MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.9 Composition of numbers: 20–100

Teacher guide | Year 1

Teaching point 1:

There is a set counting sequence for counting to 100 and beyond.

Teaching point 2:

Objects can be counted efficiently by making groups of ten. The digits in the numbers 20–99 tell us about their value.

Teaching point 3:

Each number on the 0–100 number line has a unique position.

Teaching point 4:

The relative size of two two-digit numbers can be determined by first examining the tens digits and then, if necessary, examining the ones digits, with reference to the cardinal or ordinal value of the numbers.

Teaching point 5:

Each two-digit number can be partitioned into a tens part and a ones part.

Teaching point 6:

The tens and ones structure of two-digit numbers can be used to support additive calculation.

Overview of learning

In this segment children will:

- learn how to count to 100 and beyond, by making use of the pattern and structure of our number naming
- count collections of objects efficiently by making groups of ten
- represent numbers from 20–100 using base-ten equipment (for example, Dienes or straws), numerals and number names
- explore the position of two-digit numbers in the number system – identifying the previous and next multiple of ten, placing them on a number line, and making links to measures
- compare the size of two-digit numbers
- use their understanding of the structure of two-digit numbers to partition them into tens and ones
- apply partitioning into tens and ones to calculations such as $20 + 3 = 23$ and $64 - 4 = 60$.

This segment will give children a sense of the regularity of number naming up to 100 *before* they begin work on the irregularly named teen numbers in segment *1.10 Composition of numbers: 11–19*. Dual counting is used to emphasise the regularity – i.e. *'Two ten one, two ten two, two ten three…'* and *'Twenty-one, twenty-two, twenty-three…'*. Children begin with some simple counting practice in *Teaching point 1*, with the dual naming revealed later when children have some understanding of the structure of the numbers. Even within 20–100, some number names are more clearly linked to the value of the tens digit (4, forty; 6, sixty; 7, seventy; 8, eighty; 9, ninety) – where possible, plan to use these numbers as teaching examples. The number names with a less apparent link to the value of the tens digits (2, twenty; 3, thirty; 5, fifty) can then be used as practice examples after the initial concept has been introduced. The representations provided below provide a mix.

In *Teaching point 2*, the focus is on the cardinality of two-digit numbers, with children developing efficient strategies for counting larger quantities by making groups of ten, building on knowledge of counting in tens (segment *1.8 Composition of numbers: multiples of 10 up to 100*). Children then represent two-digit numbers with base-ten equipment, with numerals, and with number names. Throughout, the emphasis is on the link between the representations and the tens and ones structure of the numbers, encouraging a depth of understanding of our decimal number system.

There are many different forms of base-ten representations (both concrete equipment and common pictorial representations), and a range of representations are suggested throughout this segment. Children benefit from being able to see the links between different representations of the same thing – a certain amount of variation is necessary if children are to understand the unifying ideas behind representations. However, this must be balanced with the need to avoid overloading children by jumping too quickly from one representation to another. You will need to closely watch how children are learning in order to make considered judgements about the introduction of new representations. Children will need to be familiar with some core representations, such as Dienes, which will be used consistently through the year groups for different areas of mathematics.

We often do not recognise that a large part of understanding number comes from a deep understanding of how numbers are ordered; understanding number is not just about the link to objects. In *Teaching point 3*, children develop their understanding of the position of two-digit numbers in the number system, and move from this to thinking about number as length. They then bring together ideas of both cardinality and ordinality as they learn to compare the size of two two-digit numbers in *Teaching point 4*.

The remainder of the segment builds towards calculation. First, children use the part–part–whole model to represent the partitioning of two-digit numbers into tens and ones 'parts', and the aggregation of tens and ones 'parts' to make 'whole' two-digit numbers. The part–part–whole models are then linked to the corresponding equations, so that by the end of the segment children can complete addition and subtraction calculations using their knowledge of composition of number.

An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks

Teaching point 1:

There is a set counting sequence for counting to 100 and beyond.

Steps in learning

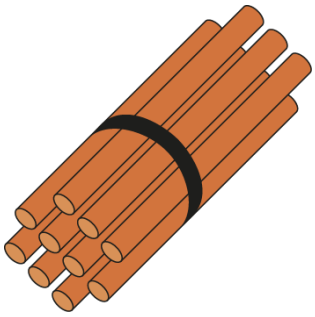
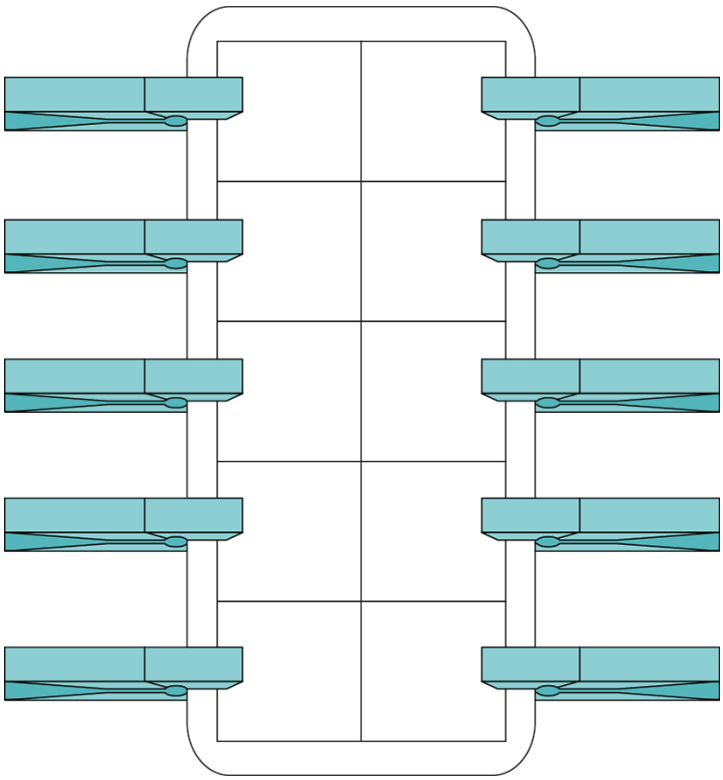
	Guidance	Representations
1:1	Provide children with practice counting to 100, and beyond, before beginning the detailed examination of the numbers 20–100 in *Teaching point 2*. It is likely that children will already have experienced counting within 100 in several ways: • You may have practised ordinal counting (oral counting with no object reference), with the whole class practising saying the sequence of number names – *'One, two, three…'* – possibly up as far as 100 or even beyond. In this type of practice children get used to the sound of the number names, and the pattern in which one follows the next. • In other contexts, you may have used the counting sequence to enumerate a quantity of objects (with the focus on cardinality) – for example, to count the number of children, or pieces of equipment. In this type of counting children get used to linking number names with quantities. • You may also have practised counting whilst pointing to numbers written in digits (for example, on a hundred square, number line or Gattegno chart). Here children get used to linking number names with the digits that represent them. Note that each type of counting is reinforcing a different type of number knowledge. Before and during the segment, practise each type of counting. Classroom routines often provide useful contexts for the first two types described above; for example:	Hundred square: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 Gattegno chart: 1000 2000 3000 4000 5000 6000 7000 8000 9000 100 200 300 400 500 600 700 800 900 10 20 30 40 50 60 70 80 90 1 2 3 4 5 6 7 8 9

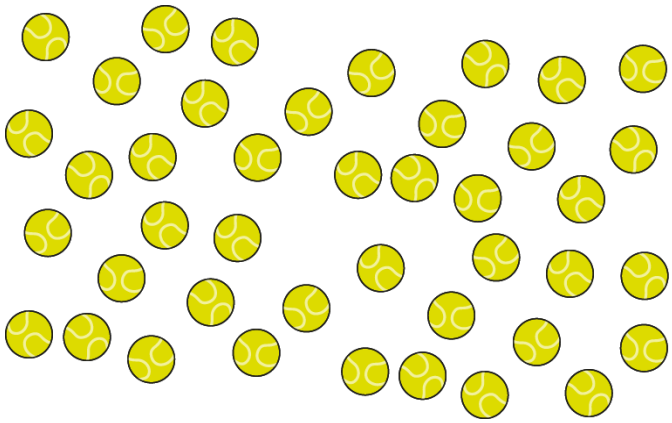
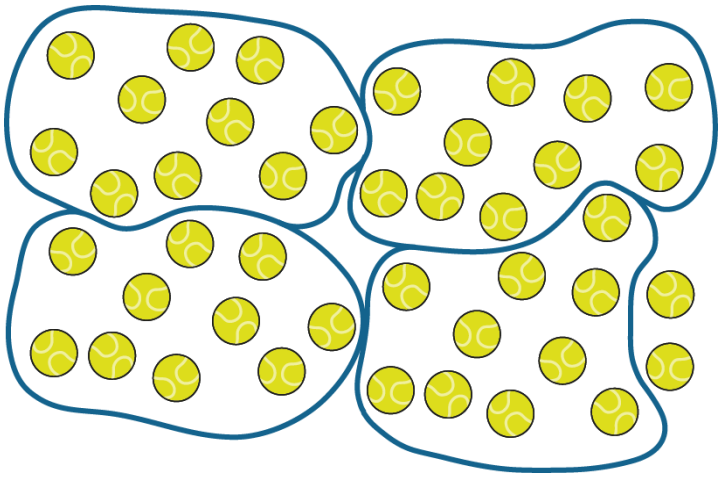
	• How many seconds does it take for a child to take the register to the office and come back? • A daily count – count the number of days since the start of the school year. When using the Gattegno chart you will need to double tap, for example 40 then 3, for 43. Also ensure that children gain experience of the gestural patterns, by having them tap the numbers out for themselves. During counting practice, continue to just over 100; often it is working over the boundary that children find most challenging, and counting *'ninety-nine, one hundred, one hundred and one, one hundred and two…'*, trailing off soon after, will prepare children for 100 not being an 'end'.	

