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Welcome to another issue of our Primary Magazine, which has now been serving primary teachers for 91 issues with a varied collection of articles related to maths education and mathematics professional development - all of which are available in the [Primary Magazine Archive](#).

## Contents

In [Digging Deeper](#) each month we will explore an element of mathematics teaching linked to current developments and research: in this issue we look at some of the things we can learn from the end of KS1 and KS2 tests papers.

[Aspects of...](#) provides a number of bitesize ideas related to a specific element of mathematics; this month: aspects of Y6.

[Seen and Heard](#) provides a specific example of a child's response to mathematics in a classroom to stimulate thinking and provoke questions about how you would react to similar events in your own classroom. In this issue we consider approaches to a two-step problem.

**Next month** we will have articles focused on what can be learnt from the end of KS1 and KS2 tests papers and preparing for Y6. But first, as always, we have a [News](#) section, bringing news from the NCETM and beyond to keep you up to date with the fast-changing world of mathematics education.



## News



We've just published three new sets of videos, each showing a primary maths lesson that displays elements of [teaching for mastery](#). Each of the three lessons is split into a collection of smaller individual video clips, corresponding to different parts of the lesson, and followed by an interview with the teacher. In each case, there's also a range of downloadable accompanying materials, showing the resources used by the teacher and unpicking the subject knowledge underpinning each lesson.

The lessons cover [difference as a form of subtraction \(Year 1\)](#), [place value with decimals \(Year 4\)](#), and [line graphs \(Year 6\)](#).



We've added another three case studies to our collection of case studies supporting teaching for mastery. [The first](#) focuses on Hannah Gray, a Y2 teacher working with the Matrix Maths Hub on a project investigating the effects, teaching, learning and planning, of using a high quality textbook. In [the second](#) Emma Patman, Maths Lead at a primary school in Nottinghamshire talks about how her classroom practice has been transformed by her exposure to teaching for mastery. Finally, [the third](#) looks at Liam Colclough, head teacher at a primary school in Sheffield, who believes that encouraging and developing a 'growth mindset' in pupils to be fundamental.



Over the last 18 months or so, we've published a number of papers outlining our current thinking on subjects including [Primary Marking](#), [Mathematics Textbook Design](#), and Calculation for Primary Schools, all of which - and more - you can find [here](#). Our latest paper, [The Essence of Maths Teaching for Mastery](#), tries to encapsulate, in nine short bullet points, the essential elements that characterise maths teaching for mastery: we'd be interested to hear your views.



The Royal Society is promoting the 2017 Science on Stage festival next summer, which is being held in Hungary. They're looking for 12 primary and secondary maths and science teachers to form a delegation from the UK, who will exhibit a project they've been involved in. The deadline for online applications is **12 October**: more information is available on the [Royal Society website](#).

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## Digging Deeper

### End of KS1 and KS2 tests

One of the key observations emerging from examination of teaching practices in Shanghai is that there is a focus on teaching relational understanding, 'knowing both what to do and why' (Skemp 1976). This leads to an expectation that, when working on mathematics, children will 'notice' things and make decisions based on what they notice.

Looking at the test papers for the end of both KS1 and KS2, it is clear that children who look to notice things and use what they notice, and what they know, to make decisions will have an advantage over children with an instrumental understanding who have memorised what to do and follow this route regardless of the numbers and the context involved.

#### Paper 1: arithmetic (KS1 and KS2)

Despite their title, demonstrating fluency in these papers is dependent on reasoning. In order to provide space to reason children need to step back from the questions before engaging with finding an answer, so that they allow themselves the opportunity to notice things. Including decision-making as a key element of all mathematics lessons will support children with taking this approach.

It is worth considering, as a whole staff, for which of the questions on the KS2 arithmetic paper you would expect children to use a written method and why. This can be linked to examining particular questions from both the KS1 and KS2 arithmetic papers, discussing how you would expect children to be tackling them if they are demonstrating fluency, and then looking at different responses from children in your school in order to consider the adjustments to teaching that might be needed in order to support children to use what they know and understand.

For example:

#### KS1 Q5

$$4 + 5 + 6 = \boxed{\phantom{000}}$$

- Do the children look at the whole calculation before starting to calculate?
- Do they notice that they know  $4 + 6 = 10$  and that it is easy to add 5 to 10?

