MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

1 The structure of the number system

1.1 Place value, estimation and rounding

Guidance document | Key Stage 3

Making connections

The NCETM has identified a set of six ‘mathematical themes’ within Key Stage 3 mathematics that bring together a group of ‘core concepts’.

The first of these themes is *The structure of the number system*, which covers the following interconnected core concepts:

1.1 **Place value, estimation and rounding**

1.2 Properties of number

1.3 Ordering and comparing

1.4 Simplifying and manipulating expressions, equations and formulae

This guidance document breaks down core concept *1.1 Place value, estimation and rounding* into four statements of knowledge, skills and understanding:

1.1.1 Understand the value of digits in decimals, measures and integers

1.1.2 Round numbers to a required number of decimal places

1.1.3 Round numbers to a required number of significant figures

1.1.4 Estimate calculations by rounding

Then, for each of these statements of knowledge, skills and understanding we offer a set of key ideas to help guide teacher planning.

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Overview

While an understanding of our base-ten place-value system for integers and decimals should be well established at Key Stage 2 (see 'Prior learning' section, below), several important ideas emerge at Key Stage 3.

It is important that students are aware of the general structure of the place-value system as based on powers of ten and begin to see how this naturally extends to decimals. Students need to progress beyond recalling place-value column headings when answering questions such as *'What does the 8 represent in 43 872?'* and appreciate that 43 872 has 438 hundreds and, later, that 43 872 is, in fact, 438.72 hundreds or 438.72×100. This learning will support students' work on significant figures and standard form, as students who can express numbers (including very large and very small numbers) in these different ways are more likely to have a feel for the size of such numbers and where they fit in the number system.

It is also important to emphasise the use of measures in real-life contexts. This will support students in understanding that measuring is always to a certain degree of accuracy.

This teaching will then support students' understanding and facility with estimating and rounding – essential skills for working with real-life situations involving contextualised data.

Prior learning

Before beginning to teach *Place value, estimation and rounding* at Key Stage 3, students should already have a secure understanding of the following from previous study:

Key stage	Learning outcome
Upper Key Stage 2	• Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit • Round any whole number to a required degree of accuracy • Identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1 000 giving answers up to three decimal places

You may find it useful to speak to your partner schools to see how the above has been covered and the language used.

You can find further details regarding prior learning in the following segments of the NCETM primary mastery professional development materials[1]:

- Year 4: 2.13 Calculation: multiplying and dividing by 10 or 100
- Year 5: 1.26 Composition and calculation: multiples of 1 000 up to 1 000 000
- Year 6: 1.30 Composition and calculation: numbers up to 10 000 000

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Checking prior learning

The following activities from the NCETM primary assessment materials[2] offer useful ideas for assessment, which you can use in your classes to check whether prior learning is secure:

Reference	Activity
Year 6 page 9	*Think about the number 34 567 800.* *Say this number aloud.* *Round this number to the nearest million.* *What does the digit '8' represent?* *What does the digit '7' represent?* *Divide this number by 100 and say your answer aloud.* *Divide this number by 1 000 and say your answer aloud.*
Year 6 page 10	*Estimate the answer to 4 243 + 1 734 by rounding the numbers to:* • *the nearest 1 000* • *the nearest 100* • *the nearest 50* • *the nearest 10.*

Key vocabulary

Term	Definition
decimal	Relating to the base ten. Most commonly used synonymously with decimal fractions where the number of tenths, hundredths, thousandths, etc. are represented as digits following a decimal point. The decimal point is placed at the right of the ones column. Each column after the decimal point is a decimal place. Example: The decimal fraction 0.275 is said to have three decimal places. The system of recording with a decimal point is decimal notation. Where a number is rounded to a required number of decimal places, to 2 decimal places for example, this may be recorded as 2 d.p.
significant figures	The run of digits in a number that are needed to specify the number to a required degree of accuracy. Additional zero digits may also be needed to indicate the number's magnitude. Examples: To the nearest thousand, the numbers 125 000, 2 376 000 and 22 000 have 3, 4 and 2 significant figures respectively; to 3 significant figures 98.765 is written 98.8

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Collaborative planning

Below we break down each of the four statements within *Place value, estimation and rounding* into a set of key ideas to support more detailed discussion and planning within your department. You may choose to break them down differently depending on the needs of your students and timetabling; however, we hope that our suggestions help you and your colleagues to focus your teaching on the key points and avoid conflating too many ideas.