Teaching point 2:

Objects can be counted efficiently by making groups of ten. The digits in the numbers 20–99 tell us about their value.

Steps in learning

	Guidance	Representations
2:1	Children need to first understand the identity of one ten and ten ones if they are to go on to use more abstract representations, such as Dienes or (later on) place-value counters, with more than just procedural understanding. The idea of grouping ten of one thing into one of something else underpins all place-value equipment. Segment *1.8 Composition of numbers: multiples of 10 up to 100* included detailed work on exploring how one ten is equal to ten ones, and briefly introduced some common base-ten equipment (Dienes, base-ten number boards, tens frames). Building on this, begin this teaching point with children making their own base-ten equipment, giving them practical experience of grouping some ones into tens and leaving other ones as ones. Use a range of resources, as shown opposite, asking children to: • *'Show me one ten.'* • *'Show me one one.'* • *'Show me ten ones.'* (Encourage children to show you one group of ten here, though it is of course also correct if they show you ten separate ungrouped ones.) Useful resources include: • sticks or straws grouped into bundles of ten • small pots/cups, each containing ten counters • bead strings with ten beads	 

	• small net bags, each containing ten objects • multilink cubes in sticks of ten • pieces of card or laminated individual tens frames with ten clothes-pegs attached • hands. Encourage children to use the generalised statements: • ***'This is one one.'*** • ***'This is ten ones. It is also one ten.'***	
2:2	To introduce the idea of using groups of ten for efficiency, show children a picture of many objects (opposite we see 42 tennis balls) and ask *'How many are there?'* First, model counting in ones and being distracted or losing count during the process, having to start again at the beginning. Then count again, this time counting ten at a time, circling each group as you go. Emphasise that by organising into groups of ten, it doesn't matter if you lose count – you could model losing count again, and only having to go back to the start of the current group of ten. Remind children that ten ones is the same as one ten, and 'count' your total by saying *'I have one, two, three, four groups of ten and one, two extra ones'*. Finally, count the images in ones again, confirming the total as *'forty-two'*. Summarise: *'We can work out how many there are by counting in ones, or by counting the groups of ten and the extra ones.'*	Counting in ones:  Counting groups of ten and the extra ones: 

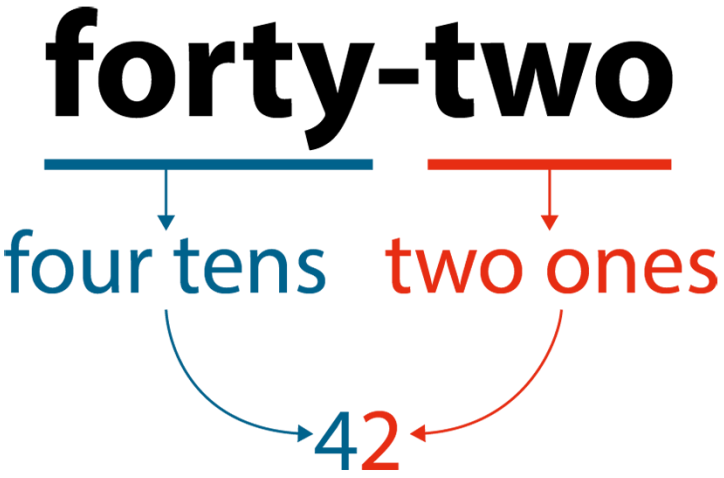
<table>
<tr>
<td>2:3</td>
<td>Continuing with the previous example, record the number of groups of ten and the number of extra ones in a table – first using descriptive column headings (for example, ‘Groups of ten’ and ‘Extra ones’), before simplifying to ‘10s’ and ‘1s’. Draw attention to the link between the number you have all said (‘forty-two’) and the digits in the table.

Finally write the number, no longer in the table, emphasising that this is the correct convention for writing 42:
• ‘This is how we write the number forty-two.’
• ‘We read from the left...’
• ‘Forty-two… forty-two...’ (point to the digits in turn).

Note that teen numbers do not follow this ‘left to right’ reading convention, hence them not being taught until the next segment.</td>
<td>
<table>
<tr><th>Groups of ten</th><th>Extra ones</th></tr>
<tr><td>4</td><td>2</td></tr>
</table>
• ‘The 4 shows we have four groups of ten.’
• ‘The 2 shows we have two extra ones.’
• ‘We have four groups of ten and two more ones.’

<table>
<tr><th>10s</th><th>1s</th></tr>
<tr><td>4</td><td>2</td></tr>
</table>
42
</td>
</tr>
<tr>
<td>2:4</td>
<td>Now spend some time examining the number names in more detail. Remind children of the number names for the multiples of ten (see segment 1.8 Composition of numbers: multiples of 10 up to 100, step 2:5), before demonstrating how the number names are written when the number of ones is not zero.

For each number you explore, link the number name, the digits that represent it and the tens-and-ones structure. Return to the representation in digits, emphasising that the digits are written in the order that the parts of the name are spoken, using the stem sentence: ‘This is the number ___. We write the ___ then the ___.’

