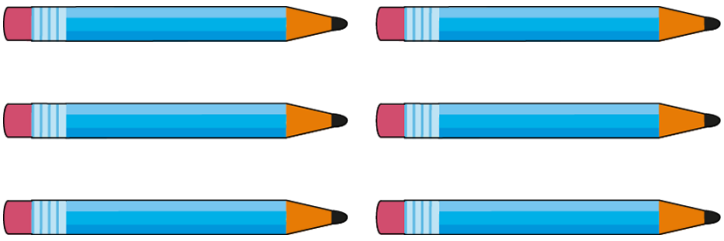
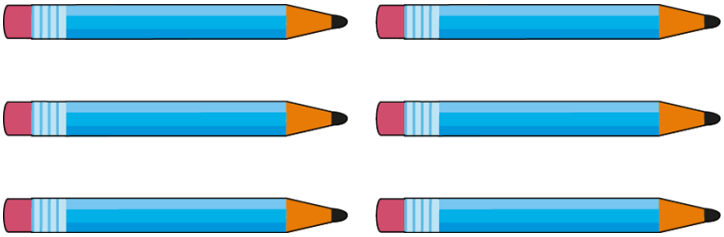
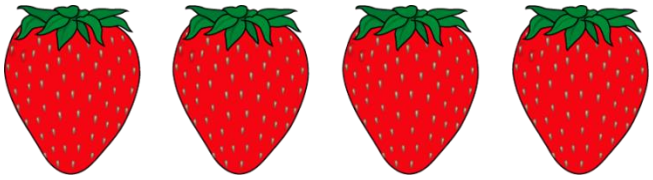
Steps in learning

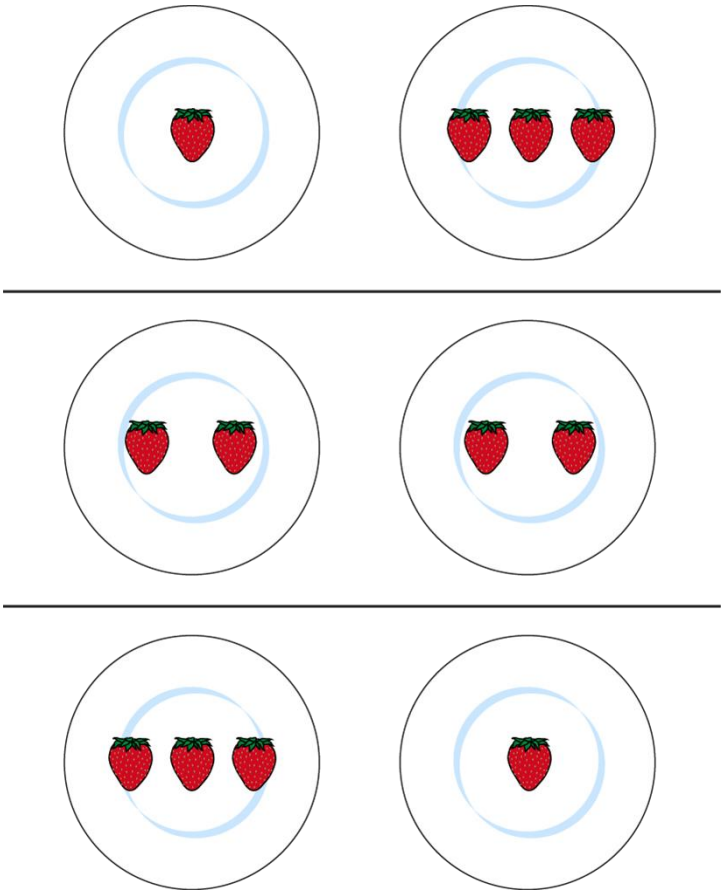
	Guidance	Representations
3:1	Begin this teaching point by using concrete representations to introduce the concept that the 'whole' can refer to a full group of discrete objects – for example, a full tray of six cakes**.** Children should be able to identify 'whole' and 'not a whole', for example, a 'whole tray of cakes' or 'not a whole tray of cakes'. At this stage you should use boxes/trays with spaces for the objects to fit in, so the children can easily see if any are missing. Show children the representation of the whole and ask *'How do we know this is the whole group of ___?'* Children should be able to apply what they learnt studying one object to explain, for example, that if the tray is full of cakes, they have the whole group of cakes. There are no cakes missing. Use the stem sentence: ***'This is a whole group of ___, because I have all of them; none are missing.'*** Model drawing an imaginary ring around the whole group so that the children can see that the whole is made up, for example, of all of the cakes in the tray. Ensure that you use a variety of contexts, and begin to introduce pictures of concrete objects as examples.	*'How do we know this is the whole tray of cakes?'*  *'The tray is full of cakes so we have the whole group of cakes; there are no cakes missing.'*
3:2	Now introduce an element of variation by showing whole groups containing non-identical objects – for example, different coloured eggs in a full tray of	