Please note: We make no suggestion that each key idea represents a lesson. Rather, the 'fine-grained' distinctions we offer are intended to help you think about the learning journey irrespective of the number of lessons taught. Not all key ideas are equal in length and the amount of classroom time required for them to be mastered will vary, but each is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

The following letters draw attention to particular features:

D Suggested opportunities for **deepening** students' understanding through encouraging mathematical thinking.

L Examples of shared use of **language** that can help students to understand the structure of the mathematics. For example, sentences that all students might say together and be encouraged to use individually in their talk and their thinking to support their understanding (for example, *'The smaller the denominator, the bigger the fraction.'*).

R Suggestions for use of **representations** that support students in developing conceptual understanding as well as procedural fluency.

V Examples of the use of **variation** to draw students' attention to the important points and help them to see the mathematical structures and relationships.

PD Suggestions of questions and prompts that you can use to support a **professional development** session.

For selected key ideas, marked with an asterisk (*), we exemplify the common difficulties and misconceptions that students might have and include elements of what teaching for mastery may look like. We provide examples of possible student tasks and teaching approaches, together with suggestions and prompts to support professional development and collaborative planning. You can find these at the end of the set of key ideas.

Key ideas

1.1.1 Understand the value of digits in decimals, measures and integers

Understanding place value is a fundamental skill and at the heart of a strong sense of number. Students need to be able to correctly say any number and understand where it fits in the number system (i.e. in an ordered list of numbers and on a number line). The focus in this set of key ideas is understanding the structure of the system (that each column value is a power of ten and that multiplying or dividing by ten shifts digits from one column to the adjacent one).

1.1.1.1 Understand place value in integers

1.1.1.2* Understand place value in decimals, including recognising exponent and fractional representations of the column headings

1.1.1.3 Understand place value in the context of measure

1.1.1.4 Order and compare numbers and measures using <, >, =

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1.1.2 Round numbers to a required number of decimal places

Students need to understand why rounding is necessary and that it is a valuable tool for estimating number to varying degrees of accuracy. Rounding to a number of decimal places is particularly useful when working with measures in real-life contexts. An important awareness is that rounding to two decimal places (for example) involves choosing between two numbers; one that is just greater than it and one that is just less than it, both of which have two decimal places. Memorising and applying a procedure for rounding a number to a specified number of decimal places without this overall awareness often results in errors.

1.1.2.1 Round numbers to three decimal places

1.1.2.2 Round numbers to any number of decimal places

1.1.3 Round numbers to a required number of significant figures

It is important for students to develop a strong sense of the size of numbers and be able to use various methods of rounding, especially when giving answers in context.

Rounding large numbers is particularly useful when estimating (for example, crowds at a football match or winnings in a lottery).

1.1.3.1 Understand the concept of significant figures

1.1.3.2* Round integers to a required number of significant figures

1.1.3.3 Round decimals to a required number of significant figures

1.1.4 Estimate calculations by rounding

Estimation is a key skill that contributes to students' fluency in calculation. Fluency demands that students have strategies for checking the validity of their answers. Students who are proficient in carrying out algorithms, but who have no idea whether the answer to a calculation is sensible or not, are not fully fluent.

1.1.4.1 Understand what is meant by a sensible degree of accuracy

1.1.4.2* Estimate numerical calculations

1.1.4.3 Estimate and check if solutions to problems are of the correct magnitude

1.1.4.4 Determine whether calculations using rounding will give an underestimate or overestimate

1.1.4.5 Understand the impact of rounding errors when using a calculator, and the way that these can be compounded to result in large inaccuracies

1.1.4.6 Calculate possible errors expressed using inequality notation $a < x \leq b$

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Exemplified key ideas

1.1.1.2 Understand place value in decimals, including recognising exponent and fractional representations of the column headings

Common difficulties and misconceptions
Students are likely familiar with place value charts and the column headings in both words (tens, ones, tenths, hundredths ...) and as decimals (10s, 1s, 0.1s, 0.01s ...) (for example, in the Year 4 *Primary Mastery Professional Development Materials, Topic 1.24*) but may need to revisit column headings written as fractions and exponents. Understanding that a mathematical object can have the 'same value but a different appearance' is a key understanding in maths, and students may find it challenging to recognise that a column headed as tenths, $\frac{1}{10}$, 0.1 or 10^{-1} will represent digits of equal value. A focus for this key idea is the structure of the place value system and the connections within it. This builds on the understanding that students developed at Key Stage 2 and moves to a more general appreciation of the multiplicative relationships between columns.