For example:
42</td>
<td> 
</td>
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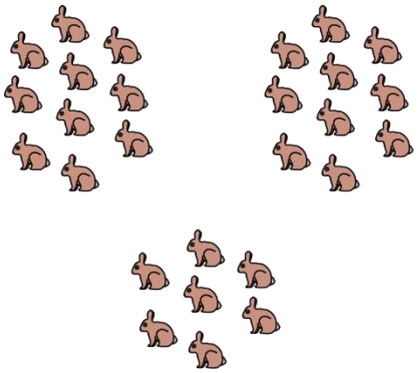
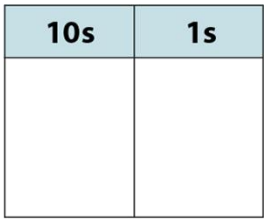
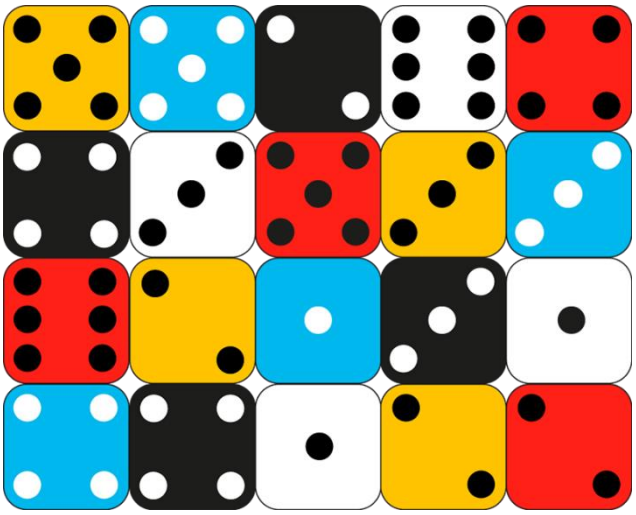
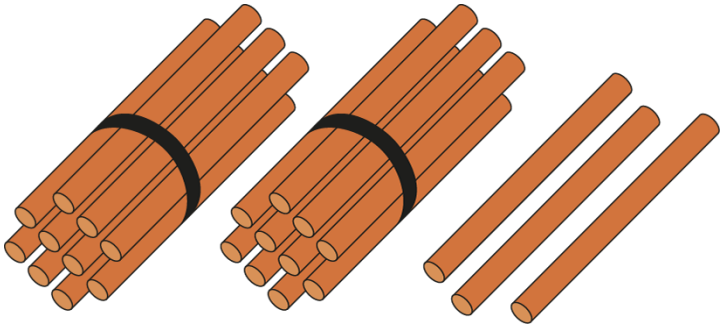
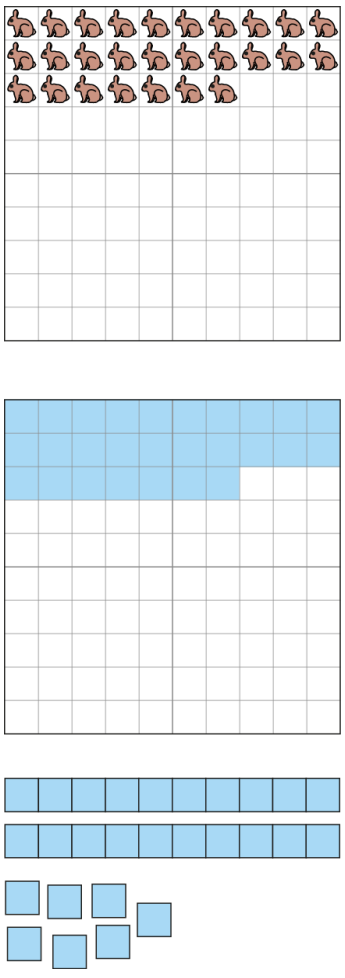
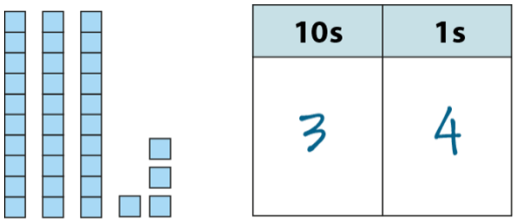
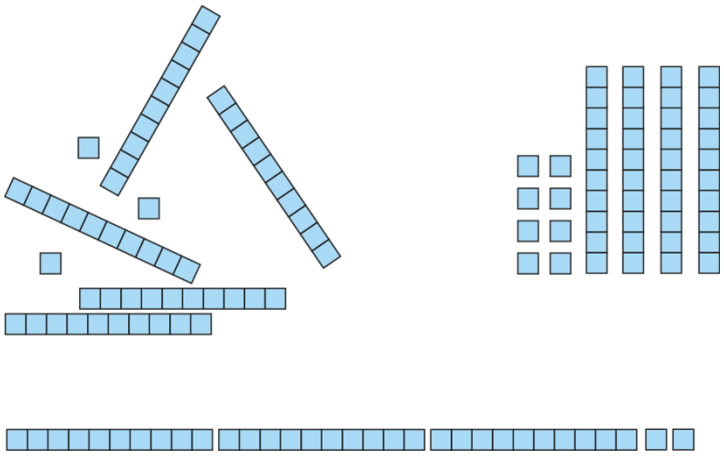
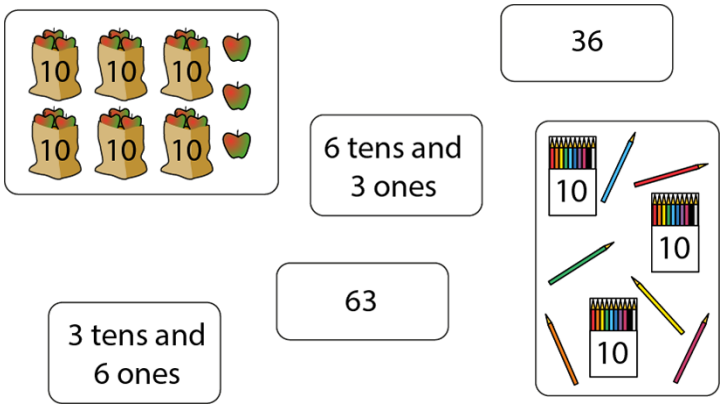
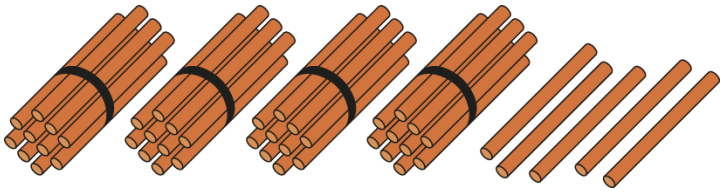
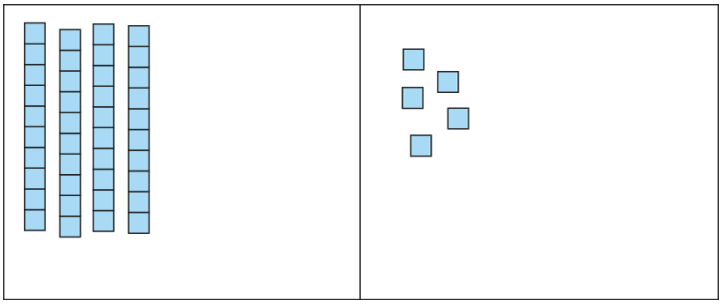
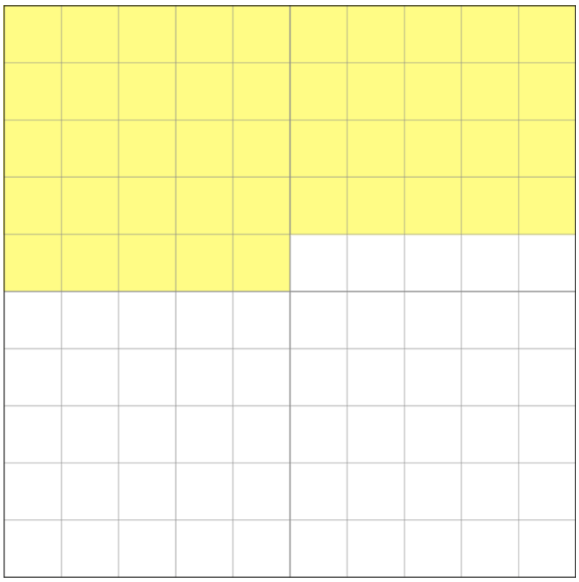
</table>

	'This is the number forty-two. We write the four then the two.' Repeat the sequence in steps *2:2–2:4* several times, with different quantities. Note that for numbers such as 63, you can hear the 'six' in 'sixty', but in 36, the connection of 'thir' with 'three' is less clear; ensure that you include numbers with all of the irregularly named multiples of ten (i.e. twenty-something, thirty-something, forty-something and fifty-something) in your sequence.	
2:5	Provide opportunities for children to practise enumerating sets of items, and expressing the number name verbally and with digits. Children can count in ones to check their answers, but make sure they are using the 'groups of ten' strategy first. Encourage children to use the generalised statement and stem sentence from step *2:4* to link the numerals they have written to the quantity value of the number. Some children may write '20' in the tens column. Be meticulous about correcting this: '*There are* two *tens which have a* value *of twenty.*' Uncorrected, this may lead to children writing, for example, 27 as '207', following the thought process: *'I can hear twenty-seven so I will write twenty and seven.'*	Enumerating items: *'How many rabbits are there?'*   Dòng nǎo jīn: • 'How many dots are there altogether?' • 'How could you count these efficiently?' 

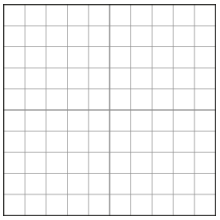
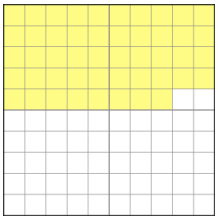
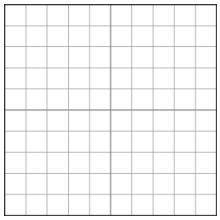
2:6	Now start to take steps to prioritise unitising. Return to the sticks or straws used earlier, placing a number of them onto the table, then bundling them into groups of ten. Show how we can work out the total number of sticks by counting the groups of ten and counting the extra ones. Then ask children to pick up a given number of sticks, both saying and writing the number as you do. Work through checking the total by counting in ones, and emphasise how the task was achieved more quickly by working with bundles of ten. This step is similar to step *2:2*, but now children are beginning to manipulate each group of ten as a single unit – the straws representation supports this.	*'Pick up twenty-three sticks.'* 
2:7	Now move one level of abstraction further, using Dienes – here, each ten can't be physically separated into ten ones. You can use the progression shown opposite to move from a pictorial representation to Dienes. Again, link to the number written as numerals. Note that the hundred grids used opposite have heavier lines across the vertical and horizontal midpoints. This is to support children with subitising and moving away from counting – as we can subitise up to about five items, including these lines helps children to see the total number of items without counting – opposite we have five and two which must be seven.	

		<table><tr><th>10s</th><th>1s</th></tr><tr><td>2</td><td>7</td></tr></table> 27
2:8	Give children practice working with Dienes, again writing the numerals using place-value charts and then writing without. Provide variation in how the Dienes are presented: • Present them with the ones and tens mixed up – children may need prompting to deal with the tens first, then the ones, rather than counting them in the 'mixed' physical order in which they are laid out. • Present them with the ones on the left and tens on the right – for the example opposite, check that children can see the Dienes represent 48, not 84. • Present them in a linear arrangement to prepare for links with measures contexts. The second arrangement introduces the concept that the order is irrelevant. This can be reinforced practically: have a group of children standing at the front of the class; each can choose whether they show one or ten fingers. However the children are arranged, the total number of fingers held up remains the same. Check by counting in ones; this continues to reinforce the important idea that, for example, five tens and five ones is still 55 ones, as well as providing practice counting in ones.	Interpreting Dienes representations:  <table><tr><th>10s</th><th>1s</th></tr><tr><td>3</td><td>4</td></tr></table> 34 Variation: 

2:9	Also provide practice using pictorial real-world objects that are grouped into tens, again providing variation in how the images are presented. Bags of classroom fruit are a really useful resource as children will be familiar with the idea that they contain ten pieces of fruit, even though they may not be able to see all the individual pieces. Support children in making connections between different representations of the same number, by presenting a range of representations to sort according to the number/quantity they represent. Note that, in the example shown opposite, both 63 and 36 have been included to challenge any remaining misconceptions regarding the order of the digits. Ask children to consider what all of the representations in a given group have in common, asking *'What's the same?'*, and drawing out the key idea that all the base-ten representations of, for example, 57 have five tens and seven ones.	*'Sort the cards into two groups according to the number they represent.'* 
2:10	Once the children are confident writing quantities for amounts shown (represented by Dienes or in real-world contexts), have them work in the opposite direction: say a number name (for example, 'forty-five') or a number structure (for example, four tens and five ones), or show a written number (for example, '45' written on the board but *not* read out loud) and ask the children to represent that number with base-ten equipment. Again, reinforce that the number represented is independent of how the equipment is arranged, but laying them out with the tens on the left and	*'Show me this number.'* 45 

	the ones on the right corresponds to the order the digits are written. You may want to give children mats to work on to order their place-value equipment, as shown opposite, but note that these should not have 'tens' or 'ones' column headers on them – in the example shown, this would imply that there are 40 tens. The base-ten equipment itself signifies the value of the representation. It is important that children are not just procedurally learning to pick up 'four of the big things and five of the little things'. Reinforce the relationship of the representation to the number by counting: • *'Ten, twenty, thirty, forty, forty-one, forty-two, forty-three, forty-four, forty-five.'* • Also count in ones *'One, two, three… forty-four, forty-five'* to show there are still 45 ones. Confirm that we could represent 45 with 45 ones, but it is more efficient to work with groups of ten. Children can also work pictorially, colouring in numbers on blank hundred grids. Check carefully that children aren't counting and shading the individual squares; the manner in which they are shading them should show that they are thinking in terms of tens and ones, shading ten at a time and then the extra ones.	*'Show me forty-five.'*  *'Colour in forty-five.'*  • *'Forty-five is made up of forty and five more.'* • *'Forty-five is made up of four tens and five ones.'* • *'Forty-five is made up of forty-five ones.'*
2:11	Once children have represented some numbers in isolation, explore sets of linked examples to prompt discussion about relationships between different two-digit numbers. Using concrete base-ten equipment, ask children to represent sequences such as: • 26, 27, 28, 29 • 83, 82, 81, 80	