	eggs, or different coloured chocolates in a full box of chocolates. Ask questions to explore whether it is still a whole group if the objects are no longer identical. You can present a dòng nǎo jīn question by providing examples where part of the whole is obscured, and asking whether the group is still whole. Encourage children to discuss and explain their reasoning.	• *'What do you notice about this box of eggs?'* • *'Is it still a whole box of eggs? Convince me.'*  Dòng nǎo jīn: • *'Is this this a whole box of chocolates?'* • *'How do we know?'*  (In this image, we can deduce that the box is full because we can see the edges of the chocolates which are covered.) • *'Is this a whole box of chocolates?'* • *'How do we know?'* 

3:3	Once children can identify a group of objects as a 'whole', move on to the concept of 'not whole' (non-concept). Show part of a whole group of objects, and ask children whether it is the whole. Again, children should be able to explain their answer, for example, *'This is not the whole tray of cakes because we don't have all of them. Some of the cakes are missing.'* The children should apply their understanding of describing 'not wholes' in relation to one object (step *1:3*). Draw attention, for example, to the fact that part of the tray has cakes in and part of the tray is empty; this is not a whole tray of cakes because only part of the tray has cakes in. Use the stem sentence: '***This is not a whole group of ___ because we don't have all of them; some of them are missing.'*** An alternative stem, which uses the term 'part' is: ***'This is not a whole group of ___ because only part of the ___ has ___ in.'*** Continue to use a variety of contexts, for example, a class list containing the names of the whole class. If we remove one or more of the children's names, we no longer have the whole class. We have a part of the whole class.	*'Is this a whole tray of cakes? Explain how you know.'*  • *'This is not a whole tray of cakes because there are some cakes missing.'* • *'This is not a whole tray of cakes because only part of the tray has cakes in.'*
3:4	Once children have fully mastered steps *3:1–3:3*, in which the whole group is clearly defined by the container (for example, a tray or a box with spaces), extend to contexts where the whole group is defined as a given number of objects, starting with concrete representations. Begin by defining and presenting whole groups, for example: • *'Charlotte has six toy cars.'* Show the six toy cars.	

	• *'Is this the whole group of Charlotte's cars?'* • *'Explain how you know.'* You can continue the previous strategy of tracing an imaginary ring around each group to help define the whole. Then introduce variation by presenting cases for which children are required to interpret the context and construct the whole group themselves, for example: • *'Ajay has four toy cars.'* • Ask someone to fetch toy cars to show Ajay's whole group of cars. Make sure that children understand that 'whole' can refer to groups of different quantities of objects. In order to demonstrate this, include examples of whole groups of different sizes and compare them – for example: • *'Charlotte's whole group has six cars.'* • *'Ajay's whole group has four cars.'* • *'Charlotte and Ajay don't have the same number of cars, but they are both whole groups.'* Use the stem sentence: ***'This is the whole group of ___. I have all of them.'***	• *'Charlotte has 6 toy cars.'* • *'Is this the whole group of Charlotte's cars?'* • *'Explain how you know.'*  *'This is the whole group of Charlotte's cars. I have all of them.'* Charlotte's group of six cars:  Ajay's group of four cars:  • *'Are these both whole groups of cars?'* • *'Does it matter that there is a different number of cars in each group?'*
3:5	Once children have understood the concept of a whole group of discrete objects, begin to identify a part of the whole group, for example: • *'There are four pencils in the whole group.'* • *'There are three pencils in the part of the group that has a ring around it.'* Include problems in which children are presented with or asked to fetch/draw	

