Welcome to another issue of our Primary Magazine. This magazine has been serving primary teachers for 75 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are accessible through the Primary Magazine Archive.

Contents

In each issue we have a selection of interesting and useful articles. New National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study. This edition focuses on a new series of articles exploring how to design learning for short multiplication.

Where's the Maths in That? shares ideas for ensuring that mathematics is taught and experienced across the curriculum. In the coming months, this series of articles that will explore opportunities for mathematics and mathematical thinking within the new science programme of study. This month the theme is *Living Things and Their Habitats for Y2*.

Finally, Maths in the Staff Room provides simple plans for CPD meetings in your school to be led by a member of staff. These are short meetings that can be used exactly as indicated or adapted to meet the CPD needs of the school. We begin a series focusing on the features of great teaching in the context of maths which was explored in full in Issue 73. In this issue we explore setting worthwhile homework.

But first, we have a News section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.

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National Centre for Excellence in the Teaching of Mathematics

News

Mastery
There's a new section of the NCETM website that pulls together all the work the Centre, and the Maths Hubs are doing in the field of mastery. In view of the developmental nature of all work in this area, the NCETM has also started three new threads of discussion in the Maths Café community. Please contribute your views and experiences: constructive criticism is always welcome.

ACME conference - 9 July
The Advisory Committee for Mathematics Education (ACME) will be holding its next conference, Mathematically Thinking, on 9 July at The Royal Society, London, and registration is now open. The newly re-appointed Schools Minister Nick Gibb will be among the speakers and the day will also feature workshops on the following themes:

1. Assessing problem solving and reasoning
2. Mathematical thinking post-16: A level and Core Maths
3. Encouraging depth of mathematical thinking
4. Early years and primary mathematical thinking.

Using Manipulatives in the Foundations of Arithmetic
You might be interested in a two-year piece of research into the use of manipulatives in primary maths classrooms, which is nearing its half-way point. The Nuffield Foundation is funding a project focusing on the effective use of manipulatives for teaching and learning arithmetic, such as Cuisenaire and Dienes, for children aged 3-9. This project is being led by Dr Rose Griffiths, Dr Sue Gifford and Dr Jenni Back. You can find further information on the Nuffield website as the project unfolds.

London Mathematical Society CPD Grants
Did you know that the London Mathematics Society (LMS) provides opportunities for schools/ teachers to bid for grants of up to £400 to support teachers with maths-specific CPD? Although there are certain conditions that need to be met, these grants are available to all teachers. Application deadlines for grants are 31 August, 30 November, 31 January and 30 April each year, so you might like to start considering an application over the summer.

NCETM National Curriculum support
Have you explored our National Curriculum Planning Tool yet? This interactive tool will support you in the following ways: your subject knowledge; making connections within and across the primary curriculum; suggest helpful papers, pupil activities, exemplification of expectations, and links to the suite of NCETM videos. There are also sections on the Bar Model, Teaching Fractions, Progression in Reasoning, and Developing a Scheme of Work - all accessible via buttons on the main National Curriculum information page.

Mathematics CPD and networking events

Don't forget to use our Professional Development Calendar if you are looking for courses or training run by high quality providers of maths CPD, or indeed any events in the next few months where you can meet and share experiences with other maths teachers. Look for CPD Standard Holders (gold rosette) and/or Accredited Professional Development Leads (purple rosette). A list of providers, classified by the same system, can be found in the CPD Providers Directory.

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New National Curriculum in Focus

New National Curriculum in Focus is dedicated to unpicking the new curriculum and how to understand and develop the requirements of the new programmes of study for mathematics. You can find previous features in this series here.

Designing learning for short multiplication

The new curriculum requires children to learn to use standard written methods sooner than has been taught in recent years. In their report Good practice in primary mathematics: evidence from 20 successful schools, Ofsted identified that in the most successful schools pupils were moved to standard written methods swiftly and once pupils were secure with interim methods were moved quickly on to more efficient methods.