	• 42, 41, 40, 39 • 40, 50, 60, 70 • 61, 71, 81, 91 • 37, 87, 67, 27 • 34, 43 • 28, 82 To help highlight the relationship between the numbers in each sequence, children can point to the numbers on a Gattegno chart before they represent the numbers with base-ten equipment. With the first six examples, some children may use efficient methods to move from one number to the next (i.e. adding or removing the necessary ones or tens), whereas other children may return the equipment after representing the first number, then start again to represent the second number (for example representing 26 then, instead of just adding another one, starting again to represent 27). To encourage use of efficient methods, referring to the next number in the sequence, ask children *'What's the same? What's different?'* In the final examples (34 and 43, 28 and 82), the numbers have the same digits, but the digits have different values (tens versus ones), resulting in an entirely different picture.	
2:12	To further encourage children to think about the relationship between numbers, use dòng nǎo jīn challenges, such as: • Prepare a bag containing a mixture of tens and ones; children work in pairs, each drawing six random 'objects' from the bag, then exploring what numbers they can each make. The child that makes the largest number wins. • *'I am thinking of a two-digit number. The first digit is four greater than the*	

<table>
<tr><td></td><td>second digit. What could my number be?'
• Provide three single-digit cards (for example, 0, 2 and 4) and challenge children to see how many different two-digit numbers they can make.</td><td></td></tr>
<tr><td>2:13</td><td>Finally, give children practice moving between digits, pictorial/concrete representations, number names and number structures, as exemplified opposite.
You could also ask children to, for example, show 42 in their books, and then compare how different children choose to represent this (some might draw dots, some might draw base-ten equipment, some might write '42', etc.)</td><td>'Complete the table.'
<table><tr><td>29</td><td></td><td></td></tr><tr><td>  </td><td>  </td><td>  </td></tr><tr><td></td><td></td><td>sixty-seven</td></tr><tr><td></td><td>___ tens
and
___ ones</td><td>___ tens
and
___ ones</td></tr></table></td></tr>
</table>

Teaching point 3:

Each number on the 0–100 number line has a unique position.

Steps in learning

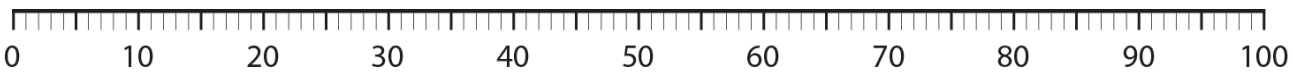
3:1

Throughout the segment, children have been practising counting forward and back in ones. The idea of dual counting was introduced in segment *1.8 Composition of numbers: multiples of 10 up to 100*, whilst counting in multiples of ten (*'One ten, two tens, three tens…'* and *'Ten, twenty, thirty…'*). Use the representations introduced in step *1:1* above (number line, hundred square, Gattegno chart) to support dual counting between 20 and 100, counting both forwards and backwards:

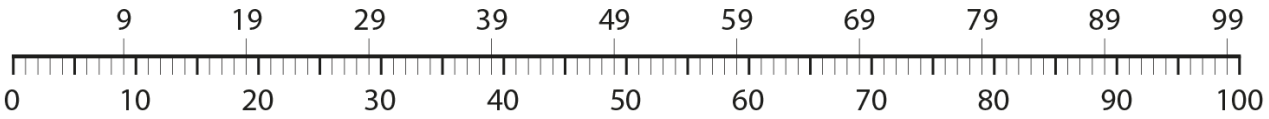
- *'Two ten one, two ten two, two ten three…'*
- *'Twenty-one, twenty-two, twenty-three…'*

Focus particular attention on counting forward and backwards over the tens boundaries. If you are using a number line with only the multiples of ten marked, you may initially want to mark the numbers with nine ones to support backwards counting. The Gattegno chart is also a useful resource for supporting counting over these boundaries.

Number line with multiples of ten labelled:



Marking numbers with nine ones:



3:2

During counting, stop at regular points and ask children to identify 'one more' or 'one less'. Then provide missing number problems. Before moving on, ensure that children are fluent in identifying one more and one less over the tens boundary as well as within each ten.

'Fill in the missing numbers.'

37		39		41	42		44

3:3

Next, look at identifying the two multiples of ten which a given number sits between. Children will not be rounding numbers for some time yet, but getting familiar with previous and next multiples of ten will prepare them. Rounding to the nearest ten (and the nearest 100) is covered in segment *1.22 Composition and calculation: 1,000 and four-digit numbers.*

Begin by identifying groups of numbers on the number line, asking, for example:

- *'Where are the twenties numbers?'*
- *'Where are the fifties numbers?'*

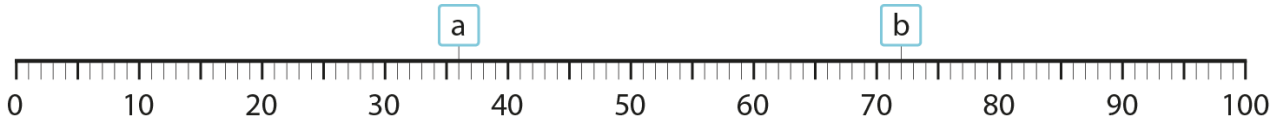
Then, work through the following progression (as exemplified below):

- Initially look at points (such as 'a' and 'b' shown below) marked on a number line (numbers unlabelled) and identify previous and next multiples of ten with this scaffold.
- Then move on to working with number lines with the numbers labelled.
- Finally look at numbers in isolation without the scaffold of a number line.

Make sure you use the language of 'previous' and 'next' multiple of ten; avoid saying things like *'the multiple of ten that is more/less'* which can lead to confusion with the number that is ten more/ten less than the given number. Use the following stem sentences:

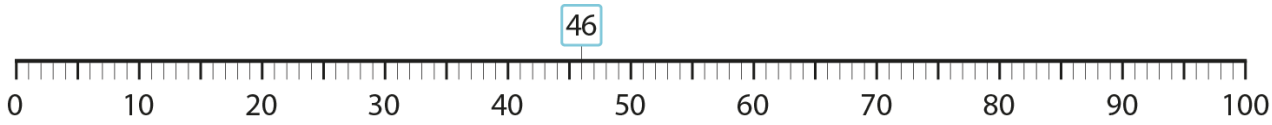
- ***'___ is between ___ and ___.'***
- ***'___ is the previous multiple of ten.'***
- ***'___ is the next multiple of ten.'***

Identifying previous and next multiple of ten – unlabelled numbers:



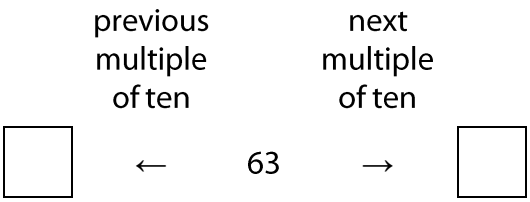
- *"a' is between thirty and forty; thirty is the previous multiple of ten; forty is the next multiple of ten.'*
- *"b' is between seventy and eighty; seventy is the previous multiple of ten; eight is the next multiple of ten.'*

Identifying previous and next multiple of ten – labelled numbers:



'Forty-six is between forty and fifty; forty is the previous multiple of ten; fifty is the next multiple of ten.'

