MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Multiplication and Division

2.30 Multiplicative contexts: area and perimeter 2

Teacher guide | Year 6

Teaching point 1:

The area of a parallelogram can be calculated by multiplying the base by the perpendicular height; all parallelograms with the same base and perpendicular height will have the same area.

Teaching point 2:

The area of a triangle can be calculated by multiplying the base by the perpendicular height and then dividing by two.

Teaching point 3:

Shapes with the same area can have different perimeters; shapes with the same perimeter can have different areas.

Teaching point 4:

When a shape has been transformed by a scale factor, the perimeter is also transformed by the same scale factor.

2019 pilot

Overview of learning

In this segment children will:

- revisit the properties of parallelograms and triangles
- build on their conceptual understanding of area and perimeter
- learn to identify the perpendicular height of parallelograms and triangles
- apply their prior knowledge of finding the area of a rectangle to finding the area of a parallelogram
- learn that a rectangle or parallelogram is made of up two triangles
- apply this knowledge to find the area of a triangle
- compare the area and the perimeter of a shape
- apply a scale factor to side-lengths, perimeter and area.

This segment builds on knowledge of area and perimeter covered in segment *2.16 Multiplicative contexts: area and perimeter 1*, when children learnt that perimeter is the distance around the edge of a two-dimensional shape and is measured in units of length, and that area is the measurement of the surface of a flat item and is measured in square units. Later in the segment children will also build on their prior knowledge of scale factors covered in segment *2.27 Scale factors, ratio and proportional reasoning*.

Teaching point 1 begins by recapping how to find the area of a rectangle and then moves on to looking at the properties of a parallelogram. Children then learn how to identify the perpendicular height of a parallelogram. It is important that children are able to correctly identify the perpendicular height of a variety of parallelograms in different sizes and orientations. Children are encouraged to use visualisation to understand how a parallelogram can be rearranged into a rectangle by moving a triangular section from one side to the other. This knowledge is then applied to finding the area of a parallelogram, and division is used to find an unknown base measurement or perpendicular height when the area is provided.

Teaching point 2 revisits the properties of different types of triangles. Children then learn how to identify the perpendicular height of a triangle. Next, two triangles are combined to make a rectangle or parallelogram and this understanding is used to find the area of a triangle. It is important that children are secure in their understanding of how to find the area of a rectangle or parallelogram so that they can make the connection that the area of a triangle will be half of this. Division is then used to find an unknown base measurement or perpendicular height when the area is provided.

Teaching point 3 compares shapes with the same perimeter but different areas, and with the same area but different perimeters. *Teaching point 4* revisits scale factors and applies them to perimeter and area as well as side-lengths.

Children should be encouraged to use the correct units of measurement throughout.

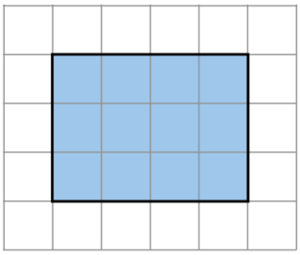
Note: in representations, measurements have been drawn proportionally correct but scaled to fit the available space.

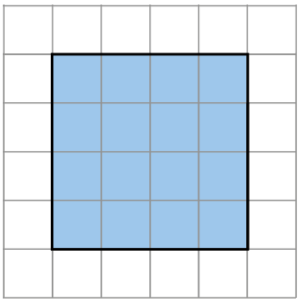
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Teaching point 1:

The area of a parallelogram can be calculated by multiplying the base by the perpendicular height; all parallelograms with the same base and perpendicular height will have the same area.

Steps in learning

<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr><td>1:1</td><td>In this teaching point children will build on their knowledge of how to find the area of rectangles and rectilinear shapes, covered in segment 2.16: Multiplicative contexts: area and perimeter 1. To re-familiarise children with finding the area of a rectangle and with the method of counting square units to find the area of shapes, show children examples of rectangles on square grids and ask questions such as:
• 'What is the area of each shape?'
• 'What methods could you use to find the area?'
• 'Which shape has the larger area?'
Remind children of the generalisation introduced in segment 2.16, Teaching point 6: 'To find the area of a rectangle multiply the length by the width.'</td><td>Finding the area of rectangles on square grids:
Shape A

• 'The area is twelve square units.'
• 'You could count the squares.'
• 'You could multiply the length by the width.'
$3 \times 4 = 12$
• 'You could multiply the width by the length.'
$4 \times 3 = 12$

Shape B

• 'The area is sixteen square units. This shape has a larger area than Shape A.'
• 'You could count the squares.'
• 'You could multiply the length by the width.'
$4 \times 4 = 16$</td></tr>
</table>

1:2

This step will focus on the properties of parallelograms. Parallelograms can be created by using two pieces of translucent ribbon that have parallel sides. Whenever the two pieces of ribbon are rotated and overlaid, the overlapping shape will always be a parallelogram.

Provide children with pieces of translucent ribbon and ask them to create parallelograms of different sizes. Ask children to identify the properties of parallelograms, drawing out that parallelograms are quadrilaterals where:

- opposite sides are parallel
- opposite sides are equal in length
- opposite angles are equal.

Show children a variety of quadrilaterals, including parallelograms, and ask them:

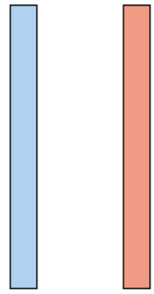
- *'Is this shape a parallelogram?'*
- *'How do you know?'*

Encourage children to refer to the properties of parallelograms listed above.

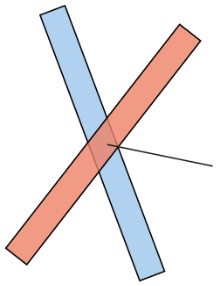
Some children may confuse parallelograms with other shapes. If such confusion arises, work through the properties of a parallelogram one by one, to identify which criteria are not met by the other shapes.

Children should also be able to identify that rhombuses, squares and rectangles are all parallelograms. Explain that a shape can fit into more than one category, for example, it can be both a square and a parallelogram. Look at examples of rhombuses, squares and rectangles, and use the following stem sentence: ***'A ___ is a parallelogram because ___.'***

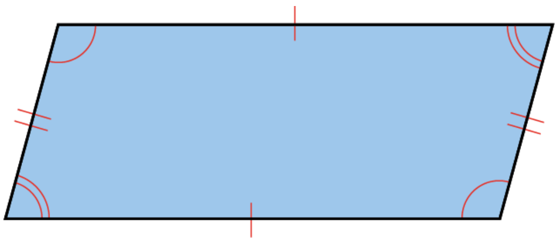
Two pieces of translucent ribbon with parallel sides:



Rotate and overlay the ribbons – the overlapping shape is always a parallelogram:

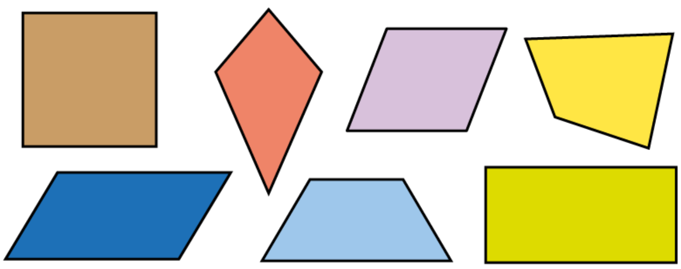


The properties of a parallelogram:

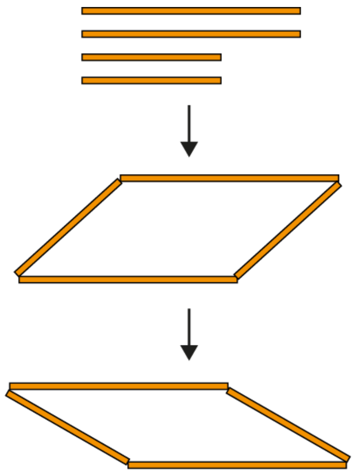
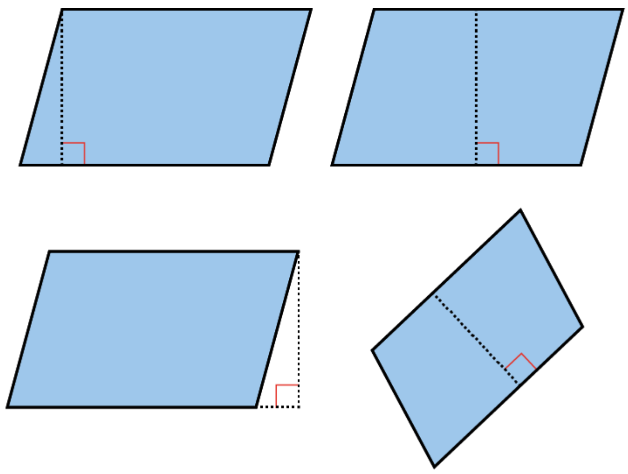
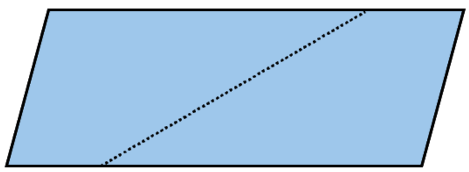


- Opposite sides are parallel.
- Opposite sides are equal in length.
- Opposite angles are equal.