	the whole group of objects, and must then identify part of the group. To encourage depth of thinking, ask questions such as: • *'What is the smallest number of ___ you could have in your part?'* • *'What is the largest number of ___ you could have in your part?'* Zero is not included at this point; it is considered too abstract at this early stage where children are learning what a 'part' is within the context of partitioning a group of objects. Continue to ask *'How do you know?'* to develop children's reasoning skills. Use stem sentences in the form: • ***'There are ___ ___ in the whole group.'*** • ***'There are ___ ___ in this part of the group.'***	• *'There are four pencils in the whole group.'* • *'There are three pencils in the part of the group that has a ring around it.'*  *'Here are some pencils. Draw a ring around the whole group of pencils.'*  *'Draw a ring around part of the whole group of pencils.'*  • *'What is the smallest number of pencils you could have in your part?'* • *'What is the largest number of pencils you could have in your part?'*
3:6	For further challenge provide a dòng nǎo jīn problem in which you define the number of objects in the whole group and challenge children to find how many different ways they can split the group into parts. Here, children can begin to work systematically to find all of the combinations. At this stage it is not necessary to take commutativity into account – for now consider 'three and	Dòng nǎo jīn: *'I have four strawberries. This picture shows my whole group of strawberries.* *How many different ways can you put the four strawberries onto two plates?'* 

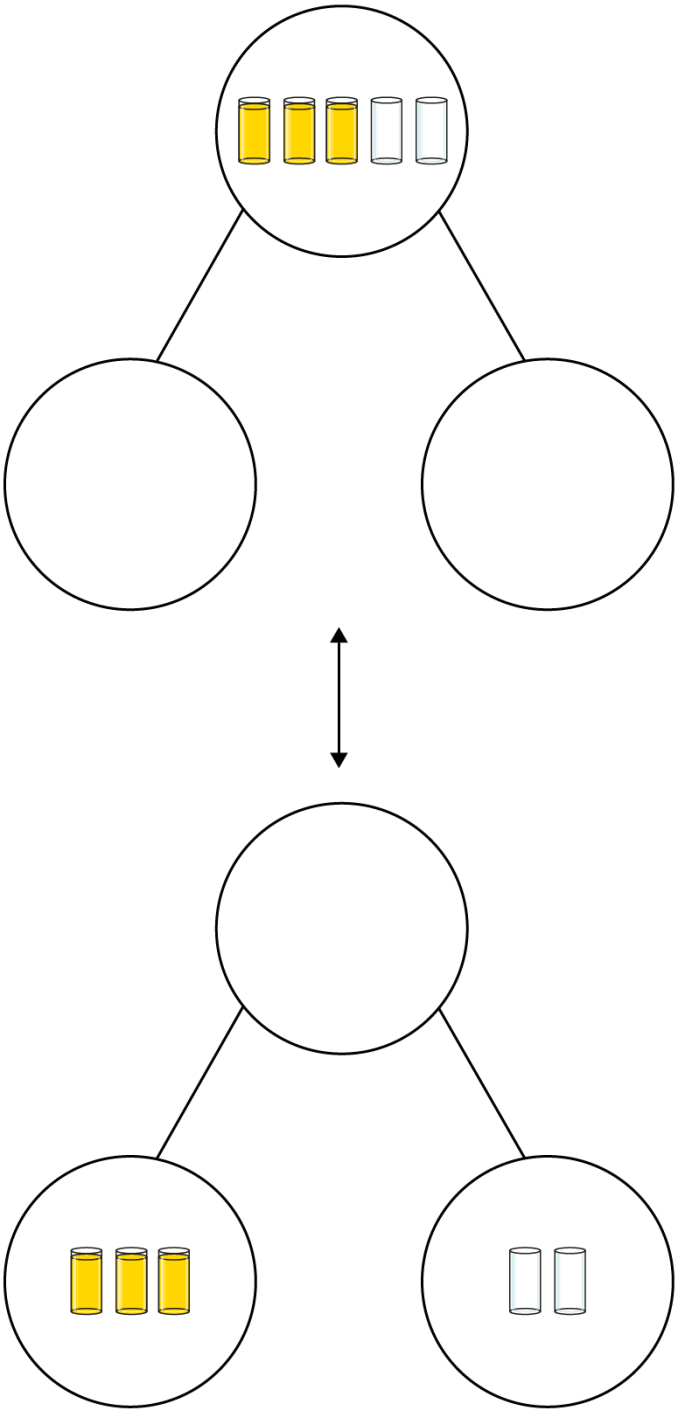
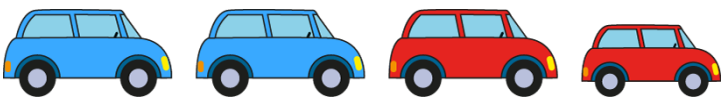
	one' as being different to 'one and three' even though the objects are identical.	Systematic solution:  *'There are three ways to split the group of four strawberries.'*