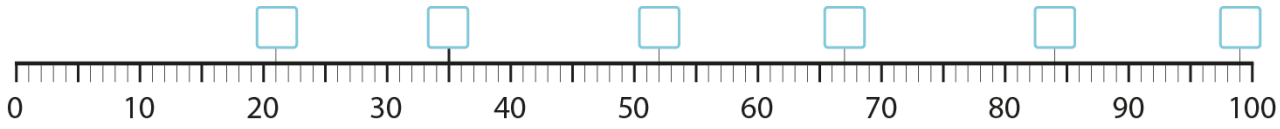
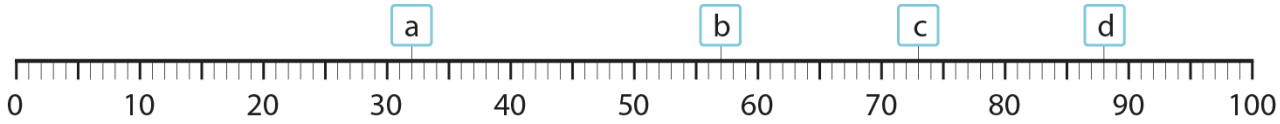
Identifying previous and next multiple of ten – unscaffolded:



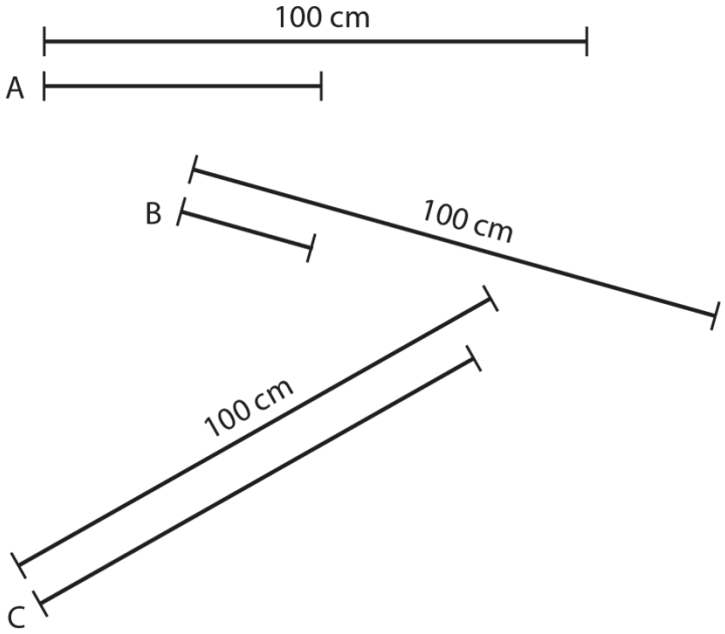
3:4

Now work towards identifying a given number pointed to on the number line. First, spend some time looking at the layout of the number line in detail:

- Note how the multiples of ten are marked.
- Note the emphasis on the tick mark in the middle of each section, between multiples of ten (e.g. 15, 25).

	• Draw attention to the smaller tick marks indicating the ones. Show a number line with a given number 'pointed at' but not identified (e.g. 52 as shown below). Initially show the children how to look at the previous multiple of ten (fifty), modelling language such as *'I know this is going to be a fifties number.'* Then count up from the previous multiple of ten, in ones, or apply knowledge of the zero to ten number line, to identify the number indicated. Return to noting that the emphasised middle tick mark always has a ones value of five, and demonstrate how this knowledge can be used to identify numbers such as 35 without counting, by looking first at the previous multiple of ten. Once children can easily find whole tens, and numbers with five ones, look at numbers which can be identified as one more or less than these marker numbers (e.g. 29, 31, 34 and 36), then two more or less (e.g. 28, 32, 33 and 37). Practise identifying a range of marked numbers between 20 and 100, ensuring that you always use a number line with the fives tick marks emphasised.
	'Identify these numbers.'  Dòng nǎo jīn: • *'Which of these numbers has a ones digit which is larger than the tens digit?'* • *'Which of these numbers has two digits that add up to ten?'* 
3:5	Now move on to placing a given number on a number line, drawing on learning from the previous step. For example: '*Place these numbers on the number line.*' 21, 29, 32, 38, 43, 72, 83, 94 Continue to use number lines with the tick marks for the fives emphasised. Look carefully at the strategies children use. Make sure that all are, at the very least, locating the previous multiple of ten and counting forward from there in ones. Some children may not need to count at all. They may be able to use strategies such as these to place, for example, 48: • Locate 40; know that 48 must be 45 (the midpoint between 40 and 50) and three more. • Locate 50; know that 48 must be two less than 50. You can promote strategies like this by verbalising a similar thought process as you show how *you* place numbers on the number line. If you see any children counting in ones from the start of the number line, give them additional support to use the previous multiple of ten as a starting point before counting up from there.

	To promote and assess depth of understanding, present dòng nǎo jīn problems such as: • *'Mark on all of the numbers which have the same ones digit and tens digit.'* • *'Mark this number on a number line:* *It is bigger than twenty.* *It is smaller than thirty.* *The ones digit is odd.* *The digits add up to three.'*
3:6	Children should now be able to identify the position in the number system of a given number between 0 and 100. The final two steps of this teaching point work to develop children's understanding of number as length – for example, knowing that something which 'runs' from 0–32 has a length of 32 units. Bead strings or bars are a useful representation for making a link between number as quantity (for example, 32 things, or three lots of ten things and two more), number as length (for example, a length of 32 units) and the position of a number on the number line. However, the link is clearest when the bead string representation is presented in a straight line. When classroom bead strings are used by children, the clarity of this linear structure (with its groups of tens) can easily be lost as the bead strings hang down or curl into piles on the table. Large classroom bead *bars* (which are rigid) expose the linear model clearly; if you do not have access to a bead bar, it is likely that an *image* of a bead string, displayed on the whiteboard, will expose the maths more clearly than concrete bead strings. This is a good example of thinking about not just *which* resources effectively expose a particular mathematical concept, but also making sure that the *way* in which a resource is used supports this. Using an image of a bead string, or a classroom bead bar, ask children how we could use the coloured groups of ten to find a given number of beads easily. If you are using a bead bar you will be able to separate the beads; if you are using an image on the whiteboard, then circle the group of beads. It is really important to note that here we are not talking about the 'number 32' bead (or the 32nd bead); we are now looking at a group of 32 beads.
	Bead bar: *'Show me thirty-two beads.'* Bead string image: *'Show me thirty-two beads.'* 
3:7	Begin talking about 'length', by drawing a line under an image of a bead string, asking *'How many beads long is this?'* Now move on to estimating some lengths relative to 100 centimetres. You could draw out markings in the playground as shown below, with pre-drawn 100 centimetre reference lines;

	when introducing the task, show a metre stick to demonstrate that the reference lines are 100 centimetres. Note that estimation should then be carried out relative to the pre-drawn 100 centimetre line and should not involve use of the metre stick; this keeps the focus on developing children's understanding of where two-digit numbers sit in the number system (rather than their ability to manipulate and read a metre stick). At this stage, the focus is also not on introducing the centimetre unit, and the idea that 100 centimetres is equivalent to one metre – more in-depth work on measures is required for this. Before children begin working on estimations themselves, model the process. Use knowledge of estimating the position of multiples of ten (see segment *1.8 Composition of numbers: multiples of 10 up to 100*) as a starting point, before refining the estimate down to the ones digit, for example: • *'I know that fifty is in the middle of zero and one hundred. I think this line is a bit more than fifty. I think it is fifty-three centimetres long.'* • *'This is quite near to one hundred. I think it is an eighties or nineties number. I think this line is about eighty-five centimetres long.'* Encourage the children to talk in this way in pairs as they visit each line on the ground. Once children have made estimates, review the length of each line as a class, asking children to explain their thinking. You can show or tell children the exact length of each line, but make sure they in no way see the exact length as the 'right answer' – reasonable estimates should be celebrated for showing good proportional understanding.
	'How many beads long is this?'  *'Look at lines A, B and C. Can you guess how long they are by comparing them to the 100 cm lines?'* 

Teaching point 4:

The relative size of two two-digit numbers can be determined by first examining the tens digits and then, if necessary, examining the ones digits, with reference to the cardinal or ordinal value of the numbers.

Steps in learning

4:1

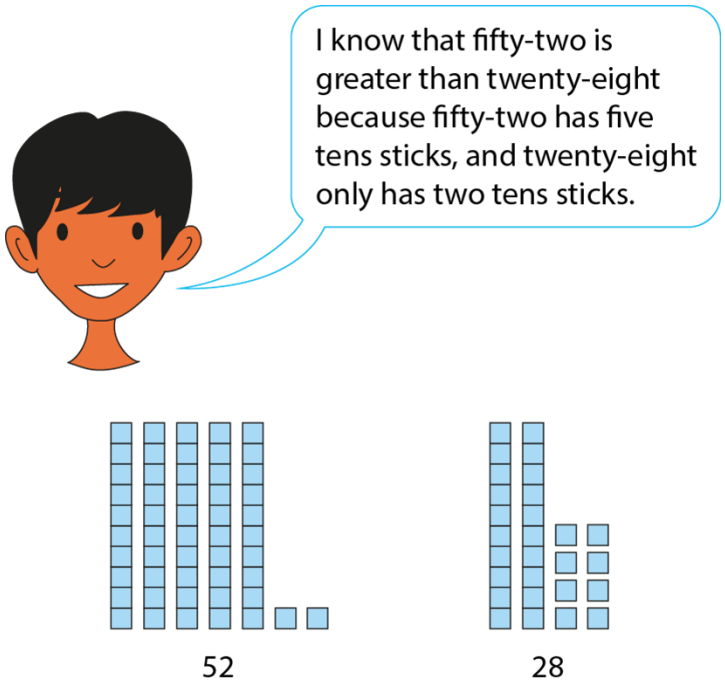
Once children can confidently identify and represent two-digit numbers based on cardinality (for example, using Dienes, hundred squares, etc.) or based on ordinality (number lines, Gattegno charts), use these representations to compare numbers.