- *'Is this a parallelogram?'*
- *'How do you know?'*



- *'A square is a parallelogram because it has opposite sides that are parallel and equal in length.'*

	Next provide children with four straws or sticks of two different lengths. Ask them to create a variety of parallelograms. By the end of this step children should be comfortable with the following generalisation: ***'A parallelogram is a quadrilateral with opposite sides that are parallel and equal in length.'***	Using straws to create parallelograms: 
1:3	Next, focus on the perpendicular height of a parallelogram. Show children a parallelogram and draw a vertical line at 90° to the base. Explain to children that this is called the *'perpendicular height'*. Draw the perpendicular height in a variety of places, including examples where the line is drawn outside the shape. Now rotate the parallelogram to show that the perpendicular height is drawn from the base of the shape, and not necessarily from the horizontal plane. Include examples of lines that are *not* the perpendicular height, for comparison. Remind children that the line must be at 90° to the base of the shape.	The perpendicular height of parallelograms:  A line that is not the perpendicular height: 

1:4

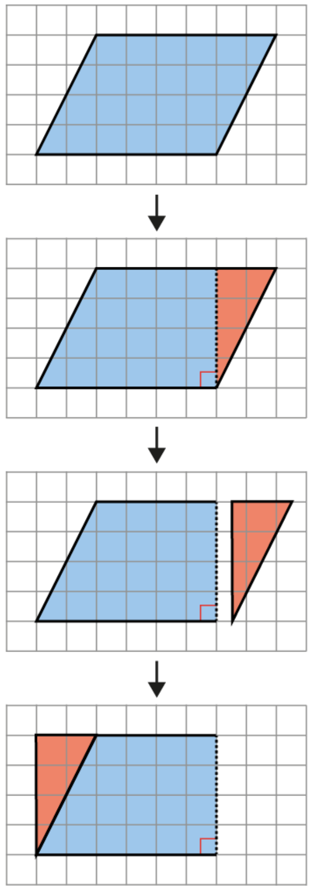
Now use a transparent square grid to show how you can change a parallelogram into a rectangle. Cut a parallelogram out of paper and place it under the transparent grid. Encourage children to notice that the parallelogram is made up of one rectangle and two triangles. Next show children that if we cut off one of the triangles, we can move it to the other side of the shape to make a new rectangle.

Provide children with a variety of paper parallelograms that fit under the grid to make a rectangle with sides that measure in whole units. Ask children to identify the two triangles, and then to cut and move one of the triangles to create a new rectangle.

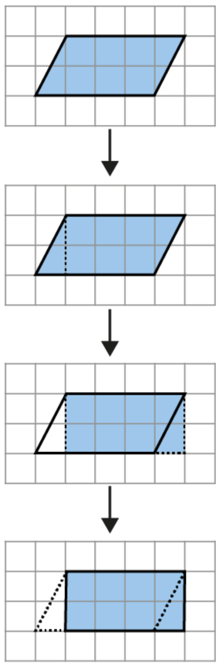
Once children are confident with this strategy, progress to drawing lines to indicate the movement of the triangle, rather than cutting it. Children will need to visualise the new rectangle rather than physically creating it. A firm understanding of the previous stage is therefore essential.

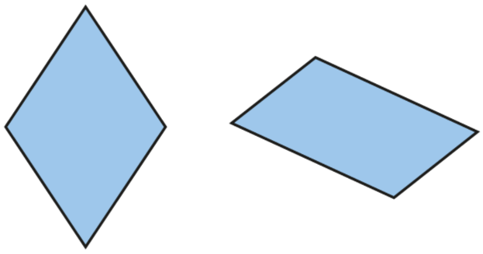
Work towards the following generalisation: ***'A parallelogram can be made into a rectangle that has the same area.'***

Changing a parallelogram into a rectangle by cutting shapes:



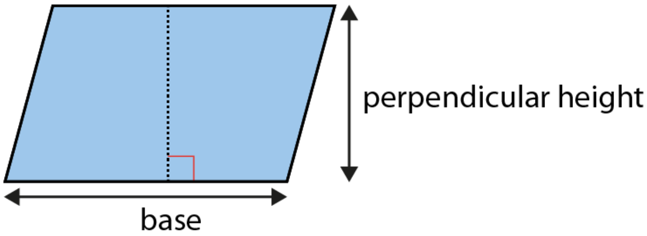
Changing a parallelogram into a rectangle by drawing lines:



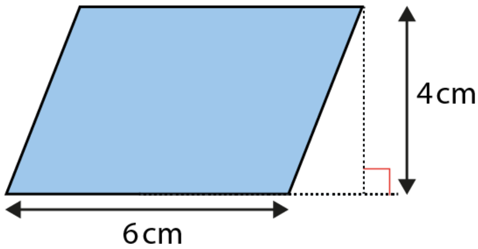
<table>
<tr><td></td><td></td><td>'Draw a line on each shape to show where you could cut it so that the two pieces of the shape could be joined together to make a rectangle.'
</td></tr>
<tr><td>1:5</td><td>Now build on step 1:4 to calculate the area of a parallelogram. Remind children of the generalisation from the previous step, using the visual reminder that we can make both a parallelogram and a rectangle using the same piece of paper, without adding or removing anything.

Children already know that they can find the area of a rectangle by multiplying the length by the width (recapped in step 1:1). Demonstrate that the perpendicular height of a parallelogram is the same as the length of the equivalent rectangle, and the base of the parallelogram is the same as the width of the equivalent rectangle. The same method can therefore be used to find the area of a parallelogram as is used to find the area of a rectangle: multiplying the base by the perpendicular height.

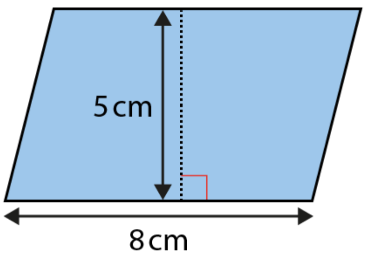
Work through several examples using the following stem sentences:
• 'The base is ___.'
• 'The perpendicular height is ___.'
• 'The area is ___.'

Include a variety of examples where the perpendicular height is marked inside or outside the shape. Remind children that the base and the perpendicular height must form a right angle.</td><td>Finding the area of a parallelogram:


• 'To find the area of a parallelogram multiply the base by the perpendicular height.'

• 'Find the area of this parallelogram.'


• 'The base is 6 cm.'
• 'The perpendicular height is 4 cm.'
• 'The area is $6 \times 4 = 24$ cm^2.'

• 'Find the area of this parallelogram.'


• 'The base is 8 cm.'
• 'The perpendicular height is 5 cm.'
• 'The area is $8 \times 5 = 40$ cm^2.'</td></tr>
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<td></td>
<td>Ensure children are able to identify:
which measurement is the base
which measurement is the perpendicular height.
Remind children that area is measured in square units, such as square centimetres (cm^2) or square metres (m^2).