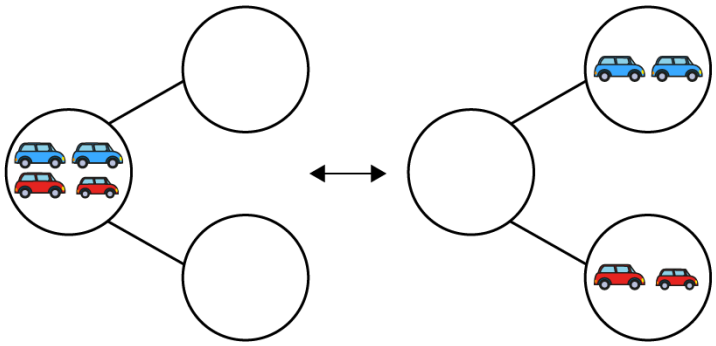
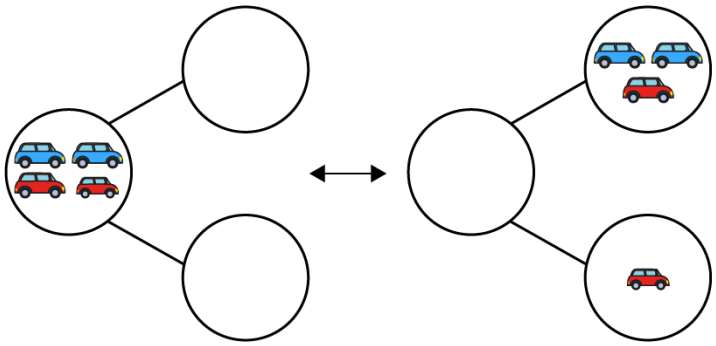
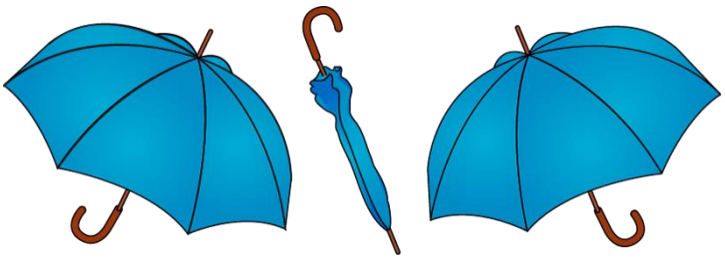
Teaching point 4:

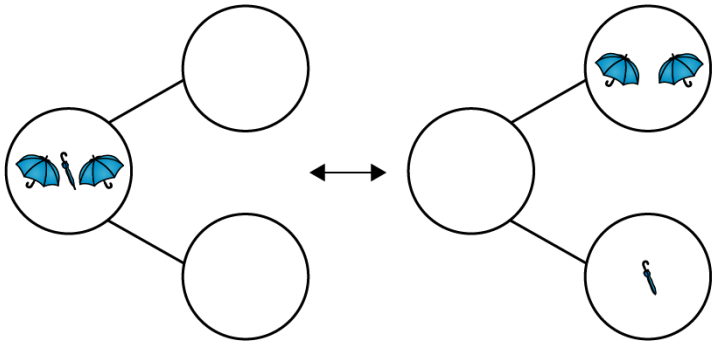
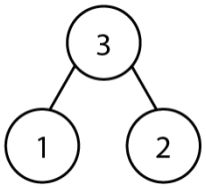
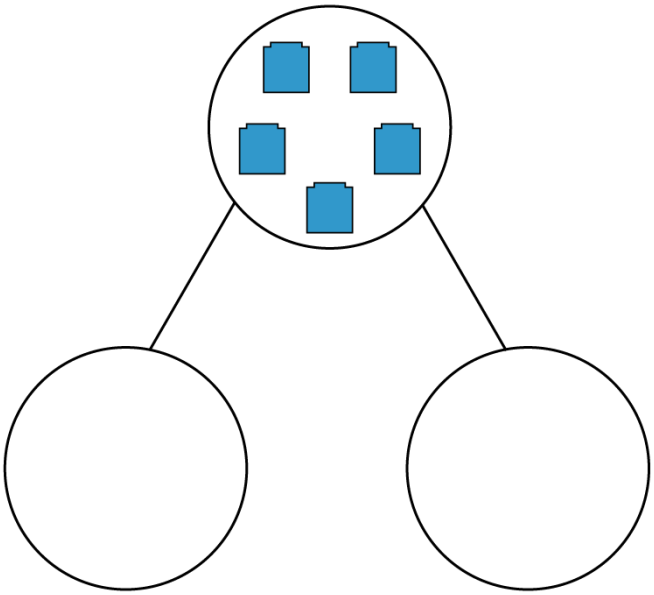
A whole group of objects can be composed of two or more parts and this can be represented using a part–part–whole 'cherry' diagram. The group can be split in many different ways. The parts might look different; each part will be smaller than the whole group and the parts can be combined to make the whole group.

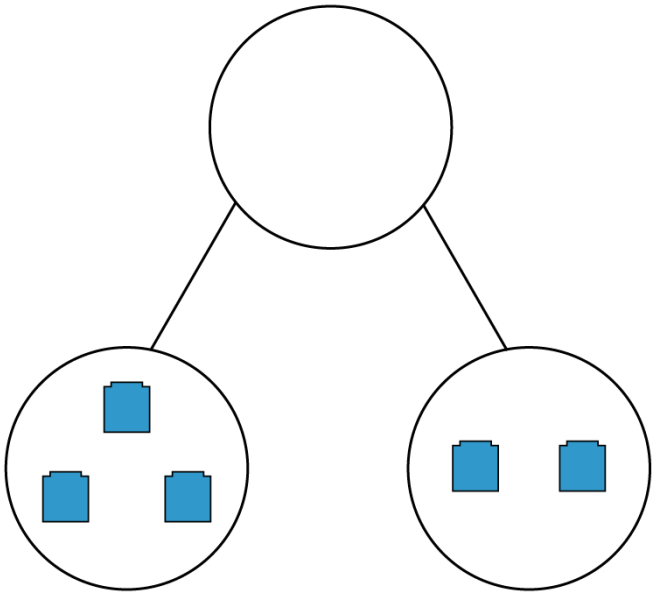
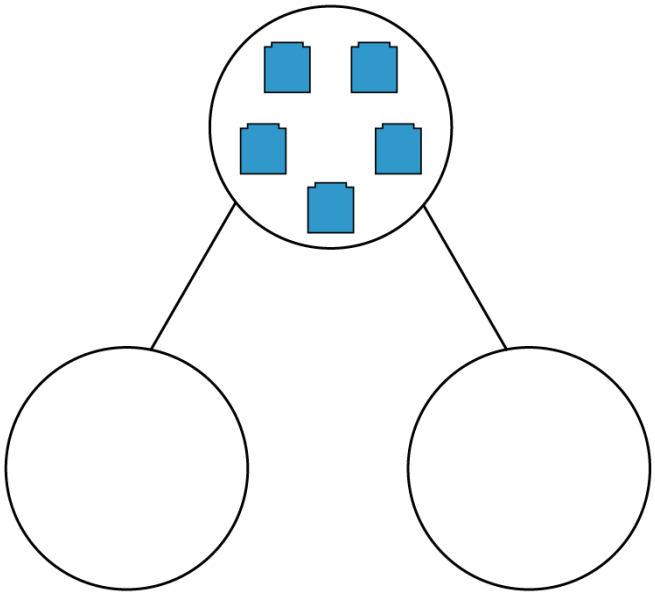
Steps in learning