What is important is that, if pupils are to be expected to move to a standard written method more quickly than previously expected, we need to ensure that that they do this not only with procedural fluency but with conceptual understanding.

So can this be achieved for short multiplication?

Let's consider the Y5 statement from the programme of study:

> *multiply numbers up to four digits by a one-digit number … using a formal written method*

Before pupils can begin to learn to do this there are a number of skills and concepts that need to have been developed in order to carry out short and long multiplication:
- Recall fluently multiplication facts
- Visualise and understand how a four-digit number can be partitioned and recombined into multiples of 1000, 100, 10 and 1 with both concrete and abstract representations. (i.e. base 10 (concrete), place value counters or arrow cards)
- Visualise the relative quantity of the numbers
- Know the value of a digit because of its position in a number
- Understand the effect of multiplying by 10, 100 and 1000
- Know and use the fact that multiplication is commutative
- Know and use the fact that multiplication is distributive over addition
- Decide when it is more efficient to calculate mentally.

Scaffolding learning through procedural and conceptual variation

One particular feature of the teaching seen in Shanghai has been the use of teaching with conceptual and procedural variation. You can read in more detail what this means in the National Curriculum in Focus article in Issue 74.

So what might conceptual and procedural variation look like in the context of teaching the written method for multiplication of four-digit numbers by a single-digit number? Written multiplication involves finding partial products and adding them together to find the whole product. Pupils will need to

recognise that there could be up to four partial products to find the total in a four-digit number multiplied by a single-digit number.

In KS1 and lower KS2 an effective representation of multiplication is to consider this as an array moving to an empty area model.

13 x 4 might be represented using concrete or visual representations below:

Using concrete representations (Dienes apparatus)

This representation enables pupils to see how multiplication is distributive over addition. E.g. they might see that 13 x 4 is the same as "four tens and four threes" (or another vertical partitioning)

This representation also enables pupils to see how multiplication is associative. E.g.13 x 4 is the same as "two thirteens, twice" (13 x 2) x 2.

Using an area model
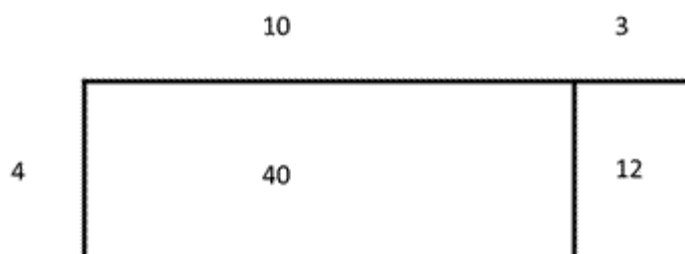
This model provides a more sophisticated and abstract representation of the Dienes representation above and children should be comfortable with the former before moving to this representation.

The area model is then easily translated to the multiplication of larger numbers. However as numbers become bigger (e.g. four-digit) it would become difficult to represent this to scale and therefore the grid representation then becomes an abstraction of this.

E.g.
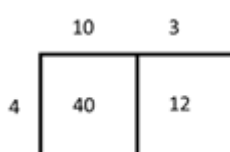
Place value counters can also be a helpful representation of multiplication – for example, 432 x 3 might be represented in this way with place value counters. It is important to recognise that although the counters