By the end of this step children should be confident with the following generalisation: 'To find the area of a parallelogram multiply the base by the perpendicular height.'</td>
<td></td>
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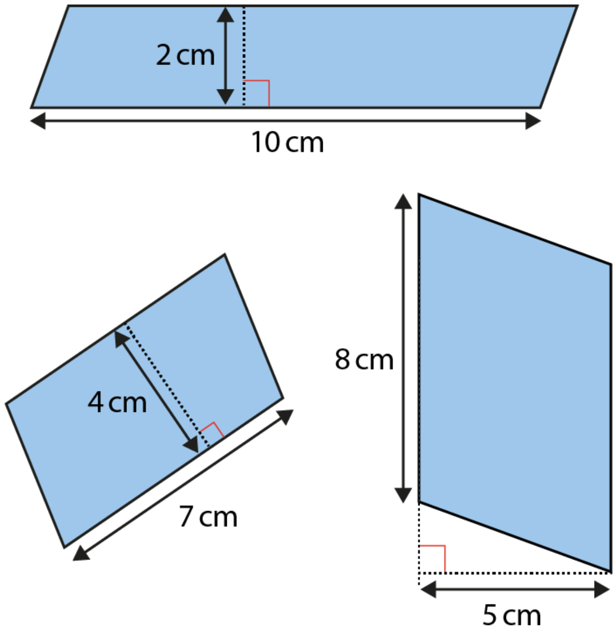
<tr>
<td>1:6</td>
<td>Provide children with intelligent practice finding the area of parallelograms when the base and perpendicular height are provided. Include parallelograms in different orientations to deepen children's understanding of how to identify the base and the perpendicular height.

You may also like to explore the idea that all parallelograms with the same base and height will have the same area.</td>
<td>'Find the area of these parallelograms.'


'A parallelogram has a base of 10 cm and a perpendicular height of 4 cm.'
'What is the area of this parallelogram?'
'Draw as many variations of this parallelogram as you can.'</td>
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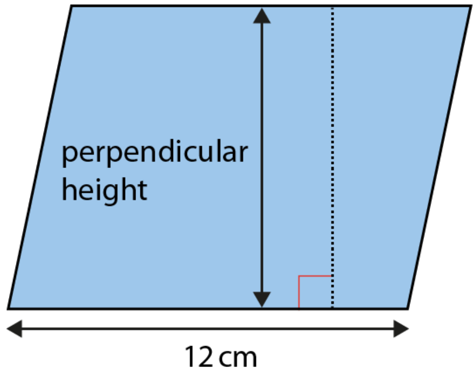
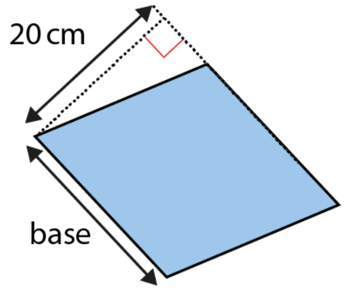
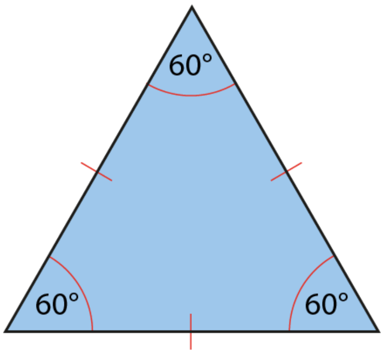
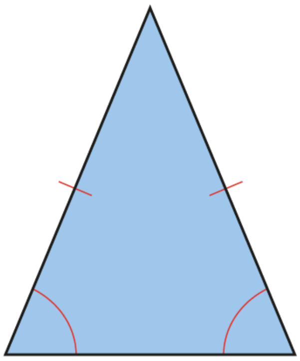
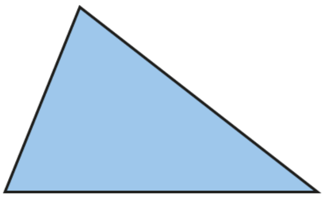
</table>

1:7	Next develop this knowledge to find unknown measurements. This builds on the teaching in segment *2.16 Multiplicative contexts: area and perimeter 1*, step *6:7*, where children found unknown side-lengths of rectangles. Show children a parallelogram with the area and the base provided, and ask them how they could work out the perpendicular height. Guide them towards the conclusion that they could use division to find the unknown height: area ÷ base = perpendicular height Work through another example where the area and the perpendicular height are provided, but the base is missing: area ÷ perpendicular height = base	• *'The area of this parallelogram is 108 cm². What is the perpendicular height?'*  area ÷ base = perpendicular height 108 ÷ 12 = 9 • *'The perpendicular height is 9 cm.'* • *'The area of this parallelogram is 440 cm², what is the length of the base?'*  area ÷ perpendicular height = base 440 ÷ 20 = 22 • *'The base is 22 cm.'* Dòng nǎo jīn: *'Jacob's lawn is in the shape of a parallelogram. He has bought grass seed that costs £0.24 per square metre. He has spent £48.00.'* • *'Simon says that the lawn could be a square with sides that are 20 m in length.'* • *'Sean says that the lawn could be a parallelogram with base 100 cm in length.'* • *'Suzie says that the lawn could be a parallelogram with perpendicular height 2,000 cm in length.'* *'Who do you agree with? Who do you disagree with?'*

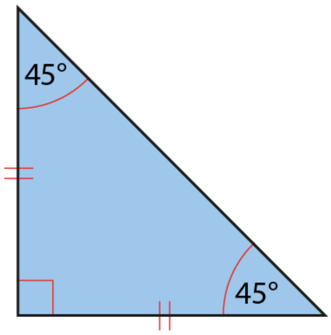
Teaching point 2:

The area of a triangle can be calculated by multiplying the base by the perpendicular height and then dividing by two.

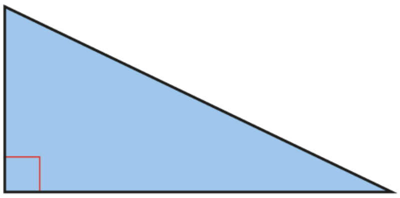
Steps in learning

	Guidance	Representations
2:1	To start this teaching point, ask children what they know about triangles, drawing out the following properties: A triangle has: • three sides • three angles, which always add up to 180°. Next explore the properties of different types of triangles, using images to illustrate the various properties. Show children a variety of triangles and ask them to categorise each one using the appropriate language from the definitions opposite and on the next page. By the end of this step children should be confident with the following generalisation: ***'A triangle is a 2D shape with three sides and three angles. It can be classified by the length of its sides and the size of its angles.'***	The properties of triangles: • *'Equilateral triangles have three equal sides and three equal angles.'*  • *'Isosceles triangles have two equal sides and two equal angles.'*  • *'Scalene triangles have no equal sides and no equal angles.'* 

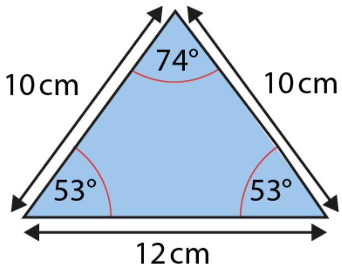
- *'Right-angled isosceles triangles have two equal sides, two equal angles and a right angle.'*



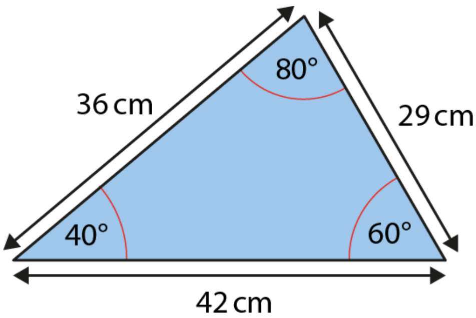
- *'Right-angled scalene triangles have no equal sides, no equal angles and a right angle.'*



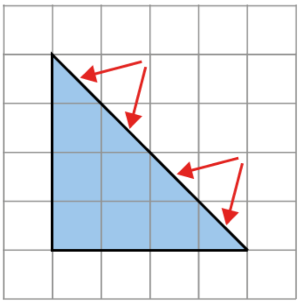
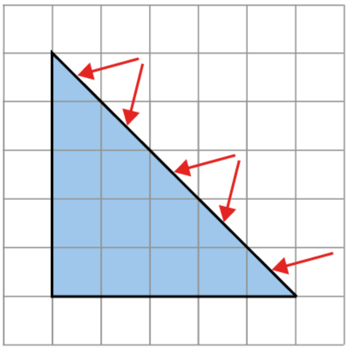
Identifying types of triangles:



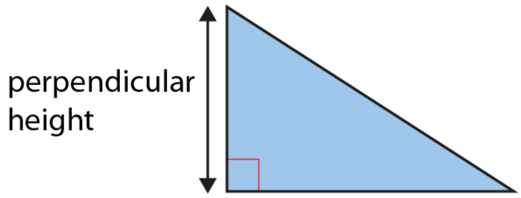
- *'This is an isoscles triangle because it has two equal sides and two equal angles.'*

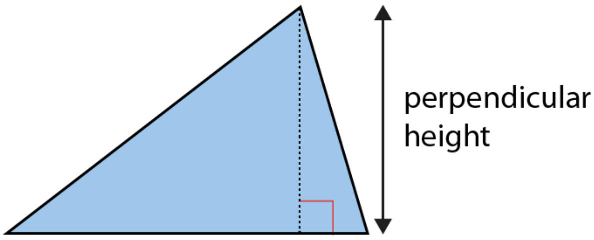


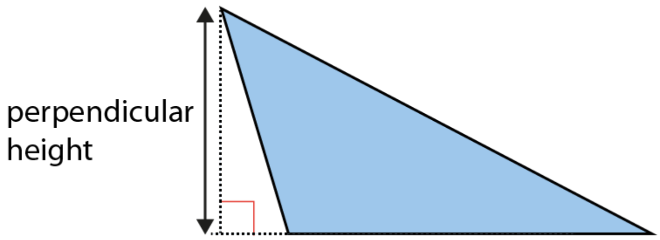
- *'This is a scalene triangle because it has no equal sides and no equal angles.'*

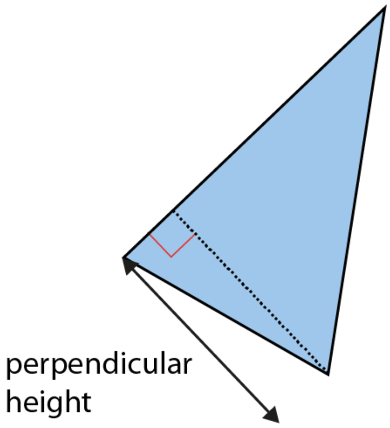
2:2	In segment *2.16 Multiplicative contexts: area and perimeter 1*, step *4:5*, children learnt that square grids can be used to find the area of shapes by combining partial squares and then counting the squares. To recap this, show children an example of a right-angled triangle placed under a transparent grid, and demonstrate that the half squares can be combined to make whole squares and then counted. Use the stem sentence: ***'The area is ___ square units.'*** Work towards the following generalisation: ***'We can count squares to find the area of a triangle.'***	*'Find the area of each triangle.'*  • *'The area is eight square units.'*  • *'The area is twelve and a half square units.'*
2:3	Next we will focus on the perpendicular height of a triangle. This builds on step *1:3* where we looked at the perpendicular height of a parallelogram. Explain that the perpendicular height is a vertical line drawn at 90° to the base, and extending to the highest point of the triangle. Draw the perpendicular height on a variety of triangles, and include examples where the line is drawn outside the shape. Include rotated triangles to show that the perpendicular height is drawn from the base of the shape, and not necessarily from the horizontal plane, and that any of the sides can be used as the base. Include examples of lines that are *not* the perpendicular height, for comparison. Remind children that the line must be at 90° to the base.	

The perpendicular height of triangles:

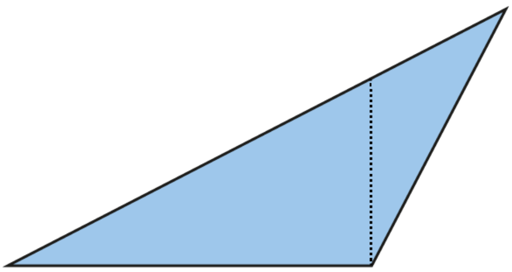


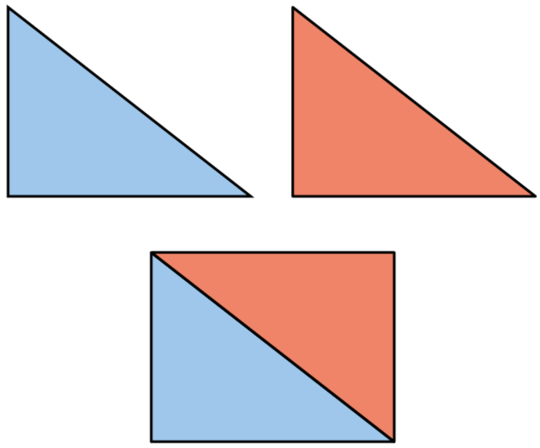
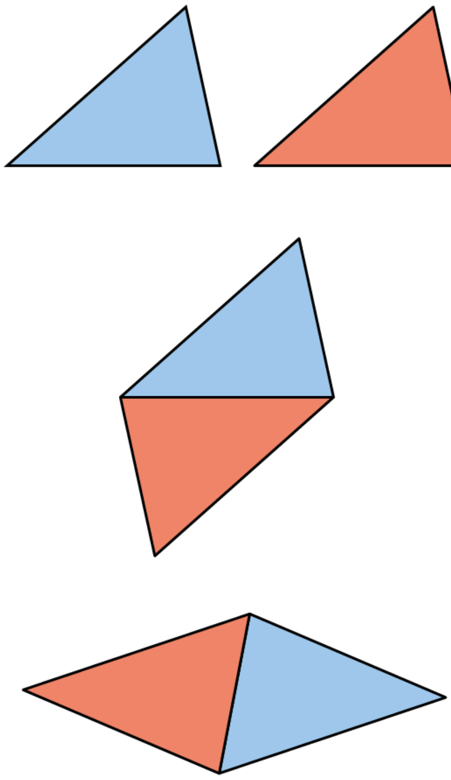






A line that is not the perpendicular height:



2:4	Now show children two right-angled triangles of different colours. Rotate one of them so that the two triangles can be placed together to make a rectangle. Next provide children with paper rectangles and ask them to cut them in half to make two right-angled triangles. Work towards the following generalisations: • ***'Two right-angled triangles that are the same can be joined to make a rectangle.'*** • ***'A rectangle can be divided into two right-angled triangles.'***	Combining two triangles to make a rectangle: 
2:5	Repeat the step above with non-right-angled triangles. Rotate the two triangles in different ways to make different parallelograms. Now provide children with squared paper and scissors, and support them in drawing and cutting out two identical triangles. Encourage them to combine their two identical triangles to make a range of parallelograms. You could also provide children with paper parallelograms to cut in half to see that two triangles can be made. Work towards the following generalisations: • ***'Two triangles that are the same can be joined to make a parallelogram.'*** • ***'A parallelogram can be divided into two triangles.'***	Combining two triangles to make a parallelogram: 

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<td>**2:6**</td>
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Next, explore how to find the area of any triangle. First remind children how to find the area of a parallelogram (step *1:5*): ***'To find the area of a parallelogram multiply the base by the perpendicular height.'***

Then remind children that a parallelogram is comprised of two triangles (step *2:5*):

- ***'Two triangles that are the same can be joined to make a parallelogram.'***
- ***'A parallelogram can be divided into two triangles.'***

Therefore, to find the area of a triangle multiply the base of the triangle by the perpendicular height and then divide by two:

area = base × perpendicular height ÷ 2

Work through several examples using the following stem sentences:

- ***'The base is ___.'***
- ***'The perpendicular height is ___.'***
- ***'The area is ___.'***

Include examples where the perpendicular height is marked inside or outside the shape. Remind children that the base and the perpendicular height must form a right angle.

Ensure children continue to use square units when measuring area. Remind them that when they are calculating area, they are finding the number of squares needed to cover the area.