#### KS1 Q9

$$56 - \boxed{\phantom{000}} = 51$$

- Do the children notice that the two numbers shown in the calculation are close together?
- Do they notice that knowing  $1 + 5 = 6$  will help them here?

KS1 Q16

$$69 + 11 = \boxed{\phantom{00}}$$

- Do the children notice that 69 is only one away from 70?
- Do they notice that adding one and adding ten is the same as adding eleven?

KS2 Q7

$$89\,994 + 7\,643 =$$

- Do the children notice that 89 994 is only six away from 90 000?
- Do they notice that adding 7 643 (or 7 637) to 90 000 is easy?

KS2 Q18

$$122\,456 - 11\,999 =$$

- Do the children notice that 11 999 is only one away from 12 000?
- Do they notice that subtracting 12 000 from 122 456 is easy?

KS2 Q33

$$\frac{3}{5} \div 3 =$$

- Do the children notice that  $\frac{3}{5}$  can be thought of as 'three lots of one fifth'?
- Do they notice that dividing three of something by three is straightforward?

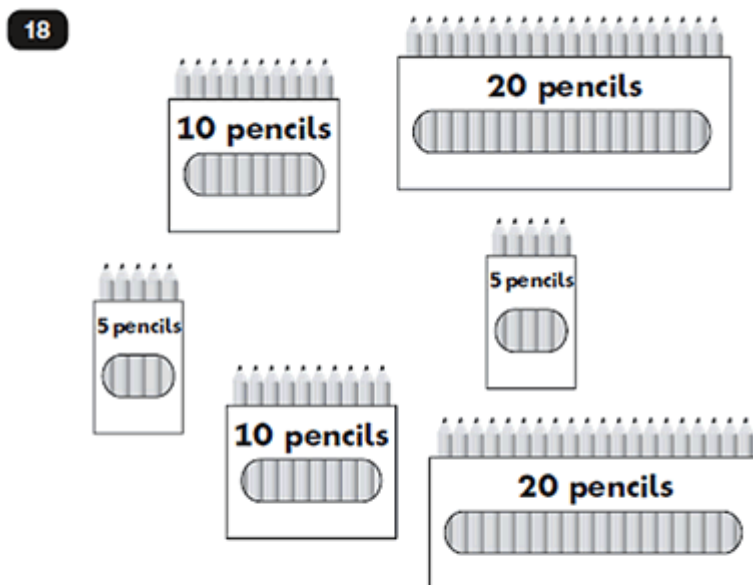
**Papers 2 and 3: reasoning**

For some KS2 children, the organisation of papers 2 and 3 was a challenge because the content that was being tested did not get progressively harder; instead content from years three to six was dotted around the papers. This is reflected in the mark scheme (page 4) with the table showing the content domain covered by each question. The start of the table for the two reasoning papers (below) shows that in paper 2 the second question covers content from year 5 and in paper 3 the second question covers content from year 6, whilst question 3 on each paper tests content from lower KS2.

Paper 2: reasoning		Paper 3: reasoning	
Qu.	Content domain reference	Qu.	Content domain reference
1a	3N2a	1	3C1
1b	3N2a	2a	6N5
2	5N2	2b	6N5
3	3C2	3	4M4b
4a	4S1	4a	6A2
4b	5S1	4b	6A2
5	5C5c	5	5F8
6	4G2c	6	4F10b
7a	6F2	7a	4G4
7b	6F2	7b	4G4

Again, at both KS1 and KS2, the numbers used in the questions on the reasoning papers invite the children to notice things. For example:

### KS1 Q18



Kemi and Ben share these pencils equally.

How many pencils do they each get?

- Do the children notice that they can share the boxes between Kemi and Ben; they don't need to find out how many pencils there are in total?

KS2 Paper 2 Q9

9

6 pencils cost £1.68



3 pencils and 1 rubber cost £1.09



What is the cost of 1 rubber?

- Do the children notice that if they know the cost of six pencils then they know the cost of three pencils by halving, they don't need to find the cost of one pencil?

Supporting children to develop relational understanding and expecting them to notice things, related to what they know and understand, and then make decisions based on what they notice is at the heart of teaching for fluency, reasoning and problem-solving, the aims of the National Curriculum. The result will be children who understand the mathematics and can demonstrate this understanding in a test situation.

