Primary
& Early Years
Magazine 83

National Centre
for Excellence in the
Teaching of Mathematics

Welcome to another issue of our Primary Magazine, which has now been serving primary teachers for over 80 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are available in the Primary Magazine Archive.

Contents

In this issue we have the third of three articles which look at assessing the aims of the National Curriculum; this month the focus is on problem solving.

Maths in the Staff Room suggests ways in which collective teacher discussions - both formal and informal - can form part of the ongoing process of professional learning, and help increase the effectiveness of maths teaching across the school. This month's article looks at over-generalisation related to mathematical structure, under the title 'Always, sometimes, never'.

Seen and Heard provides a specific example of a child's response to mathematics in a classroom to stimulate thinking and provoke questions about how you would react to similar events in your own classroom. This month a Year 5 pupil prompts us to think about what children understand about the structures of subtraction and how this has an impact on their fluency when solving problems.

If you have a photograph, or an account of a classroom conversation, that might stimulate similar thought, please email it to us. If we publish your suggestion, we'll put a £20 voucher in the post.

But first, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

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News

Year 6 pupils in 80 primary schools will this summer take part in a pilot of what the DfE are calling an on-screen multiplication tables check, with the test due to be rolled out across the country in 2017. Pupils will be asked to give timed responses to a series of multiplication questions, with answers scored instantly. Details of how the test will be structured, and how the results will fit in to wider pupil assessment and school accountability measures, are yet to be announced, but here is how the Sunday Telegraph reported the news at the beginning of January.

The Royal Institution STEM E&E (Enrichment & Enhancement) Grant Scheme offers UK state schools a grant of up to £500 towards an activity listed on the STEM Directories, usually events or experiences that cannot be delivered with standard school resources. Applications can be submitted at any time up to **7 February**; more details, including a link to the online application form are available on the STEM Directories website.

An opportunity to explore statistics in a meaningful cross-curricular context is available through participation in the RSPB's Big Schools' Birdwatch (part of the Big Garden Birdwatch) which runs until 12 February. Resources and more information are available on the RSPB website.

There's no direct maths-learning references in the episode of the Radio 4 programme The Educators broadcast on 23 December, which focused on Margy Whalley, the co-founder of Pen Green Children's Centre and Research Base in Corby, Northamptonshire, but it nevertheless makes interesting listening for anyone working in a primary school and interested in the learning experiences children acquire in the home from the moment of birth onwards. For thirty years, the centre has been educating parents about the way their children behave and learn, and using the insights of parents and nursery staff to understand the learning process of every child. Ranked outstanding in every one of its Ofsted reports, Pen Green has influenced other centres and early years provision in the UK, and plays an ongoing role in early years research.

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National Curriculum in Focus

National Curriculum in Focus *is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series here*

Assessing the Aims: Part Three – Problem solving

This is the third of three articles focused on assessment of the aims of the National Curriculum

The National Curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately

- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

As stated in the NCETM Teaching for Mastery booklets:

> *"Progress in mathematics learning each year should be assessed according to the extent to which pupils are gaining a deep understanding of the content taught for that year, resulting in sustainable knowledge and skills. Key measures of this are the abilities to reason mathematically and to solve increasingly complex problems, doing so with fluency, as described in the aims of the National Curriculum."*

As teachers and schools grapple with decisions about assessment it will be important for them to consider how the aims are reflected in their:

- assessment principles
- assessment criteria, and
- assessment practice.

The third of the aims, problem solving, gives purpose to mathematical learning. As the NRICH article Problem Solving and the New Curriculum states:

> *"...the whole point of learning maths is to be able to solve problems"*

The article goes on to quote from *How to Solve It*, by George Pólya, who described problem solving as:

- seeking solutions not just memorising procedures
- exploring patterns not just memorising formulas
- formulating conjectures, not just doing exercises.

So, problem solving involves application of the other two aims of the national curriculum; pupils need to recognise what it is they know and understand that will be useful and make connections in order to make sense of a problem and to find a solution.

In order for problem solving to be assessed, children will need to communicate their process as well as their solution and this will include using mathematical images, pictures and symbols alongside explanation. Pupils may need support to develop the use of different representations when solving problems. In particular, the use of drawings, to both support thinking and communicate thinking, needs to be modelled and should be a focus of some lessons.

Assessment will include looking for efficient and elegant solutions and the application of reasoning and will often necessitate talking to the children, using probing questions to get underneath their thinking. This is vital as, sometimes, children can get correct solutions to problems using inefficient methods or by using incorrect reasoning. The focus when assessing problem-solving needs to be as much on the method as on the outcome. This starts with how the children make use of what they know from how the problem is presented. Problem-solving should also provoke children to consider 'What if?' leading to the generation of further problems.