Work towards the following generalisation: ***'To find the area of a triangle multiply the base by the perpendicular height and then divide by two.'***

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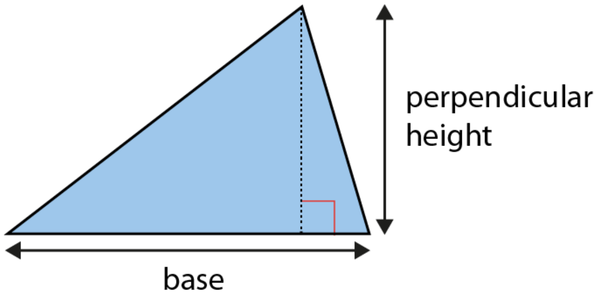
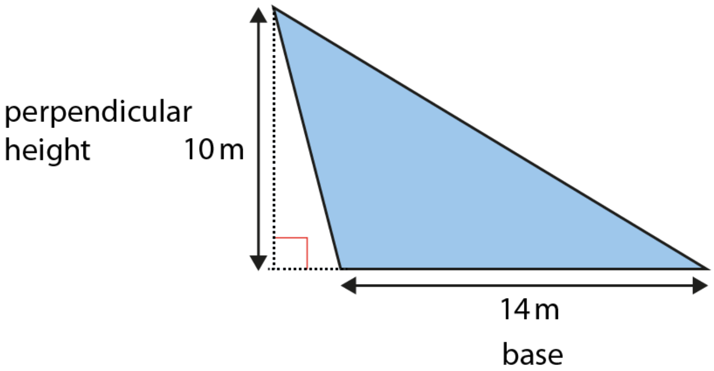
Finding the area of a triangle:



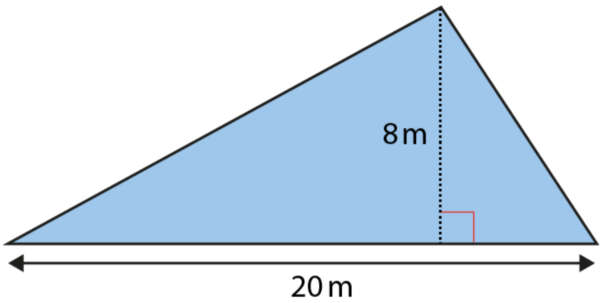
area = base × perpendicular height ÷ 2

- *'Find the area of this triangle.'*



- *'The base is 14 m.'*
- *'The perpendicular height is 10 m.'*
- *'The area is* 14 × 10 ÷ 2 = 140 ÷ 2 = 70 m²*.'*

- *'Find the area of this triangle.'*



- *'The base is 20 m.'*
- *'The perpendicular height is 8 m.'*
- *'The area is* 20 × 8 ÷ 2 = 160 ÷ 2 = 80 m²*.'*

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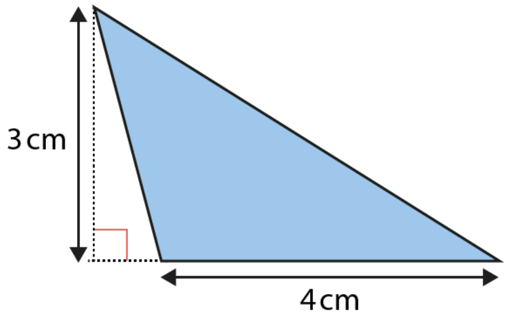
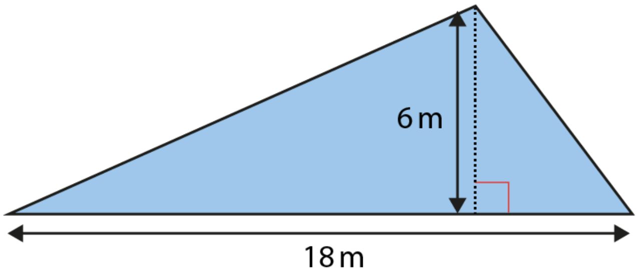
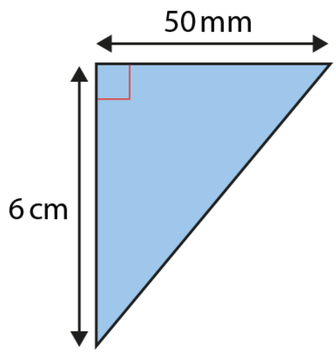
<table>
<tr>
<td>2:7</td>
<td>At this point provide children with intelligent practice finding the area of triangles when the base and perpendicular height are provided. Include triangles in different orientations to deepen children’s understanding of how to identify the base and the perpendicular height. Ensure children refer to the correct units of measurement throughout.</td>
<td>‘Find the area of these triangles.’







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<tr>
<td>2:8</td>
<td>Next develop this knowledge to find unknown measurements of triangles (refer to step 1:7). Show children a triangle with the area and the base provided, and work through the following steps to show how the unknown perpendicular height could be found. In the below steps, the first triangle on the next page is used.

We know that:

area of the triangle × 2 = area of the rectangle</td>
<td></td>
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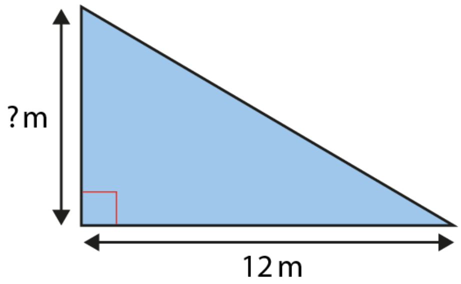
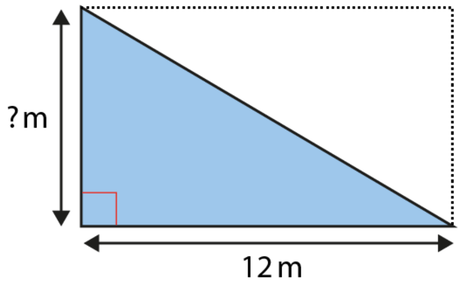
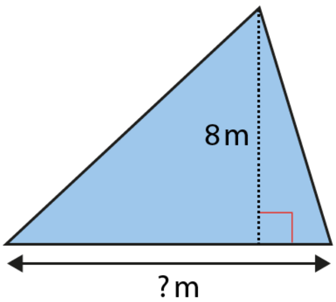
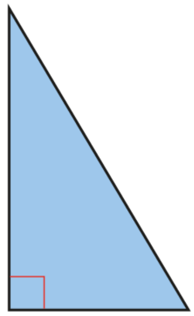
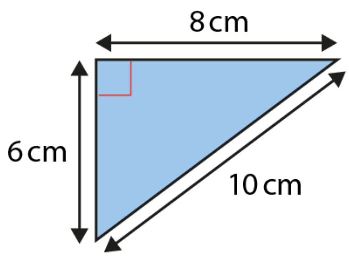
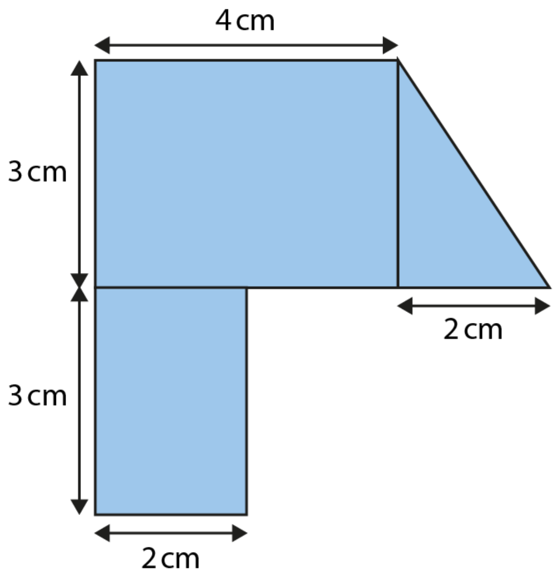
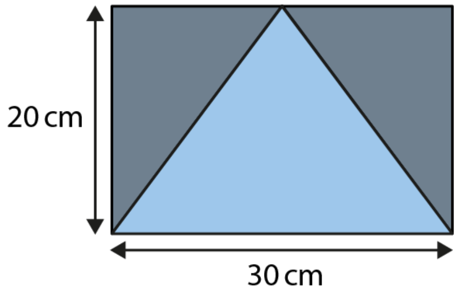
	Therefore if we make this triangle into a rectangle we know that: area of the rectangle $= 42\ \text{m}^2 \times 2$ $= 84\ \text{m}^2$ Now that we know the area of the rectangle is 84 m² and the length is 12 m (the base of the triangle), the height of the rectangle (and the perpendicular height of the triangle) can be found by dividing the area by the base: $84 \div 12 = 7$ We can confirm this by rearranging the calculation to: $12 \times 7 = 84$ Therefore the perpendicular height of the triangle is 7 m. Now summarise: to find an unknown base or perpendicular height of a triangle, multiply the area by two and divide by the known base or perpendicular height: area × 2 ÷ base = perpendicular height area × 2 ÷ perpendicular height = base Work through another example where the area and the perpendicular height are provided but the base is unknown. Remind children that the area can be doubled to make a rectangle or a parallelogram.	Finding unknown measurements: • *'The area of this triangle is 42 m². What is the perpendicular height?'*   $42\ \text{m}^2 \times 2 = 84\ \text{m}^2$ $84 \div 12 = 7$ • *'The perpendicular height of the triangle is 7 m.'* • *'The area of this triangle is 44 m². How long is the base of the triangle?'*  $44\ \text{m}^2 \times 2 = 88\ \text{m}^2$ $88 \div 8 = 11$ • *'The base of the triangle is 11 m.'*