What students need to understand	Guidance, discussion points and prompts
Know and understand powers of 10. *Example 1:* *Write down as many three-digit powers of 10 as you can.*	**V** *Example 1* invites students to think about what makes a number a power of 10: considering what it is, what it *also* is, and what it's not. By working with student-generated examples the teacher is able to gather 'boundary cases' while also assessing the students' current understanding. For example, a student might offer 010 as a three-digit power of 10 and, in discussing the validity of this, the key features of powers of 10 can be highlighted. If not forthcoming from students, the teacher might offer 0.10, or 0.01 and so the task provides an opportunity to expand the range of examples that students can draw on.
Interpret powers of 10 written using index notation. *Example 2:* *Which of these calculations **cannot** be answered by placing a single digit in the box?* a) $10 \times 10 \times 10 \times 10 = 10^{\square}$ b) $10 + 10 + 10 + 10 = 10^{\square}$ c) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 10^{\square}$ d) $10 \times 5 = 10^{\square}$	While *Example* 1 focuses on understanding the exponential nature of the powers of 10, here we focus on understanding the notation. **V** *Example 2* offers an opportunity for students to think about what is and what is not a power of 10, and to use correct notation to record this. Part b) draws attention to the multiplicative nature of exponentiation. Part c) offers an opportunity to discuss the role of the base and the exponent in the notation (drawing a distinction between 2^{10} and 10^2).

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e) $1 \times 10 \times 10 \times 10 = 10^{\square}$ f) $1 = 10^{\square}$	**V** Parts e) and f) introduce a 1 to the beginning of the expansion. This inclusion of the multiplicative identity may help students make sense of the fact that $m^0 = 1$. By describing part e) as *'one multiplied by ten three times'*, part a) can then be described as *'one multiplied by ten four times'* and part f) is *'one multiplied by ten zero times'*. **PD** It is common to describe a number such as 10^3 as *'ten multiplied by itself three times'*. Do you think that the description of *'one multiplied by ten three times'* offers a greater insight for students, muddies the water further, or makes little difference?
Example 3: *10^9 is a power of 10.* *a) How would you find the next power of 10 (and what is it)?* *b) How would you find the previous power of 10 (and what is it)?*	This task is an opportunity to explore the structure of powers of 10, drawing attention to the multiplication that underpins them. Moving from one power of 10 to the next is a key idea when working with place value. **PD** An *'always, sometimes, never'* question can be useful for exploring mathematical structure with students. For example, *'Is it always, sometimes or never true that multiplying two powers of 10 together will generate another power of 10?'* • Would you agree that this *'always, sometimes, never'* question and *Example 2* draw attention to the same mathematical structure? • When might you ask one question and when would you choose the other?
Example 4: *Fill in the gaps:* *$10^3 = 1 \times 10 \times 10 \times 10 = 1\,000$; 10^3 is 1 000 times greater than 1.* *$10^2 = 1 \times 10 \times 10 = 100$; 10^2 is ___ times greater than 1.* *$10^1 =$ ___ $= 10$; 10^1 is 10 times greater than 1.* *$10^0 = 1$; 10^0 is equal to 1.* *$10^{-1} = 1 \div 10 = 0.1$; 10^{-1} is 10 times ___ than 1.* *If the pattern continues, what would the next row be?*	**L** In *Example 4*, students continue the pattern and should notice the way the powers of 10 extend to negative numbers. The use of a written description is intended to make explicit the structure underpinning that pattern.

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Example 5:

$10^3 = 1\,000$; 10^3 is 10 times greater than 10^2

$10^2 = 100$; 10^2 is ___ times greater than 10^1

$10^1 = 10$; 10^1 is 10 times greater than 10

$10^0 = 1$; 10^0 is 10 times greater than ___

$10^{-1} = 0.1$; 10^{-1} is 10 times ___ than 10^{-2}

Example 5 builds on *Example 4* but this time the language draws attention to the relationship between consecutive powers of 10 when represented using index notation. This understanding is key when working with column headings in place value charts.

Connect different representations of column headings.

Example 6:

Fill in the gaps so that each column shows a different way of writing the same value.