Present two methods for comparing, say, 52 and 28, for example:

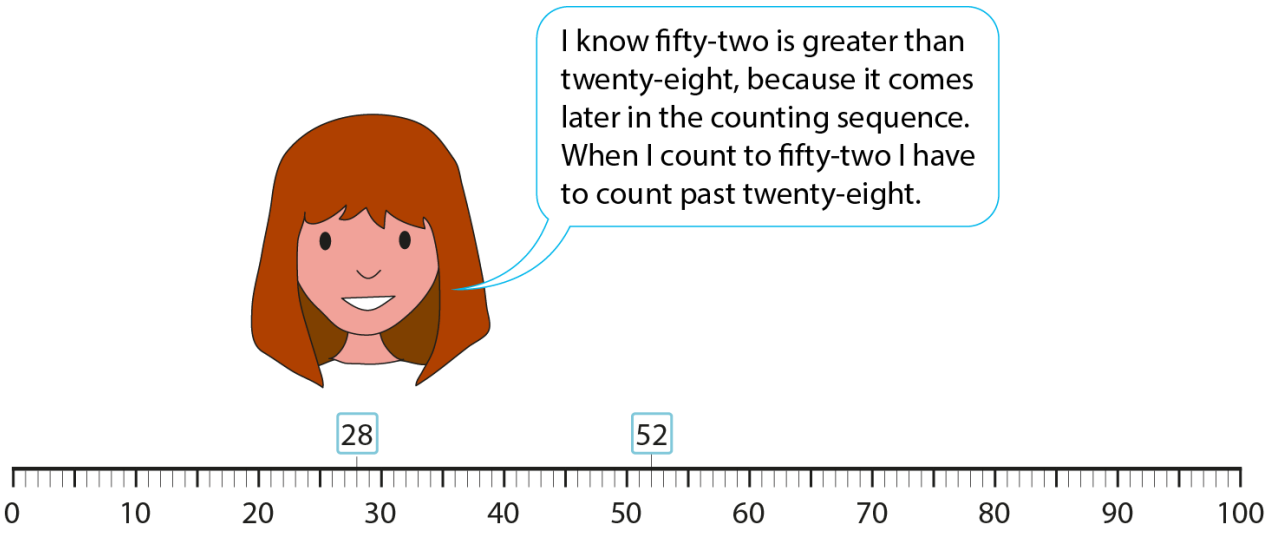
- Javed says: *'I know that fifty-two is greater than twenty-eight because fifty-two has five tens sticks, and twenty-eight only has two tens sticks.'*
- Nancy says: *'I know fifty-two is greater than twenty-eight, because it comes later in the counting sequence. When I count to fifty-two I have to count past twenty-eight.'*

These are two explanations children may use based on their experience of cardinality and ordinality during this segment. Discuss the reasons that the two children have given.

Comparison supported by cardinal representation:



Comparison supported by ordinal representation:



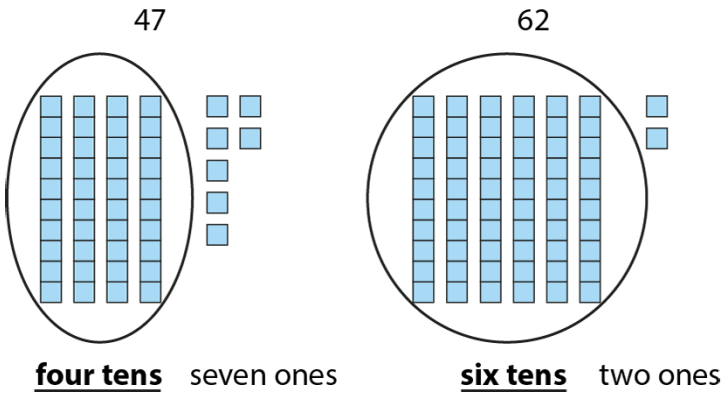
4:2

Now begin to explore how we can compare any two two-digit numbers by examining the digits, *without* having to build the numbers with Dienes or count through the numbers.

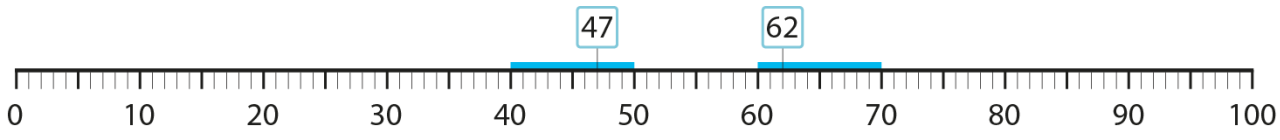
Making reference to both cardinal and ordinal representations, demonstrate how, when the tens digits are different, we can compare two-digit numbers by looking *only* at the tens:

- Begin by comparing two numbers with different tens digits.
- Then demonstrate that we can change the ones digits without affecting the comparison (remove or add ones to the Dienes representation, slide the numbers along on the number line).

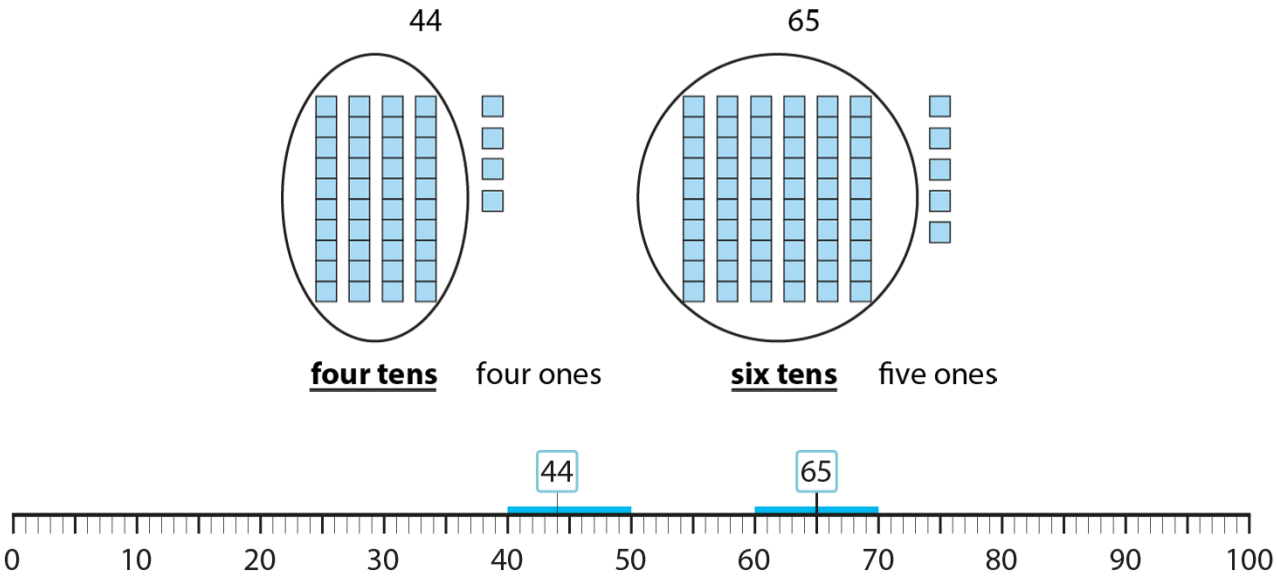
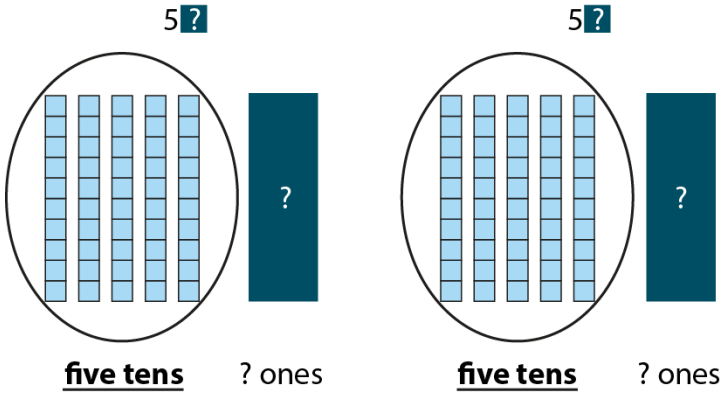
Numbers with different tens digits:

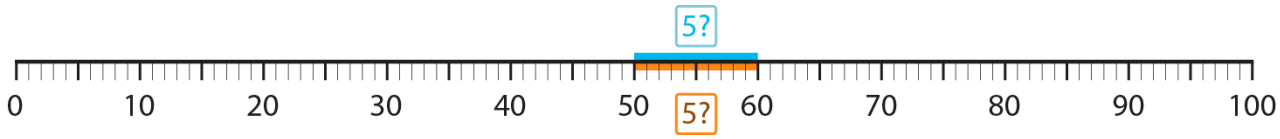


'Sixty-two is greater than forty-seven because it has more tens.'

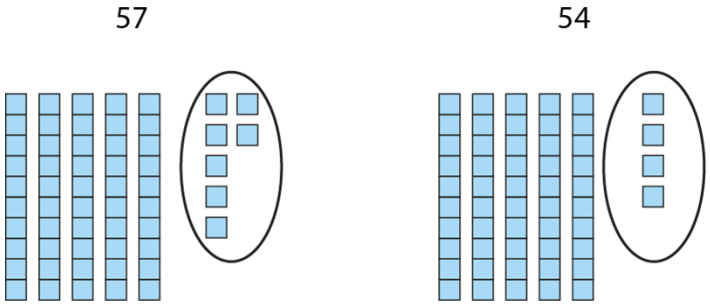


'Sixty-two is greater than forty-seven because it is further along the number line.'