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are arranged in an array, the 10 counter represents 10 'ones' pushed together in this representation. The 100 counter represents 100 'ones' pushed together.
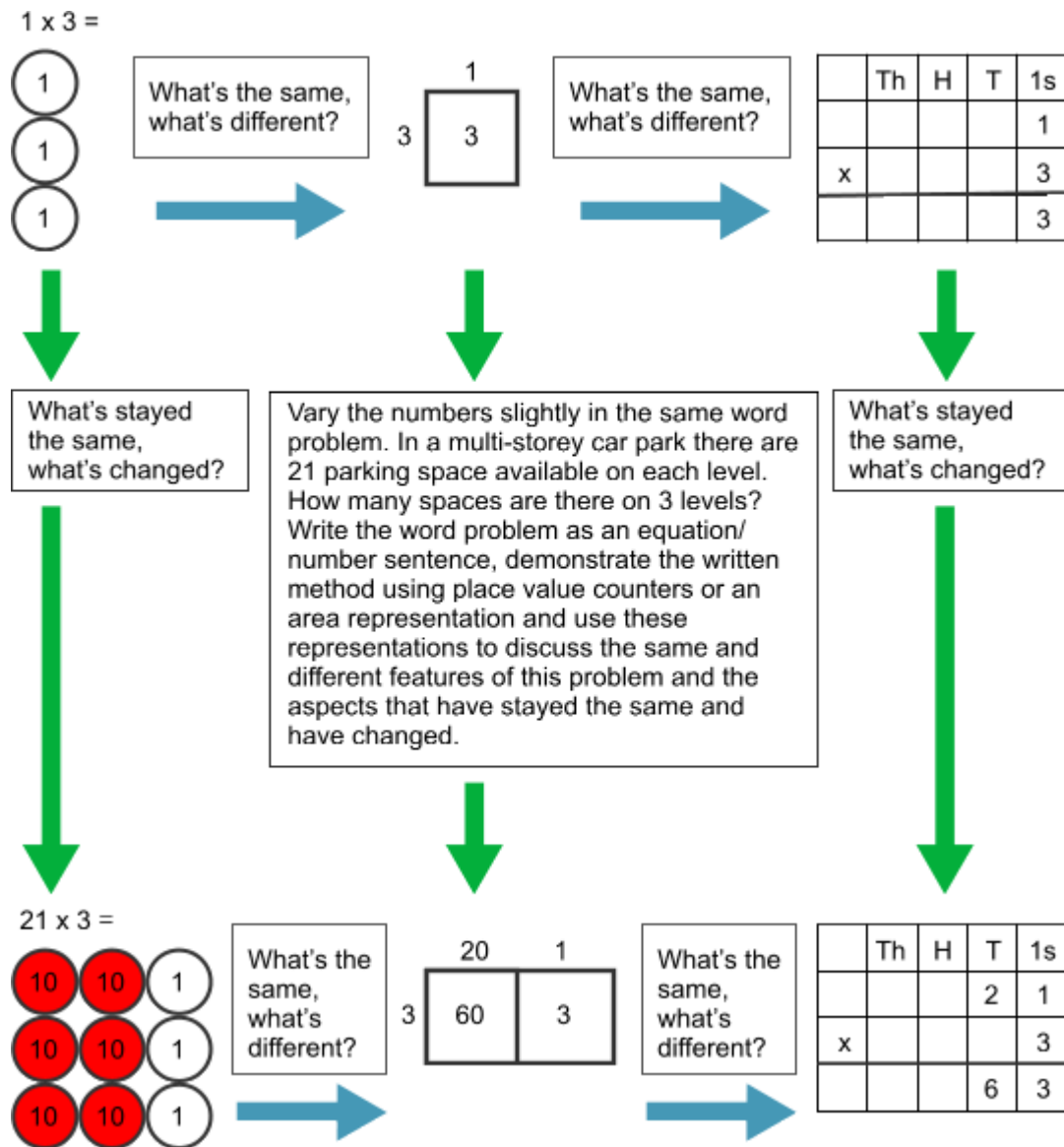

In many primary schools pupils have become confident users of the grid representation to solve multiplication problems. However this method becomes cumbersome as the multiplicand and multipliers increase in size. The number of partial products to add together can mean that children make mental calculation errors. The grid representation can support pupils' conceptual understanding of multiplication and mental calculation skills but it can also help to move pupils' towards the standard written method for multiplication (in this case short multiplication).

So how should we help children to learn to become fluent with the written method and retain their conceptual understanding? Below is a sequence of problems that can be used to illustrate how to shift from the grid representation as a jotting for mental calculation to the standard written methods for short multiplication. An assumption is made that the key skills and concepts mentioned above have all been acquired in order for the pupils to work on this process. Also, as for addition and subtraction we also have instances when a number needs to be regrouped. E.g. in 2324 x 2 each successive digit multiplied by 2 will yield a single digit number and no regrouping is required. However in 2326 x 2 we find that the ones digit multiplied by 2 will yield a two-digit number which complicates the written method and requires numbers to be regrouped.