		Dòng nǎo jīn: *'The area of this triangle is 54 cm². What could the base and height be?'* 
2:9	Provide children with varied practice to deepen their understanding. Dòng nǎo jīn: *'True or false: If I know the lengths of two of the sides of a triangle, I can calculate the area.'*	• *'Sonnie says the area of this triangle is 40 cm².'* • *'Brittany says the area of the triangle is 30 cm².'* • *'Tom says the area of the triangle is 24 cm².'* *'Who is right? Explain your answer.'*  Dòng nǎo jīn: • *'What is the total area of this shape?'* 

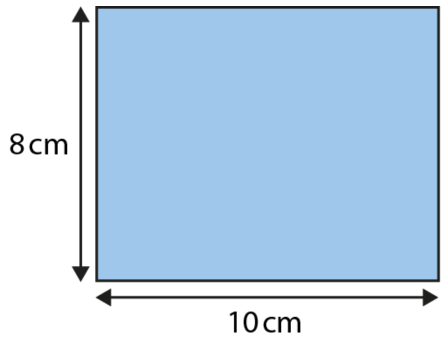
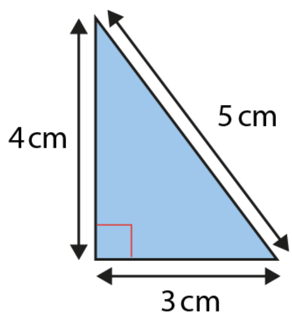
		• *'What is the total area of the shaded sections?'*

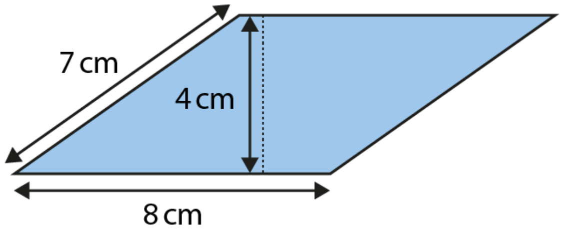


Teaching point 3:

Shapes with the same area can have different perimeters; shapes with the same perimeter can have different areas.

Steps in learning

	Guidance	**Representations**
3:1	Begin by recapping the concept of perimeter covered in segment *2.16 Multiplicative contexts: area and perimeter 1, Teaching point 1*. Show children a variety of shapes and use the following stem sentence to describe the perimeters: ***'The distance around the edge of the ___ is its perimeter.'*** Work as a class to find perimeters of rectangles, triangles and parallelograms using addition and multiplication strategies (refer to segment *2.16* if needed). Now ask children to find the areas of the same shapes, using the strategies covered in *Teaching points 1* and *2*. Remind children that perimeter is measured in units of length (segment *2.16, Teaching point 2*) and that area is measured in square units, such as square centimetres (cm^2) or square metres (m^2) (current segment, step *1:5*). Also challenge children to find the area and perimeter of irregular shapes.	Comparing perimeter and area:  • *'The distance around the edge of the rectangle is its perimeter.'* perimeter $= (10 \times 2) + (8 \times 2) = 20 + 16 = 36$ cm area $= 10 \times 8 = 80\ cm^2$  • *'The distance around the edge of the triangle is its perimeter.'* perimeter $= 3 + 4 + 5 = 12$ cm area $= 4 \times 3 \div 2 = 6\ cm^2$

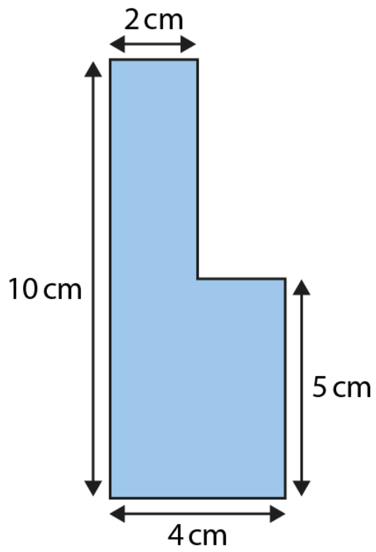


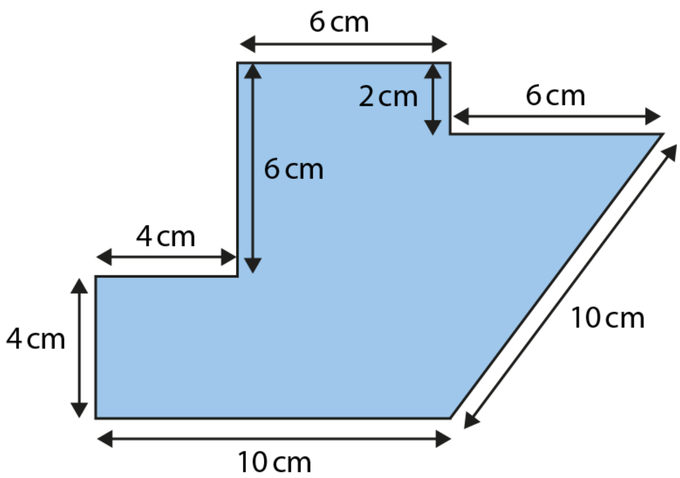
- *'The distance around the edge of the parallelogram is its perimeter.'*

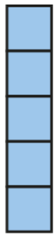
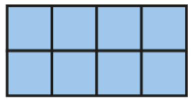
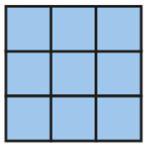
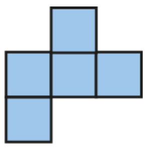
  perimeter $= (7 \times 2) + (8 \times 2) = 14 + 16 = 30$ cm

  area $= 8 \times 4 = 32$ cm^2

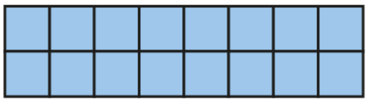
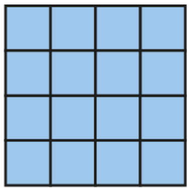
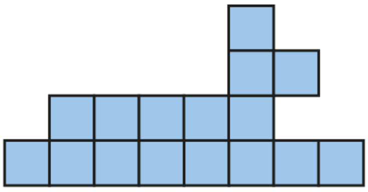
- *'Find the area and perimeter of each shape.'*





3:2	Next explore the fact that shapes with the same perimeter can have different areas. For this step you could use matchsticks or counting sticks to create rectangles and rectilinear shapes. This will enable children to see that a given perimeter is conserved when the shape is changed (because the same number of sticks are used). You may also like to provide children with a square grid to assist with creating rectangular and rectilinear shapes. Present children with a range of rectangular and rectilinear shapes that all have the same perimeter but different areas. Ask children: • *'What's the same?'* (the perimeter) • *'What's different?'* (the area) Now provide each child with 16 matchsticks or counting sticks and ask them to create as many rectilinear shapes as they can with a perimeter of 16 units. Ask them to write down the area of each shape they create. Work towards the following generalisation: ***'Shapes can have the same perimeter but different areas.'***	Same perimeter, different area:  perimeter = 12 units area = 5 units^2  perimeter = 12 units area = 8 units^2  perimeter = 12 units area = 9 units^2  perimeter = 12 units area = 5 units^2