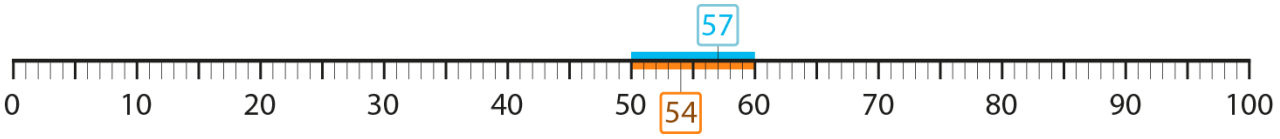
	Changing the ones digit: *'What happens if we change the number of ones?'* 
4:3	Now investigate what happens if the tens digits are the same, demonstrating that we can no longer identify which number is larger or smaller by looking only at the tens. To highlight the fact that it is no longer sufficient to look only at the tens digits, begin by hiding the ones digits – the children won't be able to say which number is greater; then reveal the ones digits to enable comparison. Make a range of comparisons – both with equal and unequal tens digits – to bring together the ideas in steps *4:2* and *4:3*.
	Numbers with equal tens digits: *'Which number is greater?'* 



'We can't tell which number is greater by looking at the tens digit.'



five tens **seven ones** five tens **four ones**



4:4

Now compare a range of numbers, looking only at the digits. You can use the >, < and = symbols introduced in segment *1.1 Comparison of quantities and measures*. After making each comparison, confirm the 'answer' using Dienes or the number line. Work towards the simplified generalised statement:

'To compare two-digit numbers, we need to compare the tens digits; if the tens digits are the same, we need to compare the ones digits.'

4:5

Provide children with varied practice. Initially they can use Dienes or a number line as a scaffold, but should progress to working only with the numbers written as numerals.

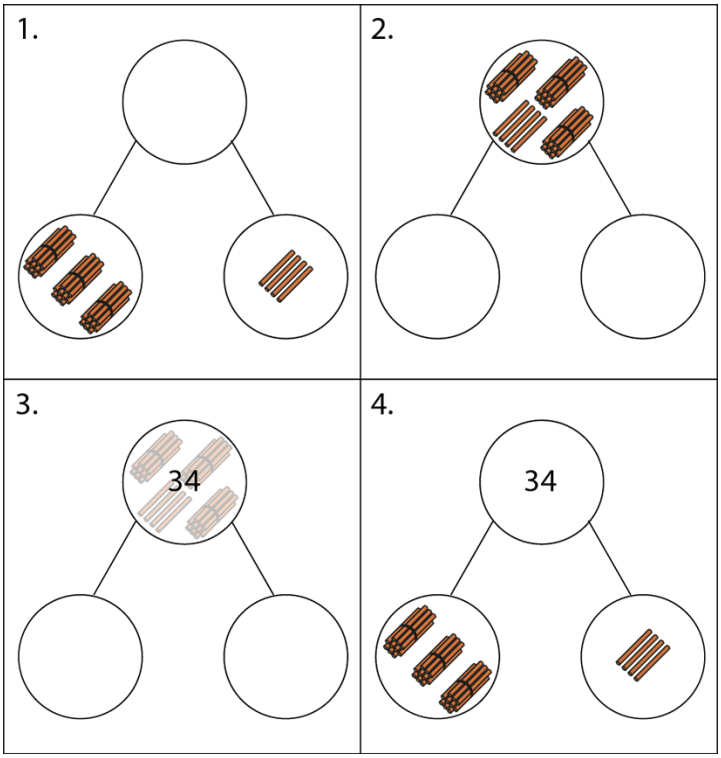
'Fill in the missing symbols.' (< > or =)

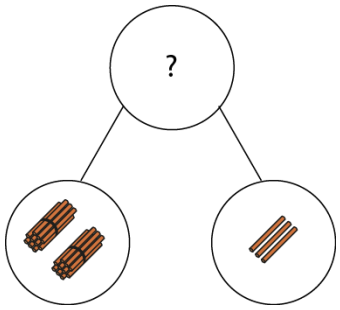
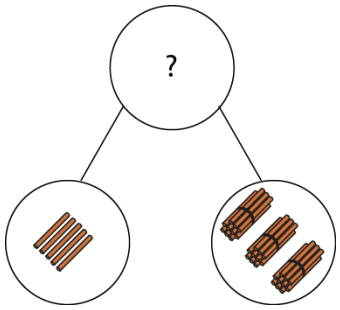
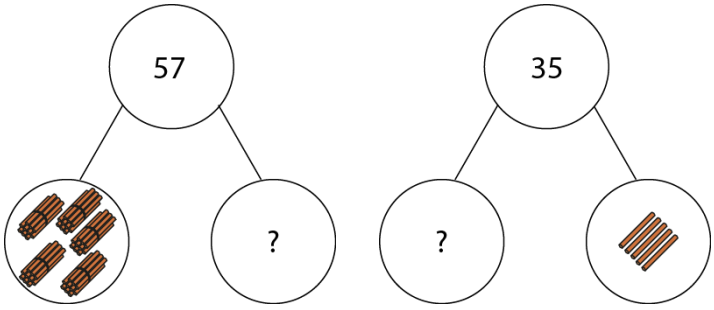
26 ◯ 28	50 ◯ 49	32 ◯ 62	51 ◯ 41	25 ◯ 35	65 ◯ 64
27 ◯ 28	50 ◯ 50	42 ◯ 62	24 ◯ 42	95 ◯ 59	38 ◯ 38
28 ◯ 28	50 ◯ 51	52 ◯ 62	40 ◯ 4	8 ◯ 70	49 ◯ 50
29 ◯ 28	50 ◯ 52	62 ◯ 62	52 ◯ 27	36 ◯ 61	45 ◯ 24
30 ◯ 28	50 ◯ 53	72 ◯ 62			

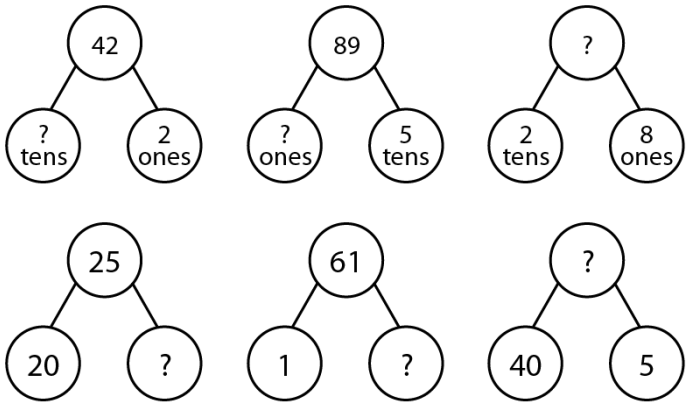
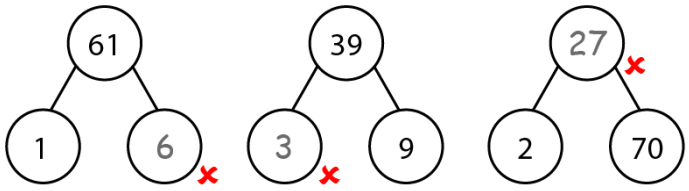
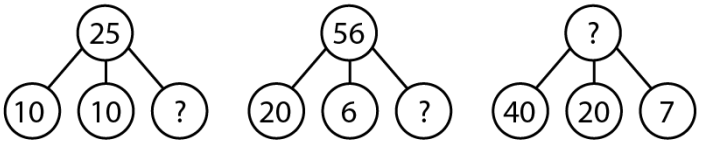
Teaching point 5:

Each two-digit number can be partitioned into a tens part and a ones part.

Steps in learning

	Guidance	Representations
5:1	*Teaching points 1–4* focused on children's understanding of two-digit numbers in the context of cardinality, ordinality and measures. Now pick up again on the tens-and-ones structure, examined in detail in *Teaching point 2*, progressing to representing this on part–part–whole diagrams. This paves the way for calculations of the form $20 + 8 = 28$ and $34 - 30 = 4$, covered in *Teaching point 6*. In step *2:6*, children were presented with a mixture of bundles of ten straws and individual straws, then asked to collect a given number of straws. Return to this activity, asking children to verbalise how they collect, say, 34 straws; draw attention to children who say you need three tens and four ones. Then model the partitioning of the straws using a part–part–whole diagram (using a similar approach to that introduced in segment *1.2 Introducing 'whole' and 'parts': part–part–whole*): • Show the tens and ones partitioned in the 'parts' circles. • Move the tens and ones together into the 'whole' circle. • Label the 'whole' with the digits. • Move the tens and ones apart again into the 'parts' circles.	
5:2	Progress to presenting missing 'whole' problems on part–part–whole diagrams (addition). When using concrete representations of the tens and ones, then move them together and apart as they are moved to/from	