The Teaching for Mastery booklets are full of examples of problem-solving questions where methods can be assessed for their efficiency, elegance and application of understanding and knowledge. They reflect the notion that problem solving involves organising thinking in order to find solutions, finding all possibilities and exploring patterns and relationships.

- **Organising thinking**
  A long brick is twice the length of a short brick.
  Which is longer:
  2 long bricks or 3 short bricks?
  3 long bricks or 5 short bricks?

Year 1

Children in year 1 may need to model their thinking with a resource; equipment such as Lego bricks and Cuisenaire rods could be used for this question. The question provides an opportunity to focus on equivalence and allows for the children to generalise. The children could be asked to generate questions where the lengths would be the same and other questions where the short bricks together would be longer than the long bricks together.

Year 2

I spend £2 on a drink and a sandwich. The sandwich costs 80p more than the drink. How much does the sandwich cost?

This is a good example of a question where looking at how the children use the information provided will reveal a lot about their understanding, in this case of additive relationships, and problem solving skills. Representing the information presented could be discussed and explored to support the development of problem-solving skills. This could be done physically with coins, or with rods and matched to a bar diagram. This problem tends to be tackled in two main ways. One starts from the fact that the sandwich costs 80p more than the drink and involves putting the 80p to one side. The other starts from the fact that two items are being bought and involves splitting £2 between the two items. Discussion of which of these makes the problem easier to solve will be vital and children should be encouraged to explore these methods with other problems and to generate their own examples of problems which could be solved in a similar way.

In total Sam and Tom cycle a distance of 120km. Sam cycles twice the distance that Tom cycles. How far does Sam cycle?

Year 4

This is similar to the Year 2 example above but focuses on multiplicative rather than additive relationships. The information could be represented with Cuisenaire rods and this would support the development of drawing a bar to model the problem.

A shop sells magazines and comics. Last week Arthur bought a magazine and a comic. He can't remember exactly what he paid, but he thinks he paid £1.76.
Yesterday he bought a magazine and four comics. He paid £4.30.

Do you think he is remembering correctly when he says that he paid £1.76 last week?

Year 6

This could be represented in a number of ways including symbolically. As Arthur thinks the magazine and one comic cost £1.76, he can subtract this from what he paid yesterday, and this will be the cost of the additional three comics:
£4.30 - £1.76 = 3 comics
£2.54 = 3 comics
£2.54 ÷ 3 = 1 comic
82.666p = 1 comic
Therefore, he could not have spent £1.76 because a comic cannot cost 82.666p.

- **Finding all possibilities**

Using only 2p, 5p and 10p coins, can you show 20p?

In how many different ways can you do this?

Are you sure you have got them all?

Explain how you know.

Year 1

Finding all possibilities involves having a systematic approach and being able to explain and reason how all possibilities have been found. When children take a random approach, they may find lots of solutions but they will have no sense of whether they have found them all and will not be able to talk about how they decided which one to do next each time. This would reveal that their problem solving needs developing.

□ □ × □ = ?

Putting the digits 1, 2 and 3 in the empty boxes, how many different calculations can you make?

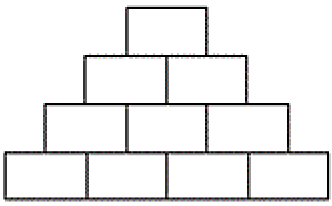
Which one gives the largest answer?
Which one gives the smallest answer?

Year 3

Put the numbers 1, 2, 3 and 4 in the bottom row of this multiplication pyramid in any order you like.

What different numbers can you get on the top of the number pyramid? How can you make the largest number?

Explain your reasoning.



Year 5

Both of these questions combine finding all the possibilities with looking for patterns and relationships. They encourage the children to notice things about the numbers and could be followed up by asking children to consider another set of numbers and to suggest how they would arrange them to make the largest and smallest answers <u>without</u> finding all possibilities; this would encourage them to generalise and focus on structure.

- **Exploring patterns and relationships**

Look at the grid. Choose a number and complete the second grid.

		50	
Count in 1s	49	50	51
Count in 10s	40	50	60

		?	
Count in 1s			
Count in 10s			

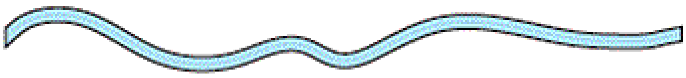
Year 1

This question involves more than just knowing one more/one less and ten more/ten less; it requires that children make decisions and use what they know to notice the relationship between the numbers and replicate this for a different number. The choice of number itself is one of the biggest challenges in this question.

A 1 m piece of ribbon is cut into equal pieces and a piece measuring 4 cm remains.

What might the lengths of the equal parts be?