Short multiplication without regrouping

Begin with a simple word problem. Eg. In a multi-storey car park there is one parking space available on each level. How many spaces are there on three levels? Write the word problem as an equation/ number sentence, demonstrate the written method using place value counters or an area representation and use these representations to discuss the same and different features of this problem.

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1 x 3 =

① 1
① 1
① 1

What's the same, what's different?

1
3 | 3

What's the same, what's different?

	Th	H	T	1s
				1
x				3
				3

What's stayed the same, what's changed?

Vary the numbers slightly in the same word problem. In a multi-storey car park there are 21 parking space available on each level. How many spaces are there on 3 levels? Write the word problem as an equation/ number sentence, demonstrate the written method using place value counters or an area representation and use these representations to discuss the same and different features of this problem and the aspects that have stayed the same and have changed.

What's stayed the same, what's changed?

21 x 3 =

(10)(10)(1)
(10)(10)(1)
(10)(10)(1)

What's the same, what's different?

20 1
3 | 60 | 3

What's the same, what's different?

	Th	H	T	1s
			2	1
				3
x				
			6	3

The variations from left to right draw the pupils' attention to the meaning of the increasingly abstract representations. Pupils should be encouraged to seek the similarities for how the partial products are represented in each case. Some children may find the jump from the grid to the written model harder to grasp when there are two (or more) partial products. In which case it might be necessary to add a further interim step of presenting each of the partial products below the solid line and then perform and addition of the partial products. It is important to stress that this is an interim step to ensure conceptual understanding and not a final written method.

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E.g.

	Th	H	T	1s
			2	1
x				3
				3

+

	Th	H	T	1s
			2	1
x				3
			6	0

And/ or moving to…

Th	H	T	1s
		2	1
	x		3
			3
	+	6	0
		6	3

…before moving on to the standard written method.

Continue with the sequence of problems to build up understanding of the written method without regrouping up to four-digit numbers. Ensuring that the word problem is used as the basis of the numbers at each step.

E.g.

321 x 3
1321 x 3

Pupils can then practise the written method using similar sequences that involve multiplying without regrouping.

E.g.

1312 x 3
1321 x 3
1213 x 3
1231 x 3
1123 x 3
1132 x 3

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The above sequence will enable teachers and pupils to focus in on the value of the digits in each product. Because the numbers involved do not require any regrouping the 6 appears in the products sometimes as ones, tens or hundreds and this can lead to a rich discussion about why.

As pupils become more fluent provide a variety of contexts for the problems they are solving using multiplication still without regrouping. Pupils should also be given the opportunity to explore numbers where the multiplicand contains a 0 in various places and notice the number of non-zero partial products there are to add together in each case.

Short multiplication with regrouping

Provide a sequence of problems using a familiar context that will reveal a change in the process that leads to pupils becoming aware of the need to regroup. The point of the exercises will be to help the pupils to generalise when there is a need to regroup. The pivot points are the points at which we want the children to notice something that changes.

E.g.

1322 x 3
1323 x 3
1324 x 3 (pivot point 1)
1325 x 3
1326 x 3
1327 x 3 (pivot point 2)