As a fraction		$\frac{1}{100}$		$\frac{1}{10000}$
As a decimal		0.01	0.001	
As a power of 10		10^{-2}		
In words	one-tenth	one-hundredth		

R Although not made explicit in the wording of the task, *Example 6* shows the four consecutive columns of a place value chart and the different ways in which the column headings might be represented. It is important the students understand that although the representations shown in the columns look different, they represent the same value.

Example 7

The value of the digit 7 in the number 5.703 could be thought of in different ways, including 7×0.1, 7×10^{-1} and $7 \times \frac{1}{10}$. These products can be written as 0.7 or $\frac{7}{10}$ and can be said as 'zero point seven' or 'seven tenths'.

a) Write similar sentences about the value of the digit 2 in the number 5.203.

b) Write similar sentences about the value of the digit 2 in the number 5.032.

V Students should be aware that the value of a number depends on both the digits that represent the number and the position of those digits.

In part a) of *Example 7*, the value of the second digit has changed and in part b) the position of that digit has changed.

Understand the multiplicative relationships between columns in the place value structure.

Example 8

*The value of the **bold red** digit in the number 17.**4**05 is four tenths.*

*In each of the following, **how many times** the size of **four tenths** is the value of the **bold blue** digit?*

*a) **4**7.105*

*b) 1**4**.705*

*c) 17.50**4***

V *Example 8* asks students to work multiplicatively between place value columns.

Parts a) and b) offer the chance to explore the multiplicative connections moving to the left. Part c) involves a multiplicative connection in which the product is smaller and so may raise additional challenges for students and provoke fruitful discussion.

R Although not used here, presenting the task in a place value chart or a Gattegno chart may support students in making connections between the position and the value of the digit.

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Example 9: *The value of the* ***bold red*** *digit in the number 17.****4****05 is four tenths.* *In each of the following,* ***how many times*** *the size of* ***four tenths*** *is the value of the* ***bold blue*** *digit.* *a) 17.****8****05* *b) 1****8****.705* *c) 17.2****4****5* *d) 17.4****8****5* *e) 17.48****2*** *f)* ***1****7.405*	**V** *Example 9* builds on *Example 8* and challenges students to work multiplicatively within and between place value columns. Part a) offers students an opportunity to work within a place value column then part b) moves across columns. This pattern is reversed in parts c) and d). The intention of the task is to make explicit the connections between place value and multiplication, and to support students in building a deeper and rounded understanding of the multiplicative connections that exist. **PD** Compare *Examples 8* and *9*. Do you think there is value in multiplicatively changing the digits within a column and across columns (as in *Example 9*), or does it detract from the focus on place value offered by keeping the same digit (as in *Example 8*)?

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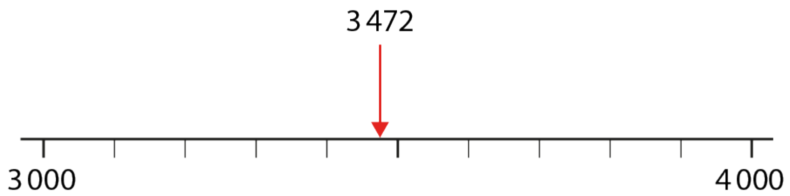
1.1.3.2 Round integers to a required number of significant figures

Common difficulties and misconceptions

Students may see the task of rounding as an algorithm to follow without appreciating the idea that they are trying to find a number (with a specified number of significant figures) to which the chosen number is closer. For example, students may keep on rounding until they achieve a number to one significant figure, thus:

3 472 → 3 470 (because two is less than five) → 3 500 (because seven is more than five) → 4 000 (because five is halfway) and not realise that 3 472 is closer to 3 000 than 4 000.

R A number line could be used to support students' understanding. Locate the number to be rounded on the line and identify the critical values of 3 000 and 4 000 either side of it. This should help students to see that 3 472 is closer to 3 000.



Students can also find it challenging to identify when a zero digit is significant.

Designing questions that contain zero digits in a variety of positions (i.e. within a number and at the end of a number) will help challenge students' understanding and enable you to identify and address misconceptions. Students should experience situations where they are asked to round to more significant figures than the number has digits (for example, rounding 96 to three significant figures). Knowing what to do in these scenarios, and how zero digits can be used, is important in developing a comprehensive understanding of the concept. For a deeper understanding, it will also be important to offer students the opportunity to think about numbers that have already been rounded (as in *Example 9*, below).