	Guidance	Representations
4:1	The purpose of this teaching point is to introduce the part–part–whole 'cherry' representation. Continue using a variety of contexts that show the whole group partitioned into two, clearly identifiable, parts. Provide laminated part–part–whole diagrams so that children can physically partition and recombine concrete resources or cut-out-pictures using the scaffold. Encourage children to practise describing the parts and wholes in full sentences; ensure that children become flexible in the way they can describe the groups, by prompting them sometimes to start with the whole group, and sometimes to start with the parts. When you or the children manipulate pictures, it is essential that the objects are moved from the 'whole' section on the diagram into the 'part' sections. For example, do not show five glasses in the 'whole' section at the same time as showing the five glasses (as three and two) in the 'part' sections – the resulting diagram would show a total of ten glasses, which could result in misconceptions. For each example, repeatedly move the objects/pictures back and forth to represent splitting into parts (partitioning) and recombining the parts. This will help children to recognise that the whole and the two parts are the same quantity. Emphasise	Describing the group in full sentences: *'Say what you see.'*  • *'There are five glasses. Three of them are full and two of them are empty.'* • *'There are three full glasses and two empty glasses. Altogether, there are five glasses.*

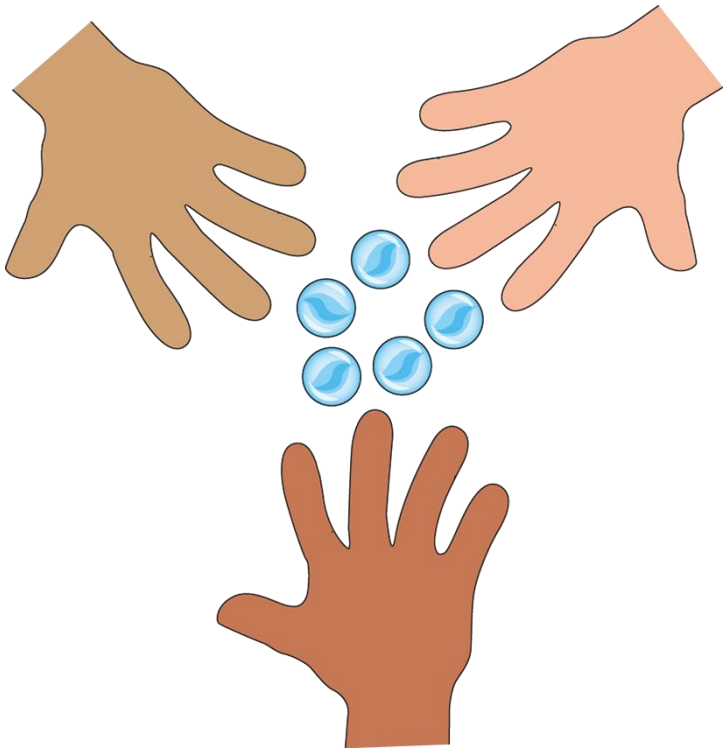
	this conservation of number and the equivalence between the parts and whole by, for example: • asking *'Have we still got five?'* after splitting into parts • adding the numeral 5 to the 'whole' position when the objects have been moved out; this will also prepare children for the symbolic recording of part–part–whole relationships in step *4:3*.	Manipulating the objects: 
4:2	Use a range of contexts and incorporate variation in terms of: • orientation of the diagram (children should understand that the orientation doesn't affect the maths being represented)	 • *'There are four cars. Two of them are blue and two of them are red.'*