In how many different ways can the ribbon be cut into equal pieces?



Year 5

This question requires that children use what they know about multiplication and multiples and could start with asking the children to think of something they know about dividing one metre equally. This could give rise to a known fact, such as 25 x 4 = 100, which could be used as a starting point and then adjusting so that there would be 4cm left over, building on this to find further possibilities. Assessment will include considering how well the children use what they already know and what they have found out to find further possibilities.

Assessing problem solving will, therefore, focus on assessing how children use what they know, organise their thinking and build on and adjust their thinking as they work through a problem. It includes application of mathematical thinking and understanding and will be underpinned by talking with children, listening to and probing their thinking.

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Maths in the Staff Room – Short Professional Development Meetings

Maths in the Staff Room *provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school. You can find previous features in this series here*

Understanding key mathematical structures. Part four: Always, sometimes, never

This continues the focus on key structures in mathematics which has featured in the last three magazines. This month, the focus can be used to challenge over-generalisation and expose misconceptions

Meeting aims

- To consider the danger of over-generalising about structure in mathematics
- To explore proof in primary mathematics
- To make explicit opportunities to embed the aims of the National Curriculum.

Timings

- Ten minutes initial input
- Ten minutes, thirty minutes, sixty minutes or ninety minutes follow-up after two weeks.

Resources

- Large sheet of paper for display in the staffroom.

Ten minute introduction

1. Explain that understanding structure is an important part of mathematics but that sometimes children apply a generalisation about structure to a different bit of mathematics where it doesn't apply, and that you are going to explore how this can be tackled by using general statements and considering whether they are always, sometimes or never true

2. Write up the statement: Adding two numbers together always makes a bigger number. Ask everyone to consider why children might think this is always true and how they can prove that it is not always true.

3. Agree that you only need one counter-example to prove that a statement is not **always** true. Ask everyone to consider how they might adjust the statement so that it is always true. Share ideas and agree a new statement which is always true.

4. Ask everyone to think of one thing which children in their year group tend to over-generalise and discuss this with a partner, agreeing a statement which they could use to expose this over-generalisation because the children might think it is always true rather than only sometimes true.

5. Use a large sheet of paper with ‘Always, sometimes, never’ in the centre. Share the statements and

add them to the large sheet. Ask everyone to think about this over the next two weeks and whenever they think of or experience an example of over-generalisation to make a matching statement and add it to the sheet. Say that you will come back to these in a future meeting.

Follow-up meeting two weeks later (you may need to prompt people to add to the sheet and model this by adding ideas during the two weeks). Have the large sheet which has ideas connected to 'Always, sometimes, never'.

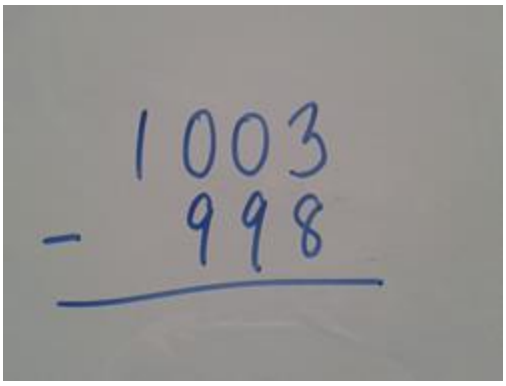
- Look at the statements. Ask each pair to choose a statement and to consider:
  - What do you think is the reason why children might believe this statement?
  - Can you generate counter-examples?
  - How might you adjust the statement to make it always true?
  - How would you 'prove' that this new statement is always true? Could you draw something or use a resource to model and demonstrate to children why it is always true?
- Ask the pairs to share
- Look at examples of 'Always, sometimes, never true' statements taken from the Teaching for Mastery booklets and the Progression maps with reasoning. For each statement agree whether it is always, sometimes or never true and discuss the thinking behind the statement.

Image Credit

Seen and Heard

Seen and Heard *will shine a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience, the aim being to stimulate thought and questions about how you would react to similar events in your own classroom*

A subject leader was talking to a group of children in Y5, as part of monitoring mathematics across the school. The children were presented with the calculation 1003 - 998. They decided to solve it using a formal written calculation and set it out as below:



A prolonged discussion about what they should do next followed and after a number of minutes the children abandoned this method. The subject leader then asked 'Are these numbers close together?' One child replied 'Yes, they are really close together, they are only five apart'.

- What does this child understand about the structures of subtraction?
- How can they best be supported to understand that five is the solution to this calculation?
- What else might the subject leader want to explore with the children?
- Are there similar questions the subject leader could use with other children across the school to investigate if this is a school-wide issue?

If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk with 'Primary Magazine: Seen and Heard feature' in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we'll send you a £20 voucher.

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