Pivot point 1 in the above is the first occurrence of a procedural change that the sequence will enable the pupils to observe. The pupils will observe that 'ten ones' has been regrouped into 'one ten'. See below for a representation using place value counters.
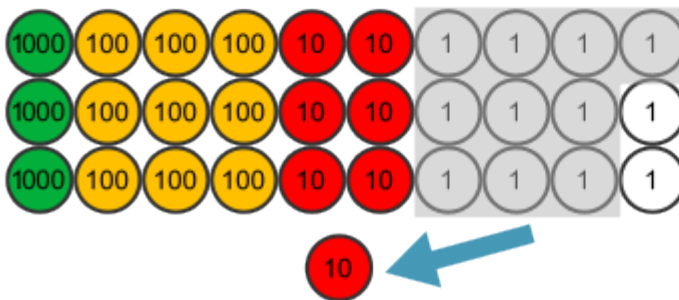
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$1323 \times 3 =$
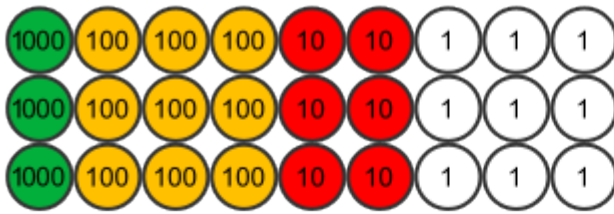
What's the same, what's different?

Th	H	T	1s
	1	3	2	3
x				3
	3	9	6	9

$1324 \times 3 =$

What's stayed the same, what's changed?

What's stayed the same, what's changed?

What's the same, what's different?

Th	H	T	1s
	1	3	2	4
x				3
	3	9	7	2
			1	

Pivot point 2 involves a further procedural change as pupils will need to observe that 'twenty ones' will need to be regrouped into 'two tens'. Pupils will then need the opportunity to practise with other sequences, observing and applying regrouping ones in to tens accordingly. Pupils will be secure with this process when they understand and articulate a generalisation that describes when regrouping of ones will occur. Pupils could be provided with a selection of multiplications and asked to notice which ones will require regrouping without them having to perform the entire calculation.

Pupils may then be moved to sequences that enable them to observe that the tens need to be regrouped into hundreds or hundreds into thousands and examples where regrouping may occur more than once in a calculation. These are other 'pivot points' to design for learning to use the written method for short multiplication. Try to think of some other examples and share the sequences in the comments box below. The starting point for a sequence of problems will depend on the children's conceptual understanding and procedural fluency of previous written methods. E.g the children may be fluent in their use of multiplying three-digit numbers by a single-digit number, in which case it may be appropriate to begin with a three-digit number and vary the thousands number. The examples above are illustrative. You may wish to use different representations or further interim steps to ensure pupils develop conceptual understanding with procedural fluency for the written method. However a key point to consider is that any interim method is best used for short periods only. They are conceptual stepping stones to something more efficient. If children stay on these methods for any length of time, they may become fixed as procedures, making it difficult to move on. The conceptual models might be revisited later to link other mathematics. The grid method can be valuable at Key Stages 3 and 4 within the context of algebra.

Pupils can deepen their understanding by solving problems involving reasoning about the written method such as [All the Digits](#) from NRICH.

Or find the hidden digits

	Th	H	T	1s
	1	⭐	4	5
x				5
	6	7	⭐	⭐

Or how many ways can you find to make this calculation work?

	Th	H	T	1s
	1	⭐	4	5
x				5
	⭐	7	⭐	⭐

Further resources
- Primary Magazine Issue 60: [Maths to Share - Multiplication](#)
- You can read more about conceptual and procedural variation [here](#).

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Where's the Maths in That? – Maths across the curriculum

*In this section of this Primary Magazine we explore how mathematics can be embedded into other subjects in the context of the new curriculum. The subject in this new series is **science** and over the next few months we will explore the different themes for the KS1 and KS2 science programmes of study and how maths can be embedded in and enhance understanding of scientific ideas. You can find previous features in this series* here*.*

In this edition we look at the theme of **Living Things and Their Habitats (Y2)** and how a scheme of work for this might incorporate mathematical skills.

The statutory requirements are that children are taught to:

Y2

- explore and compare the differences between things that are living, dead, and things that have never been alive
- identify that most living things live in habitats to which they are suited and describe how different habitats provide for the basic needs of different kinds of animals and plants, and how they depend on each other
- identify and name a variety of plants and animals in their habitats, including micro-habitats
- describe how animals obtain their food from plants and other animals, using the idea of a simple food chain, and identify and name different sources of food.