	• inclusion of examples where the two parts contain the same number of items • groups composed of objects of different sizes/shapes • partitioning a given whole in several different ways.	• *'There are two blue cars and two red cars. Altogether there are four cars.'*  • *'There are four cars. Three of them are large and one of them is small.'* • *'There are three large cars and one small car. Altogether there are four cars.'* 
4:3	Now move on to representing the whole and parts with numerals. You may have already linked the whole with a numeral to remind children of the size of the whole after partitioning (step *4:1*). Moving completely into the abstract representation can be a difficult step for children and so it is essential to link the numerals to the real context. Encourage children to describe, in full sentences, what each number represents, for example: • *'The 3 represents the whole group of umbrellas.'*	

	• *'The 1 represents the closed umbrellas.'* • *'The 2 represents the open umbrellas.'* So that children make connections between the concrete, pictorial and abstract representations, use them alongside one another, continuing to explore a variety of contexts.	Part–part–whole cherry representation with pictures:  Part–part–whole cherry representation with numerals:  • *'The 3 represents the whole group of umbrellas.'* • *'The 1 represents the closed umbrellas.'* • *'The 2 represents the open umbrellas.'*
4:4	Previously (steps *4:1–4:3*) the context prompted children to partition in a particular way (for example, open umbrellas and closed umbrellas). Now, children should physically partition the whole group in a variety of ways. Use the laminated part–part–whole diagram (step *4:1*) and a group of generalised manipulatives (for example, counters or cubes). Encourage children to explore the different ways that these objects can be partitioned. Use objects that are identical so that children can see how the group can be split into parts regardless of non-essential features, such as colour or size. Initially, scaffold the exercise, for example: • *'There are five cubes in the whole group. Five is the whole.'* • Children put the correct number of objects into the 'whole' section. • *'Three is a part; two is a part.'*	*'There are five cubes in the whole group. Five is the whole.'* 

	• Children split up the group into two parts as described. • *'Three is a part; two is a part; five is the whole.'* • Children combine the parts to make the whole again, reinforcing the idea that the parts can be recombined to make the whole, and that the starting quantity is conserved. • Repeat the previous steps partitioning in different ways.	*'Three is a part, two is a part.'*  *'Three is a part, two is a part, five is the whole.'* 
4:5	Support the transition from concrete to pictorial representations by asking children to draw the objects on printed part–part–whole cherry diagrams, before they start to draw their own diagrams from scratch. Similarly, support the transition from pictorial to abstract, by asking children to write the numerals on printed part–part–whole cherry diagrams before starting to draw their own.	

4:6	Throughout their exploration ask children the following questions to reinforce key points: • *'What do you notice about the size of the parts and the size of the whole?'* Draw attention to the fact that neither part is larger than the whole; each part is smaller than the whole. • *'What happens when you put the two parts back together?'* Draw attention to the fact that the two parts together have to equal the number in the whole group (conservation of number). • *'How many different ways can you split the whole group into two parts?'* To provide challenge and encourage children to engage with key concepts, use dòng nǎo jīn questions, such as: *'Liam says* 'I have five cakes. I can put three cakes on one plate and three cakes on another plate.' *Is he right? Explain your thinking.'* This example explores the concept that the two parts cannot total more than the whole. Throughout, reinforce use of the stem sentence: ***'___ is the whole; ___ is a part and ___ is a part.'*** Continue to emphasise that the contexts can be described both by beginning with the whole or by beginning with the parts, for example: • *'Dan has five pennies. There are three pennies in one pocket and two pennies in the other.'* • *'Dan has three pennies in one pocket and two pennies in the other. He has five pennies altogether.'*	
4:7	Finally, children should explore partitioning a group of identical objects (the whole) into three parts. This builds on splitting a single object	

	into more than two parts, originally introduced in step *2:2*. Provide contexts that challenge children to partition objects into three parts in as many ways as possible. Children will explore problems with three parts in more detail in segment *1.11 Addition and subtraction: bridging 10*. To provide additional challenge you could: • extend to more than three parts • use a dòng nǎo jīn problem that requires children to tell their own stories that represent partitioning into three or more parts.	*'Three friends won five marbles. How many different ways could they share them out?'*  *'Show your answers on a part–part–part–whole diagram.'*