V Avoid mechanistic practice of exclusively standard questions, as this can result in students stopping thinking and blindly following a procedure. Carefully design questions which draw students' attention to a specific aspect by varying one element at a time (see *Example 1*), as this will enable students to notice what is happening and develop a deeper understanding of the structure.

The use of non-standard examples, examples of errors or non-examples (such as *Examples 5* and *6*) for students to critique and reason about, and asking students to solve problems in a number of different ways, can all support students to overcome difficulties and establish a secure understanding.

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What students need to understand

Understand the value of the ones digit and how it impacts on rounding.

Example 1:

a) *Round 84 to the nearest 10.*
Round 84 to one significant figure.

b) *Round 86 to the nearest 10.*
Round 86 to one significant figure.

c) *Round 85 to the nearest 10.*
Round 85 to one significant figure.

d) *Round 95 to the nearest 10.*
Round 95 to one significant figure.

Guidance, discussion points and prompts

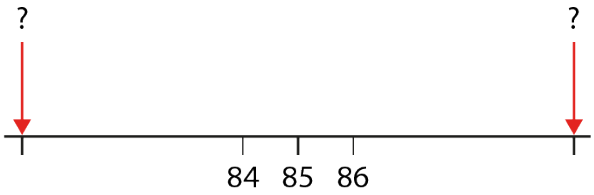
V Consider the use of variation in *Example 1*. These problems have been carefully chosen to draw out the following nuances:

- For part a), students need to round down.
- For part b), students need to round up.
- For part c), students need to think about what to do at the midpoint (five).
- For part d), students need to round up again, but this time to the hundreds boundary.

Notice that the numbers students are given to work with are all two-digit numbers. Ensure students are confident working with two-digit numbers before introducing larger numbers. Students are also only dealing with rounding to one significant figure to begin with. Once they are secure on rounding to one significant figure, further numbers of significant figures can be introduced.

R Students could be encouraged to think more deeply about these results by considering the numbers involved on a number line.

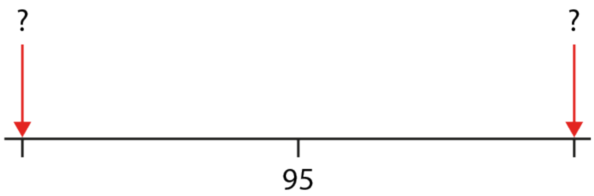
You could draw this diagram on the board:



and ask questions, such as:

- *'What are the numbers (to one significant figure) which are either side of 84, 85 and 86?'*
- *'Can you make sense of your answers to parts a), b) and c) by referring to this diagram?'*

You could draw a diagram like this:



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	and ask: • *'What numbers (to one significant figure) are either side of 95?'* • *'What other numbers would be rounded to 100?'*
Notice similarities and differences when rounding to the nearest 10, the nearest 100 and to one significant figure. *Example 2:* *a) Round 61 to the nearest 10.* *Round 61 to one significant figure.* *b) Round 185 to the nearest 10.* *Round 185 to one significant figure.* *c) Round 349 to the nearest 100.* *Round 349 to one significant figure.* *d) Round 5 419 to the nearest 100.* *Round 5 419 to one significant figure.*	**V** The choice of what to vary and what *not* to vary and the pairing of questions can draw students' attention to key ideas. In *Example 2*, students are likely to notice that the answers to part a) are the same but the answers to part b) are not. Discussing why this is so is an important element of these questions and will encourage reasoning about what the significant digit is in each case. Similarly, students will notice that the answers to part c) are the same, but the answers to part d) are not. Again, discussing why this happens will help students to gain a deep and sustainable understanding of the process of rounding. They should realise when rounding to the nearest 10 or 100 is the same as rounding to one significant figure, and when it is not. Such questions, together with appropriate teacher intervention and questioning, can help students not just to focus on the process of 'getting an answer' but to understand the concepts involved. **PD** What teacher questions and prompts might be helpful to offer alongside these examples and at what stage might they be offered? **R** Students could be asked to draw a diagram to explain why, for example, 5 419 rounds to different numbers in the two parts of part d), and be asked questions, such as: • *'What numbers would you put either side of 5 419 when thinking about rounding to the nearest 100?'* • *'What numbers would you put either side of 5 419 when thinking about rounding to the nearest one significant figure?'*

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Appreciate that the value of the first digit needs to be reflected in the size of the final answer.