3:3	Complete the first part of this teaching point with some intelligent practice.
	'A farmer has twenty pieces of 1 m perimeter fencing to make a pen for his chickens.'
	• *'What could the chicken pen look like? Draw some possible shapes.'*
	• *'The more square units there are, the more chickens the farmer can keep.'*
	    • *'How should he arrange the fencing to home the largest number of chickens?'*
	    • *'How should he arrange the fencing to home the smallest number of chickens?'*	

3:4	Next build on steps *3:1–3:3* to explore the fact that shapes with the same area can have different perimeters. For this step it will be useful to provide children with a square grid or a quantity of square shapes or counters to assist with creating rectangular and rectilinear shapes. Present children with a range of rectangular and rectilinear shapes that all have the same area but different perimeters. Ask children: • *'What's the same?'* (the area) • *'What's different?'* (the perimeter) Now provide each child with 12 square tiles and ask them to create as many rectilinear shapes as they can with an area of 12 units. Ask them to write down the perimeter of each shape they create. Work towards the following generalisation: ***'Shapes can have the same area but different perimeters.'***	Same area, different perimeter:  perimeter: 34 units area = 16 units^2  perimeter: 20 units area = 16 units^2  perimeter: 16 units area = 16 units^2  perimeter: 26 units area = 16 units^2

3:5	To complete this teaching point, provide children with practice that links to their knowledge of factors, and prime and square numbers (covered in segment *2.21 Factors, multiples, prime numbers and composite numbers*). Children could use matchsticks or squared paper to facilitate their working out.
	• *'Draw the rectangles that can be made with the following areas. Find all of the possibilities for each one.'*
	  • 3 units^2
	  • 7 units^2
	  • 9 units^2
	  • 12 units^2
	  • 16 units^2
	  • 25 units^2
	• Dòng nǎo jīn: *'If one child needs 1 m^2 of space in a swimming pool, what are all the possible rectilinear swimming pool shapes that could be built for twelve children?'*	

Teaching point 4:

When a shape has been transformed by a scale factor, the perimeter is also transformed by the same scale factor.

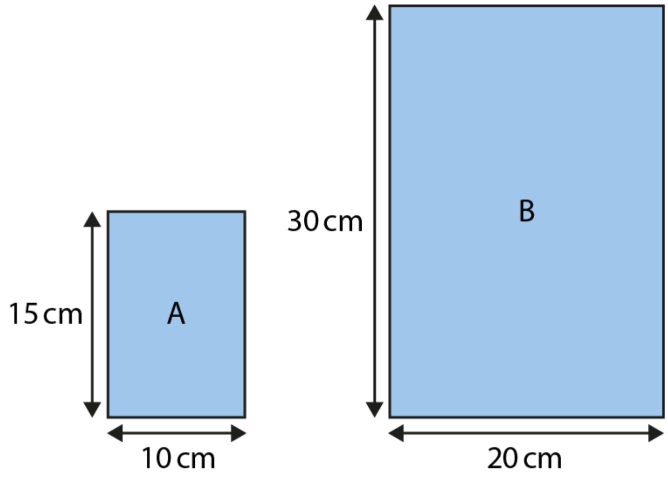
Steps in learning

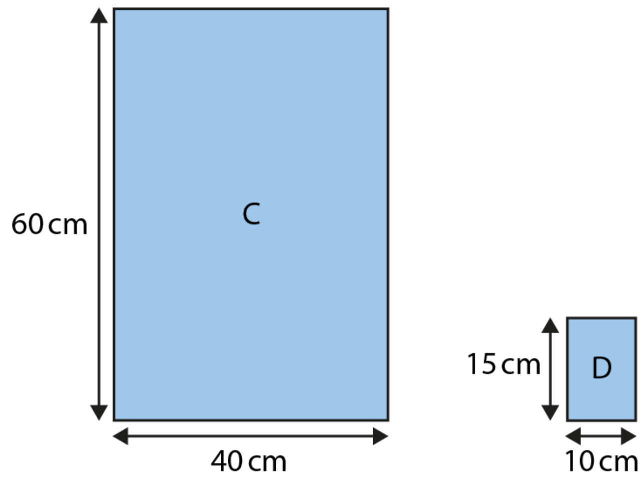
<table>
<tr><th></th><th>Guidance</th><th>Representations</th></tr>
<tr><td>4:1</td><td>
To begin this teaching point recap scale factors covered in segment 2.27 Scale factors, ratio and proportional reasoning.
Show children a shape that has been transformed by a scale factor of two. Ask:

'What's the same?'
(shape)
'What's different?'
(side-lengths, perimeter, area)

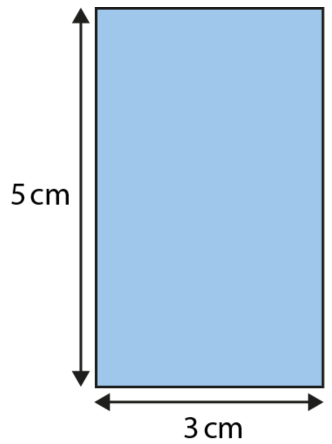
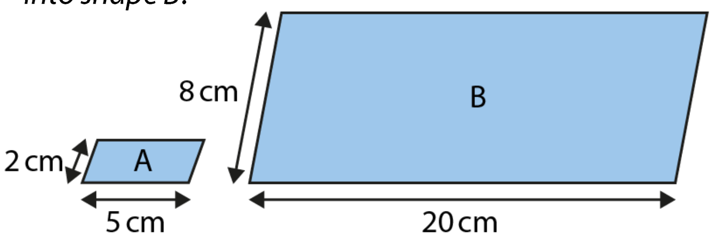
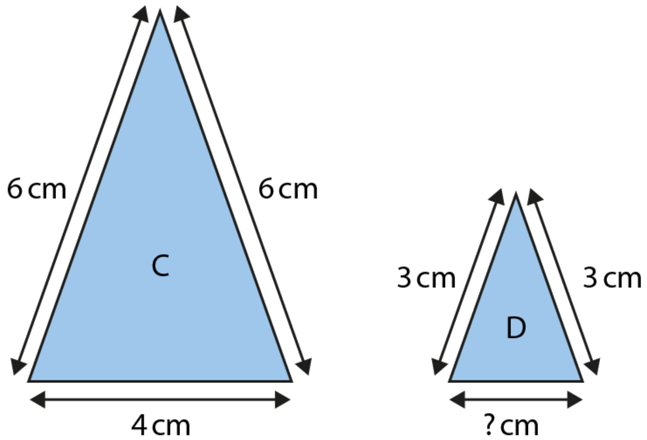
Draw attention to the fact that the side-lengths of the larger shape are double the side-lengths of the original shape. The length of each side has been multiplied by two. Describe the transformation using the following stem sentence (segment 2.27, step 4:2): 'To change shape ___ into shape ___, scale the side-lengths by a scale factor of ___.'
Repeat with a second shape that has been transformed by a scale factor of one-quarter. Ask:

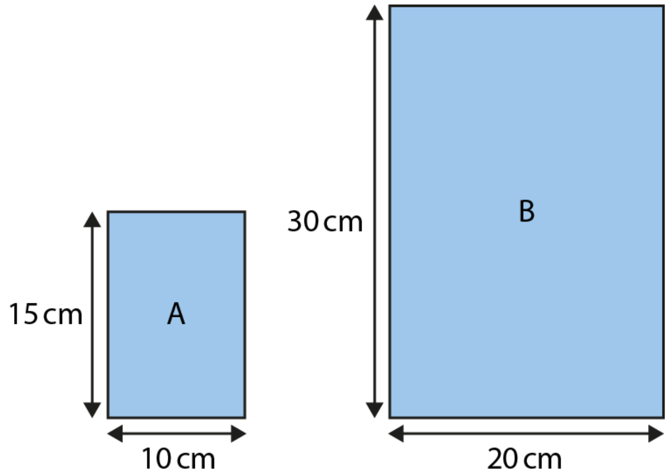
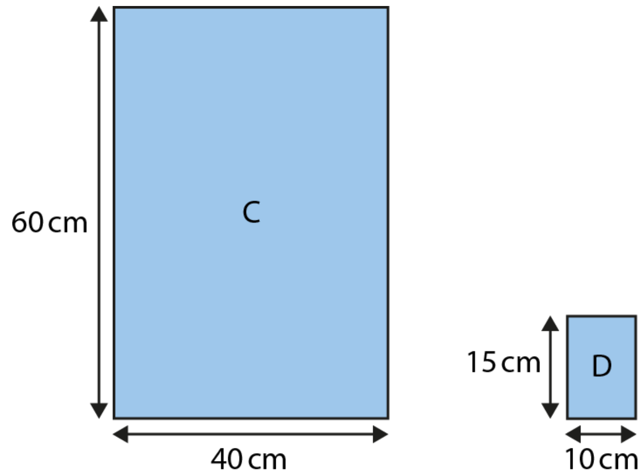
'What's the same?'
(shape)
'What's different?'
(side-lengths, perimeter, area)

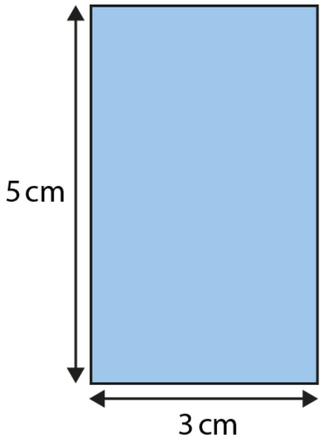
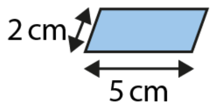
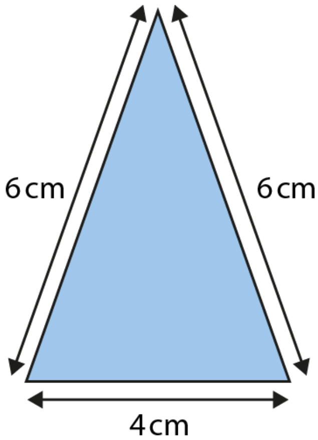
Draw attention to the fact that the side-lengths of the smaller shape are one-quarter the side-lengths of the original shape. The length of each side has been divided by four. Again, use the stem sentence above.
Remind children of the following generalisations from segment 2.27, step 4:3:
</td><td>
Applying scale factors to side-lengths – Example 1:


$10 \times \mathbf{2} = 20$
$15 \times \mathbf{2} = 30$
'To change shape A into shape B, scale the side-lengths by a scale factor of two.'
Applying scale factors to side-lengths – Example 2:


$40 \div \mathbf{4} = 10$
$60 \div \mathbf{4} = 15$
'To change shape C into shape D, scale the side-lengths by a scale factor of one-quarter.'
</td></tr>
</table>

	• ***'If the scale factor is greater than one, the shape is made larger. We can say the shape is enlarged.'*** • ***'If the scale factor is equal to one, the shape is the same size.'*** • ***'If the scale factor is less than one, the shape is made smaller. We can say the shape is reduced.'***	
4:2	Provide children with intelligent practice applying scale factors to the side-lengths of rectangles, triangles and parallelograms.	• *'Enlarge this shape by scaling the side-lengths by a scale factor of three.'*  • *'What scale factor has been used to change shape A into shape B?'*  • *'The side-lengths of Shape C have been scaled to make shape D. What is the unknown measurement?'* 

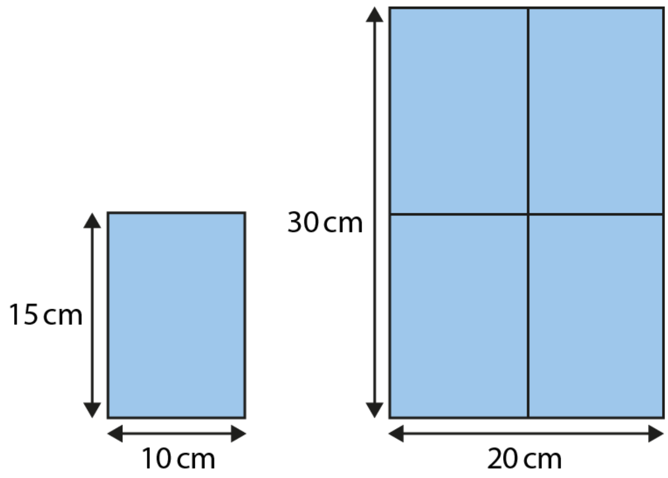
4:3	Now we will focus on applying scale factors to perimeters. Using the two examples from step *4:1*, ask children to work out the perimeter of the original and the transformed shapes. Ask children what they notice about the perimeters, guiding them to the conclusion that the perimeter has changed by the same scale factor as the side-lengths. Next revisit the shapes used in step *4:2* and ask children to calculate the perimeter of the transformed shape by applying a scale factor to the perimeter of the original shape. Work towards the following generalisation: ***'When a shape has been transformed by a scale factor, the perimeter is also transformed by the same scale factor.'***	Applying scale factors to perimeter – Example 1:  perimeter = 10 + 10 + 15 + 15 = 50 cm perimeter = 20 + 20 + 30 + 30 = 100 cm $50 \times 2 = 100$ • *'The perimeter has changed by a scale factor of two.'* Applying scale factors to perimeter – Example 2:  perimeter = 40 + 40 + 60 + 60 = 200 cm perimeter = 10 + 10 + 15 + 15 = 50 cm $200 \div 4 = 50$ • *'The perimeter has changed by a scale factor of one quarter; it has been divided by four.'*

		Using a scale factor to calculate perimeter: • *'If the side-lengths of this shape are scaled by a scale factor of three, what will the perimeter of the new shape be?'*  • *'If the side-lengths of this shape are scaled by a scale factor of four, what will the perimeter of the new shape be?'*  • *'If the side-lengths of this shape are scaled by a scale factor of one-half, what will the perimeter of the new shape be?'* 
4:4	Now look at applying scale factors to area. In this step children will learn that the generalisation that applies to perimeter in the previous step does not apply to area – we cannot apply the same scale factor to work out the area of the transformed shape.	

Looking at the first example used in steps *4:1* and *4:3*, ask children to work out the area of the original and the transformed shape. Ask children what they notice about the area and about the relationship between the two areas. Children should notice that when the dimensions are scaled by a scale factor of two, the perimeter is doubled but the area is four times the original area.

Children may like to explore what happens to the area when other scale factors are applied.

Applying a scale factor to area:



scale factor = 2

area = 10 cm × 15 cm = 150 cm^2

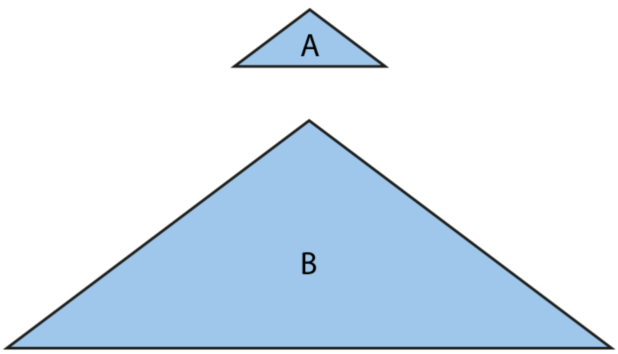
area = 20 cm × 30 cm = 600 cm^2

$150 \times 4 = 600$

4:5 Finish this segment by asking the children to find missing information, making connections to all the steps in this segment. You could use a stepped problem such as the one opposite. Note that for the parallelogram questions there are two solutions for each question, as there are two different parallelograms that could be made.

'Complete the table.'

Hint: Use the perimeter to work out all three side-lengths of triangle A.



	Perimeter	**Area**
isosceles triangle A: • base: 8 cm • perpendicular height: 3 cm	18 cm	
parallelogram made using two of isosceles triangle A		
isosceles triangle B: • base: 32 cm • side: 20 cm • perpendicular height: 12 cm		
parallelogram made using two of isosceles triangle B		

Scale factor that transforms the dimensions of triangle A into triangle B: ____