This theme involves a lot of sorting and classifying which enables pupils to use a number of diagrams to represent the information.

Use a Venn Diagram to sort living and non-living and to use the results of sorting to discuss the difference between dead and never lived. Talk about where these different classifications might fit.

Things that are living   Things that are non-living
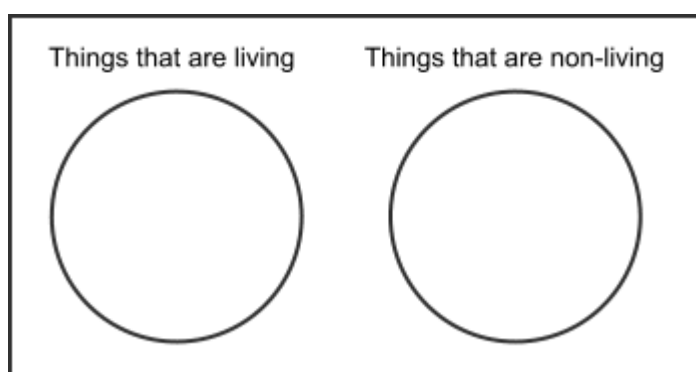
Pupils can also sort living things with two criteria by using a Carroll diagram:

	Lives in water	Does not live in water
Has wings		
Does not have wings		

Pupils' reasoning skills can be developed using a classification key. In this example from OpenLearn, the children need to consider the questions that need to be entered into the gaps in order to work out how the animals have been sorted. Before pupils have done this they will need to have used classification keys to sort living things from top to bottom so that they understand how they are structured. Further resources for grouping and classification can be found on the Science & Plants for Schools website.

Choice chambers are an interesting way of developing scientific enquiry around some of the life processes (movement, reproduction, sensitivity, nutrition, excretion, respiration and growth). Examples of questions include:

- Which foods do beetles, woodlice and snails prefer?
- What kinds of habitats do beetles, woodlice and snails prefer? (e.g. light/ dark/ damp/ dry etc)
- Which colours do beetles, woodlice and snails prefer? (e.g. bright or dull colours).

In each case discuss how to make the test fair, e.g. number of organisms in each chamber at the start of the enquiry, the time all the organisms are left in the choice chamber; and the things that will change, e.g. whether it is dark or light, dry or moist, the food provided. Results can then be presented on a pictogram or block graph.

If observing the metamorphosis of a caterpillar to a butterfly, pupils could keep a diary of their observations. Each day they note down the day and date and keep a record of observable changes.

As part of investigating about the characteristics of different living things and habitats, pupils could work together to make a class pack of top trumps cards. Pupils could play with a set already designed and then make their own from information they have researched. Bugfacts.net provides easily accessible text for KS1, although imperial measurements are given, not metric. There are lots of examples of trumps cards on the internet to download and use.

Suggested links

- National STEM Centre eLibrary - Grouping and Classification.

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Maths in the Staff Room – Short Professional Development Meetings

This section provides suggestions and resources for a professional development meeting for teachers that can be led by the maths subject leader or another person with responsibility for developing mathematics teaching and learning in the school. You can find previous features in this series here.

Maths Homework

In Issue 73 we provided a meeting to review the effectiveness of teaching mathematics in relation to the two recent reports on effective pedagogy and teaching:

- Exploring Effective Pedagogy in Primary Schools: Evidence from Research
- What Makes Great Teaching?

This meeting follows on from that first reflection and provides a more in depth understanding of how a group of teachers might address any areas of weakness from the audit carried out. In this issue we explore **maths homework**.

Meeting Aims

- Evaluate the purpose of setting maths homework for pupils
- Agree the type of maths homework that can be set to add value to learning.

Timing

- 1 x 90 minute meeting

Resources

- Examples of completed maths homework that has been set in the previous few weeks
- School homework policy or maths policy with reference to homework included.