Example 3:

a) *Round 40 to one significant figure.*
Round 46 to one significant figure.
Round 460 to one significant figure.
Round 4 600 to one significant figure.

b) *Round 800 to one significant figure.*
Round 807 to one significant figure.
Round 8 007 to one significant figure.
Round 80 007 to one significant figure.

Choose problems that draw attention to which digits are important when rounding to one significant figure.

R In *Example 3*, part a) is designed to encourage students to realise the importance of place value. The first most significant digit is the 4, but it really means 4 tens, or 4 hundreds, or 4 thousands. In part b), students must consider not only the place value of the digit 8, but also that zeros within a number are also significant and may impact on rounding.

L Consider verbalising what is going on as a class: *'When rounding to one significant figure, consider the value of the digit to the right of the first significant figure.'*

D To prompt some deeper thinking you may wish to ask whether 807 rounded to one significant figure is more accurate, less accurate or equally as accurate as 80 007 rounded to one significant figure.

Extend understanding to round to more than one significant figure.

Example 4:

Round each number to the required number of significant figures (s.f.).

		1 s.f.	***2 s.f.***	***3 s.f.***
a)	*305*			
b)	*8 953*			
c)	*18 000*			
d)	*47*			
e)	*9 999*			

Can you give an example of a number that remains the same when rounded to one, two or three significant figures?

V In *Example 4*, the numbers have been chosen carefully so that sometimes the number changes when rounded to differing degrees of accuracy and other times it stays the same.

In part a), the results when rounded to one, two or three significant figures are all different. The original number contains a zero digit, so it is important that students understand when a zero is significant and when it is not.

In part b), the results when rounded to one and two significant figures are the same. Students should experience situations when this happens and explore why it is the case, so that they do not assume they have made a mistake if a number is the same when rounded to different degrees of accuracy.

In part c), the number contains zeros at the end, so again, students should discuss whether these are significant or not.

Part d) is a two-digit number and so raises the question of what to do when rounding to three significant figures (i.e. 47.0). Exposing

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students to these difficulty points early on will help reduce misconceptions and enable a deep understanding to be developed.

Part e) is an example of a particular idea that students often have difficulties with (i.e. a trail of 9s, which might result in doing a lot of sequential rounding up rather than seeing the size of the number 9 999 and appreciating that it is 10 000 to one, two or three significant figures). A number line may help students see clearly which number to round to and avoid confusion.

Example 5:

a) *What values can each missing digit take?*

(i) *4 ?90 = 4 000 to one significant figure*

(ii) *9 ?12 = 10 000 to one significant figure*

(iii) *34 ?90 = 30 000 to one significant figure*

(iv) *6 78? = 7 000 to one significant figure*

b) *A number is 1 900 when rounded to two significant figures and 1 850 when rounded to three significant figures. What could the number be?*

V Notice the use of a non-standard question in *Example 5*, where the rounded numbers are given; the task in part a) is to work out missing digits in the original numbers. Such small changes can be very useful in supporting students to stop and think deeply about what they are doing and help to guard against unthinking mechanical practice.

D Guiding students to think carefully about rounding to one significant figure in part a), before extending their thinking to two or three significant figures in part b), will help scaffold their reasoning and enable more students to access the challenge.

Example 6:

Which of these is correct? Justify your answer.

a) *29 354 rounded to three significant figures is:*

(i) *29 400*

(ii) *29 300*

(iii) *294*

b) *6 582 rounded to two significant figures is:*

(i) *66*

(ii) *7 000*

(iii) *6 600*

c) *40 073 rounded to one significant figure is:*

(i) *40 000*

(ii) *40*

(iii) *50 000*

V Another aspect of variation is to include examples of *'what it's not'* (as well as what it is), helping students to clarify the idea in their minds.

In *Example 6*, the options are carefully designed to address various misconceptions.

In part a), (i) is the correct answer. If students choose (ii), they may have focused on the first three most significant figures and not considered the impact of the digit to the right of the 3. If students choose (iii), they may have focused on the first four digits (the first three most significant and the one to the right) – the value of the 5 indicates it should round up to 294 but the place value has been lost.