Skemp, R Relational Understanding and Instrumental Understanding. First published in Mathematics Teaching, 77, 20–26, (1976)

**Next time in Digging Deeper** we will explore the new [teaching for mastery videos](#).

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**Work out:**

- $8.4 \times 3 + 8.4 \times 7$
- $6.7 \times 5 - 0.67 \times 50$
- $93 \times 0.2 + 0.8 \times 93$

## Aspects of...

### Y6

Keeping a focus on learning in mathematics, underpinned by the aims of the National Curriculum, whilst still preparing children so that they demonstrate their understanding in a test situation, is one of the challenges in Y6. Here are seven things to consider:

#### 1 Number Talks

Use the start or the end of the day for [Number Talks](#). Choose some number talks to start from a test question and focus on reasoning and decision making. For example, use Q18 from Paper 1 2016:

$$122\,456 - 11\,999 =$$

*(KS2 SATs 2016)*

Once different ways of finding the answer have been shared and discussed, ask the children to judge which they think are efficient and which inefficient and why and then to generate their own example calculations that could be solved efficiently in a similar way.

#### 2 Focus on 'what you know'

Encourage the children to identify things they know, make a displayed class list of these and keep adding to them throughout the year. Draw the children's attention to the list on a regular basis so that it becomes a habit for them to look for things they know. Force awareness of this by presenting them with questions which use things they know and asking them to identify what it is that they know that is useful. For example:

"These questions all use one thing we know on our class list – what is the thing we know and how is it useful in each of the questions?"

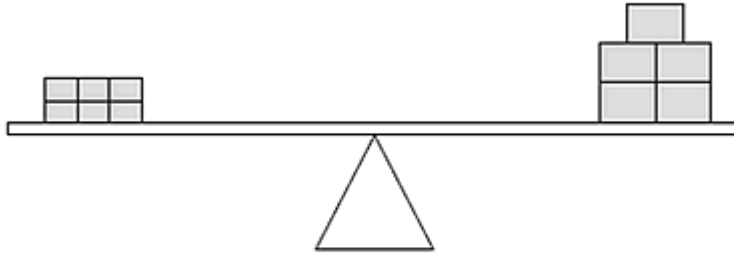
- $37 \times 2$
- $214 \div 2$
- A group of seven friends wins £14 million on the lottery; what do they each win?"

Ask the children to make up further questions which use the same fact or a group of questions which use a different fact.

#### 3 Build on what you know

Sometimes children struggle to get started with questions. Use the mantra 'What do you know? Write it down' and support the children to identify what they know (both from the content of the question and their own related knowledge) and to record it mathematically as a starting point. This gets children involved in the question and often takes them a significant way towards solving it. For example:

6 small bricks have the same mass as 5 large bricks.



The mass of one small brick is 2.5kg.

What is the mass of one large brick?

(SATs 2016)

From the question a child might record:

$$6 \text{ small bricks} = 5 \text{ large bricks}$$

$$1 \text{ small brick} = 2.5\text{kg}$$

Then from their own related knowledge they might record:

$$2 \text{ small bricks} = 5\text{kg}$$

$$\text{so } 6 \text{ small bricks} = 15\text{kg}$$

They are now halfway to solving the problem.

#### 4 Demonstrate understanding of structures and relationships

In [Digging Deeper](#) we identify that noticing things is a key part of fluency. Work with the children to notice things and then to generate their own related examples. For example, the following question involves no actual calculating:

$$326 \div 1 =$$

(KS2 SATs, 2016)

The children could generate further examples, using all the operations and combinations of operations, challenging themselves to devise what look like complicated calculations and questions but are in fact very simple, for example:

$$45\,678 \times 3\,722 \times 0$$

$$27\,696 + 38\,758 - 27\,697$$



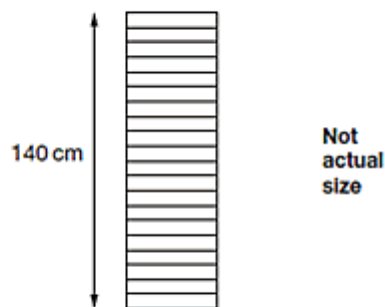
## 5 Reinforce understanding of place value

Understanding number and our number system is crucial to problem-solving and calculating. Use images such as place value charts, place value mats, place value counters and base ten to explore the multiplicative relationship between different parts of the

number system. Make a clear link between understanding whole numbers and understanding decimal numbers; base ten can act as a bridge between the two, with the thousand cube used to represent both 1000 and 1, in a relevant context such as 1000ml = 1l.