1.0 Share the aim of the session

2.0 Start with a discussion to brainstorm responses to these five questions

1 *How often do you set maths homework for your class?*

2 *What sort of maths homework do you set your class?*

Share and compare examples of maths homework that has been set. Is it clear from the homework what the purpose is? Reflect on the possible purposes below.

Examples might include
- revision
- preparation for future learning
- rehearsal
- consolidation
- enrichment

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- extension
- assessment.

3 *How are parents supported to help their children with the homework?*

Examples might include:
- None
- Information about prior learning
- Worked example provided
- Reference to maths information for parents on school website
- Advice about whether their child has already been working on this area or is new

Agree a consistent approach to supporting parents with each piece of homework.

4 *What happens to the homework after it is brought back to school?*

Examples might include:
- Nothing – stays in homework folder
- Marked for accuracy and completion and then stored
- Marked for accuracy and completion and then returned to pupils to see
- Marked formatively and returned to pupils for reflection
- Marked and used to inform teaching

Agree a consistent approach throughout the school.

5 *Does the homework make a difference to pupils' learning and progress?*

Possible Examples:
- Don't know
- No
- Probably not
- Sometimes
- Not for all
- Mostly
- Yes

The importance of this question is to drill into the purpose of setting the maths homework and should also help to refocus why maths homework is set in the school. If it is unclear whether the homework is making a difference to learning, you may wish to spend time on considering whether this needs to be considered more carefully. The next quotation may help to focus the discussion.

3.0 Share the comment from the report on effective practice in primary

"Teachers in excellent and good schools set homework that was more meaningful and more clearly linked to what the children were learning. They had a more flexible approach to setting homework, which was set to extend and deepen the children's understanding. In schools rated as poor, teachers set homework simply because they were required to, and the work itself did not appear to be expressly linked to what the children were learning in class. There were no

examples of teachers using opportunities that arose during a lesson to set homework other than what was already planned."

P21, Exploring Effective Pedagogy in Primary Schools: Evidence from Research

Outstanding Teaching Grade Descriptor

"Teachers use well-judged teaching strategies, including setting appropriate homework that, together with clearly directed and timely support and intervention, match pupils' needs accurately."

P61, Ofsted inspection guidance (Jan 2015).

4.0 Use these quotes to review any policy document that refers to maths homework or to inform the wording of a new maths homework policy statement

*What should the possible **benefits** of setting maths homework be?*

*What are the **challenges** associated with setting maths homework? And how can these be mitigated?*

If the school is to set homework, then everyone (including parents and children) needs to be clear about why it is being set and what the benefits will be when each piece of homework has been completed and then this can be monitored.

5.0 What actions can we take to improve the quality of homework and the contribution it makes to learning maths in the school?

Below are some suggestions:
- Maths homework club - involving some willing/ trained parents
- Parent maths homework ambassadors in each class - for other parents in the same class to approach for help.
- Whole school focus on a particular theme each term. E.g. times tables; number bonds; word problems; puzzles
- Termly homework booklet for pupils to work through and parents to sign (see this example from the Centre from Innovation in Mathematics Teaching (CIMT))
- Provide a selection of maths homework activities for the term/half-term (six weeks) for each year group that includes: investigations, applications (e.g. going to the shops to buy something, baking etc), and practising mathematical skills. Pupils can choose which one to do each week and report back.
- Include an information comment on every piece of homework e.g. "Next week we are going to be using multiplication facts. Use this sheet to practise your times tables." or "This week we have been comparing and ordering decimal numbers with tenths…"
- Include a completion summary e.g.
    I did this homework…
    o On my own
    o With a grown up…
    o With a brother/ sister…
    o With some help…
    o With lots of help…

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-
  - Expect parents to do five minutes of facts recall practice every day with their child (do you expect 15 minutes of reading each day? So why not maths too?)
  - Use homework formatively each week
  - Explore online homework activities.

Further support/ resources

- Ocean Maths – Maths Homework Charity
- National Numeracy Parent Toolkit
- General review of research into homework in schools and its impact
- General homework evaluation tool.

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