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	In part b), (iii) is the correct answer. If students choose (i), they have focused on the first two digits and the digit to the right and rounded 65 to 66, but incorrectly lost the place value in the process. If they choose (ii), they have only focused on the first most significant figure. In part c), (i) is the correct answer. If students choose (ii), they have considered the first two digits (the first most significant and the one to the right) – the value of the zero indicates it should stay as 40 but the place value has been lost. If students choose (iii), they may not realise that zeros within a number are significant, so think the second most significant digit is the 7, which causes the number to round up to 50 000. **R** Students could be asked to draw number line diagrams to explain why the examples above are true or not true. **PD** Can you construct some other *'what it's not'* examples that might highlight the fact that the first digit needs to be reflected in the size of the answer?
Understand 'what is the same' and 'what is different' when rounding to the nearest 10, to the nearest 100 and to one significant figure. *Example 7:* *Cara says, 'Rounding integers to one significant figure is the same as rounding to the nearest 10, 100 and 1 000.'* *Is Cara correct? Explain your answer.*	Ask students to give examples of when rounding to these three different degrees of accuracy are the same and when they are different, with reasoning.
Understand the importance of place value and maintaining size when rounding. *Example 8:* *Jobin says, '48 700 rounded to one significant figure is 50.'* *Explain why Jobin is incorrect, using a number line to support your reasoning.*	Getting students to discuss and explain why certain statements are wrong is a strategy to encourage reasoning (a fundamental aim of the national curriculum). Thinking deeply and identifying a possible misconception within a given statement (in this case, the disregard of place value) may help students to become less reliant on a procedural understanding and more aware of the concept. Considering the use of a number line to support their mathematical thinking may not only enable

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	students to see the value of using representations, but also to easily identify the misconception that has been made and consequently make it less likely that they too will make the same error.
Explore the significance of zero digits when rounding to significant figures. *Example 9:* *A number has been rounded to a number of significant figures, with the result of 76 500.* *a) Kayla says that it has been rounded to three significant figures. Lakshmi says that it has been rounded to four significant figures. Who is correct? Why?* *b) What might the original number have been before rounding?*	Students will have previously encountered numbers containing zero digits when being required to round to various numbers of significant figures (as in *Examples 3* and *4*). In order to consolidate and deepen this understanding it is important that students should also experience the reverse procedure, i.e. finding the original number given the rounded number. Students should be given the opportunity to reason fully regarding the zero digits in the answer and which of these may be significant. Providing an example, such as *Example 9*, where either statement could be correct, will challenge students' understanding.
Consider real-life applications of rounding and its limitations. *Example 10:* *The capacity of Liverpool's football stadium is 50 000 people when rounded to one significant figure. The capacity of Newcastle's football stadium is 52 338 people. Sam says, 'More people fit into Liverpool's stadium than Newcastle's.' Bano says, 'More people fit into Newcastle's stadium than Liverpool's.'* *Who is correct? Why?*	Real-life contexts are important not just for their practical use and application, but also as a motivating technique. While deeply understanding the mathematical structure of numbers and the number system is important, students also need to be able to feel that the mathematics they are learning is related to their own experiences and interests. It is therefore important that contexts take account of the interests of all your students and are varied accordingly. **PD** Can you create some other contexts suitable for your classes where the need for rounding is relevant?

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Solve problems where there is more than one answer and there are elements of experimentation, investigation, checking, reasoning, proof, etc. *Example 11:* *a) A number has been rounded to 30, correct to one significant figure. Can you give an example of what that number might be?* *b) Can you find another example?* *c) Can you describe all the numbers that round to 30 (to one significant figure)?* *d) Can you show this on a number line?*	The tactic of asking students to find one answer, then another, then another and so on, is a good way of supporting students in generalising. **V** In *Example 11*, by asking students to find one, then two and then all possible answers, they are generalising to find the upper and lower bounds. This process is then useful for *Example 12*, and for future learning.
Example 12: *When a certain four-digit number is rounded to two significant figures, the answer is 8 000.* *a) What is the greatest value the number could be?* *b) What is the smallest value the number could be?*	**R** If students are struggling, you could give them number lines to help them identify what options they could round to. Prompt them to consider strategies for finding all the options. Discuss whether the size of the initial number affects the possibilities.
Example 13: *When rounding an integer to three significant figures, the answer is 13 700. How many possibilities are there for the original integer?*	**D** For problems such as those shown opposite, you might also ask students to identify non-integer values that round to the given number. You could encourage students to go deeper by beginning to explore upper and lower bounds (as in *Example 12*); however, be mindful not to progress some students onto this new key idea before the whole class is ready to work together on it.