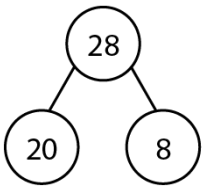
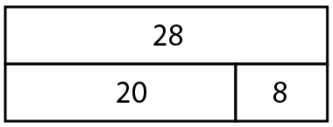
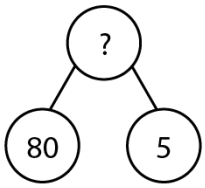
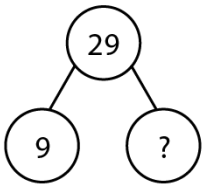
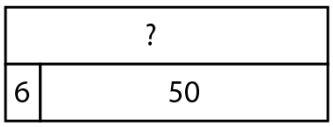
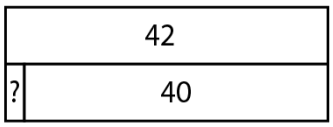
	the 'whole' circle. When using pictorial representations for the tens and ones 'parts', use numerals to represent the whole (showing an additional set of tens and ones in the 'whole' circle at the same time as showing them in the 'parts' circle is misleading as it shows double the given number of items). By now, children should have sufficient knowledge of the composition of two-digit numbers to be able to combine tens and ones. Encourage children to describe their reasoning in full sentences, using the stem sentences: • ***'There are ___ tens, which is ___, and ___ one(s), which is ___. This makes ___ altogether.'*** • ***'The ___ represents ___ tens; it has a value of ___.'*** • ***'The ___ represents ___ one(s); it has a value of ___.'*** Include some part–part–whole models where the ones are represented on the left and the tens on the right, to check that children are really thinking about the value of each digit in the number rather than just working from left to right.	 • *'There are two tens, which is twenty, and three ones, which is three. This makes twenty-three altogether: 23.'* • *'The 2 represents two tens; it has a value of twenty.'* • *'The 3 represents three ones; it has a value of three.'*  • *'There are three tens, which is thirty, and five ones, which is five. This makes thirty-five altogether: 35.'* • *'The 3 represents three tens; it has a value of thirty.'* • *'The 5 represents five ones; it has a value of five.'*
5:3	Similarly, present missing 'part' problems. Children have not yet identified missing parts in two-digit numbers, but have the building blocks in place to do so – they have been experiencing lots of partitioning and have been introduced to the partitioning structure of subtraction. Consider a whole two-digit number (for example, 57), show the tens part (for example, five tens or 50), and ask children to identify the missing part. Encourage children to reason: *'The missing part must be seven because fifty-seven is five tens (or fifty) and seven ones.'*	

	Then present a problem with the tens part missing (as in the example with 35 opposite). As before, spend time modelling and practising clear, accurate explanations.	
5:4	Once children can confidently work with the numbers represented pictorially, move on to part–part–whole models with all numbers represented as numerals. Provide children with practice completing missing 'parts' and 'wholes', varying the placement of the tens and ones 'parts', and the orientation of the diagrams. Continue to use the stem sentences that describe the value of each digit. Look out for children who make mistakes like those identified opposite; they will need to work again on the previous step – focus on emphasising the quantity that each digit represents. Once children are confident, you can present dòng nǎo jīn problems (such as those shown opposite) based on partitioning two-digit numbers into three parts, building on children's knowledge of adding multiples of ten from segment *1.8 Composition of numbers: multiples of 10 up to 100*. For now, avoid extending to partitioning a two-digit number into a multiple of ten and another two-digit number (for example, 61 into 50 and 11); this is an important skill, but will be covered in depth in segment *1.14 Addition and subtraction: two-digit numbers and multiples of ten*.	Missing part and whole problems:  Common errors:  Dòng nǎo jīn: 

Teaching point 6:

The tens and ones structure of two-digit numbers can be used to support additive calculation.

Steps in learning

	Guidance	Representations
6:1	Complete the segment by progressing from the completion of part–part–whole diagrams to the completion of equations. Look at a part–part–whole diagram (cherry or bar model) representing the decomposition of a number into tens and ones. As a class, record all of the equations which can be represented by the model. If necessary, reinforce the link between the part–part–whole diagram and the equations by using physical resources, such as sticks.	  20 + 8 = 28 28 – 20 = 8 8 + 20 = 28 28 – 8 = 20 28 = 20 + 8 8 = 28 – 20 28 = 8 + 20 20 = 28 – 8
6:2	Now present missing 'part' or 'whole' equations, supported by the corresponding missing 'part' or missing 'whole' diagrams.	  □ = 80 + 5 29 – 9 = □   6 + 50 = □ 42 – 40 = □

6:3

Now provide varied practice for the children, without the scaffold of part–part–whole diagrams.

Look out for the following particular mistakes which can indicate certain underlying misconceptions or lack of understanding:

32 = 3 + 20 ✗

Children who make this error may just have generalised a pattern of taking the digits in the order they appear, ignoring the 0, and copying them down. Address this by asking children to describe what each digit represents in their given answer (for example, *'The 3 represents three tens and the 2 represents two ones'*); then ask how many tens and ones are represented in *3 + 20*.

17 = 3 + 20 ✗

Children who make this error are likely to have overgeneralised after seeing equations of the form $a + b = c$, making the assumption that the sum always comes at the end of an equation; they have focused on adding the first and second number to get the third. Again, question children about the value of each side of the equation and whether they are the same. Emphasise that the expression on each side of the '=' symbol should represent the same value.

Missing sum/difference problems:

'Fill in the missing numbers.'

40 + 3 = ☐ ☐ = 80 + 1

30 + 4 = ☐ ☐ = 1 + 70

6 + 20 = ☐ ☐ = 3 + 20

2 + 60 = ☐ ☐ = 2 + 30

50 + 8 = ☐ ☐ = 50 + 6

80 + 5 = ☐ ☐ = 60 + 5

50 + ☐ = 58 56 = 50 + ☐

☐ + 5 = 85 65 = ☐ + 5

5 + 90 = ☐ ☐ = 4 + 70

9 + 50 = ☐ ☐ = 7 + 40

☐ + 90 = 95 74 = ☐ + 70

9 + ☐ = 59 47 = 7 + ☐

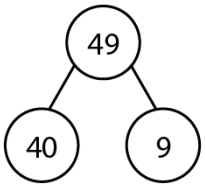
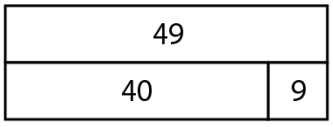
51 – 1 = ☐ ☐ = 87 – 80

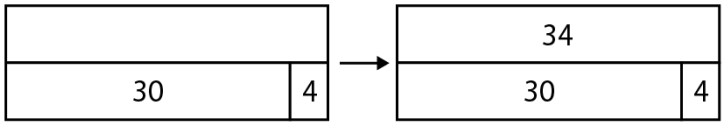
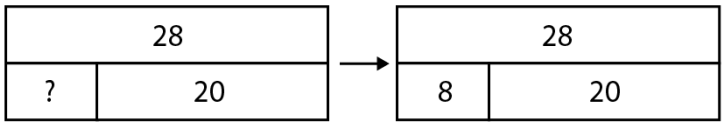
52 – 2 = ☐ ☐ = 64 – 60

53 – 3 = ☐ ☐ = 41 – 40

54 – ☐ = 50 3 = 33 – ☐

☐ – 4 = 50 6 = ☐ – 20

		25 – 20 = ☐ ☐ = 15 – 5 35 – 30 = ☐ ☐ = 58 – 8 45 – 40 = ☐ ☐ = 32 – 2 55 – ☐ = 50 40 = 46 – ☐ ☐ – 60 = 5 20 = ☐ – 1
6:4	You can link back to comparison of two-digit numbers by presenting dòng nǎo jīn problems such as those shown opposite.	*'Fill in the missing symbols.'* (<, > or =) 50 + 6 ◯ 65 17 ◯ 1 + 70 50 + 6 ◯ 56 71 ◯ 1 + 70 2 + 30 ◯ 3 + 20 40 + 6 ◯ 6 + 40 45 – 5 ◯ 56 – 6 45 – 5 ◯ 46 – 6 45 – 40 ◯ 72 – 70 45 – 40 ◯ 46 – 40
6:5	Finally, work towards children being able to solve word problems that involve adding tens and ones, including being able to draw their own part–part–whole models (cherry or bar) for support. Begin by telling a story, with the calculation answer included, and ask children to draw or complete a blank part–part–whole diagram (cherry or bar model) to represent the story. Keep the focus on the link between the story and the diagram rather than, for example, trying to encourage proportional drawings of the two parts in a bar model (the images provided opposite show precisely proportional bar models for use as a teaching tool; but children's drawings will be less precise and that is perfectly acceptable). Ask children to describe	Full story with answer: *'At first there were forty-nine grapes in a bowl. Then I ate nine of them. Now there are forty grapes left.'*   • *'The 49 represents the grapes that were in the bowl at first.'* • *'The 9 represents the grapes that were eaten.'* • *'The 40 represents the grapes that are left.'*

	what each number in the diagram represents. Now repeat, using a missing 'whole' story, and then a missing 'part' story. Encourage children to first fill in the two parts that the story tells us, then use their knowledge of number composition to find the missing number. Continue to use full sentences to relate each diagram to the story.	Missing 'whole' story: *'I have thirty maths books to mark and four English books to mark. How many books do I need to mark altogether?'*  *'You have thirty-four books to mark altogether.'* Missing 'part' story: *'I have twenty-eight pencils to sharpen. I have already sharpened twenty. How many more do I need to sharpen?'*  *'You need to sharpen eight more pencils.'*
6:6	Provide children with practice solving a range of story-based problems, including aggregation, augmentation, partitioning and reduction structures, and contexts involving measures. Examples include: • *'I had twenty conkers and then my friend gave me nine more. How many do I have now?'* (augmentation) • *'The teacher had thirty-seven pencils. He gave out thirty of them. How many did he have left?'* (reduction) • *'There are sixty-three children sharing fruit. Sixty of them like apples. How many children do not like apples?'* (partitioning) • *'I find a two-pence coin and a fifty-pence coin on the ground. How much money have I found altogether?'* (aggregation; measure) • *'I need to save forty-five pounds to buy a new bike. I already have five pounds. How much more do I need?'*	

	(augmentation with missing addend; measure) As an initial scaffold, you could provide an empty part–part–whole diagram for each question. As children gain confidence, encourage them to draw their own models to represent the stories and reveal the underlying structure.	