Provide opportunities for the children to identify how understanding place value is important in different problems, often in combination with a known fact. For example,

**13** A stack of 20 identical boxes is 140 cm tall.



Stefan takes **three** boxes off the top.

How tall is the stack now?

(KS2 SATs 2016)

and

Work out:

■  $8.4 \times 3 + 8.4 \times 7$

■  $6.7 \times 5 - 0.67 \times 50$

■  $93 \times 0.2 + 0.8 \times 93$

(NCETM 2015)

## 6 Use models/pictures to make sense of a problem so that it can be solved

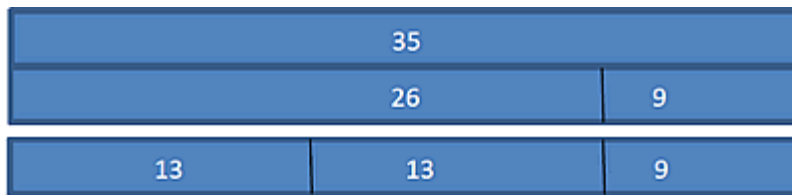
Encourage the children to draw something to model a problem and help them solve it and then discuss/share different ways and how effective they are. For example,

Sam has 9 fewer sweets than Sarah. They have 35 sweets altogether.

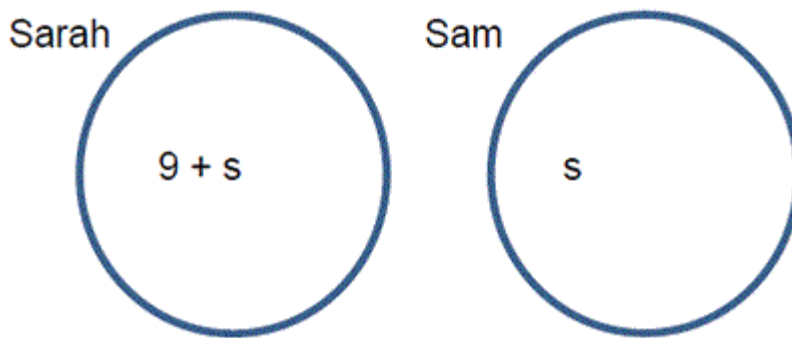
How many sweets does Sam have?

(NCETM 2015)

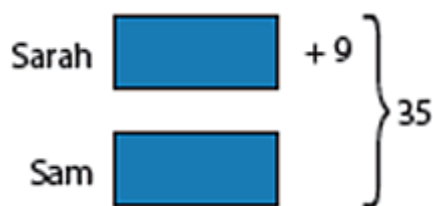
A child might draw:



Another child might draw:



A third child might draw:



The children can identify what is the same and what is different about the drawings, consider how each one helps to make sense of the problem and find a solution and try using the different models they have identified as effective to solve further problems.

## 7 Decide and justify when to use a written method

Present the children with a range of calculations and ask them to identify if there are any for which they would use a formal written method and why. Expect them to make this the last choice they make rather than the default choice, so that they start from what they notice and consider:

- Do I know the answer, because it is something I can recall or because of what I understand about the operation?

- Is the answer obvious because of what I understand about place value/our number system?
- Can I use what I know to find the answer easily?
- Can I use what I know and understand to find the answer, jotting down or modelling my route to help keep track?
- Are the numbers too awkward for working in any of the ways above and a written method is best?

Each time ask the children to generate further examples of calculations which match the ones presented, distinguishing between how they would solve each of them and justifying why.



## Seen and Heard

*Seen and Heard shines a light, via photographs and conversations from classrooms, on a specific example of the mathematics learning experience, the aim being to stimulate thought and questions about how you would react to similar events in your own classroom*

Responding to the following question from the end of KS1 test:

28



There are **40** crayons in a box.

Sam takes **17** crayons.

Kemi takes **10** crayons.

How many crayons are left?

three children recorded the following:

I counted bak 10 and  
then counted bak 17 and it  
equaled 23

23 crayons

$40 - 17 = 37$   $37 - 10 =$

27 crayons

12 crayons

- What does each child understand about how to solve the problem?
- What would you expect them to notice and know that would be useful for solving this problem?
- Where are the difficult parts for each child in reaching a correct solution? If each child had reached a correct solution, would you be happy that they are working as expected at the end of KS1? Why?
- How could you support the children in using their understanding of what happens when you subtract ten from a number to solve this problem? What images/resources might you use?
- What would you do next?

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