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1.1.4.2 Estimate numerical calculations

Common difficulties and misconceptions
It is important that students acquire a secure conceptual understanding of estimation, what it is and why it is useful, alongside developing procedural fluency. If the focus is purely on the need for rounding, then some students may find it hard to recall at which stage the rounding should take place. This can result in the misconception that estimation involves rounding the final result of a calculation, rather than rounding the numbers involved prior to calculating. Estimation builds upon prior learning of rounding (usually to one significant figure), so it is imperative that students have a deep and secure understanding of this before approaching this key idea. Some students find rounding decimals (e.g. 0.541) problematic and so it is worthwhile addressing this explicitly (as in *Example 3*). Students can find it especially challenging if the calculation requires a division by a decimal less than one, so it is worth assessing prior understanding of this skill before introducing questions such as those in *Example 4*. **PD** In Key Stage 3, students should be confident rounding to varying degrees of accuracy. This will prepare them well for Key Stage 4, when there will be more of an emphasis on making a choice of the degree of accuracy depending on the context. Students might find it easier to make sense of the concepts involved when they are given problems set in familiar contexts (see *Examples 5* and *6*). You may be able to design further questions which are better suited to the needs and interests of your students. This allows them to see clearly the usefulness of estimation and make connections to other areas of mathematics. It will also lay the foundations for future learning about how estimation can be used to check the relative size of answers and will provide opportunities for students to justify whether it is an underestimate or overestimate.

What students need to understand	Guidance, discussion points and prompts
Understand what estimation is and how it is useful. *Example 1:* a) *Livia says that estimating is just guessing the answer. Do you agree with Livia?* b) *Anton says 3.6273 + 52.99184 = 56.61914 and this can then be estimated as 60. Do you agree with Anton?* c) *Betty says 579.304 – 102.968 can be estimated as 600 – 100 = 500. Do you agree with Betty?* d) *Discuss this with your partner and come up with an accurate definition for estimation. Consider how and why estimation might be useful. Try to give some examples of when estimating might be helpful.*	**V** Students can misunderstand estimation as just guessing or approximating answers without recognising the process involved. Drawing students' attention both to what estimation is and what it is not, allows them to form a more comprehensive understanding of the concept. *Example 1* shows a common misconception that students face when estimating calculations, that is, when to round. Anton calculates and then rounds, whereas Betty rounds and then calculates. If students are given time to both consider why estimation is useful and some real-life applications of estimation, then they are less likely to calculate without rounding first.

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Understand how to estimate answers to numerical calculations. *Example 2:* a) *Estimate the answers to these calculations.* (i) $45.719 + 7824.53$ (ii) $192.8 - 34.271$ (iii) 17.935×8.47 (iv) $\frac{77.1}{1.63}$ (v) $(11.9)^2$ (vi) $\sqrt{59}$ b) *Did you get the same estimates as your partner? Discuss the methods you used.* c) *Why might someone have a different estimate to you?*	**V** *Example 2* is a series of calculations which require one operation. They require students to round numbers of varying sizes to one significant figure. Parts (i) to (iv) are the four basic operations, whereas parts (v) and (vi) are less standard. Part (vi) will require knowledge of square numbers and corresponding roots. Consider giving students time to compare methods and answers with their peers and discuss whether there might be more than one acceptable answer. In a topic such as estimation, students should explore what constitutes an appropriate degree of accuracy in different contexts. They should consider why they might be estimating in the first place and, therefore, how accurate they need to be in their rounding.
Example 3: *Martha estimates that* $\frac{14.5}{0.512} = 14.5$. *Martha is incorrect.* a) *What misconception might Martha have?* b) *What would be a more appropriate estimate?*	**V** *Example 3* is an example of a common misconception that students often have surrounding how to round decimals less than one. Since 0.5 rounds to one, many students may incorrectly assume that 0.512 will also round to one, when rounded to one significant figure. **PD** Consider spending time assessing students' understanding of rounding decimals to one significant figure and revisiting learning if required, before progressing. Where do these topics sit in your scheme of learning? How long has it been since students first learnt rounding to significant figures? What is the best method to recap prior learning?
Example 4: *Estimate the answers to these calculations.* a) $\frac{5742 \times 34.7}{875.24}$ b) $\frac{15.3 - 7.81}{0.24}$ c) $(46.32)^2 + 7326$ d) $(61.35 - 84.2)^2$ e) $\frac{9.6 + 16.51}{(7.51)^2 - 3.997}$	**V** *Example 4* is a series of calculations which require more than one operation. Students will need to recognise the multiple steps involved and consider the order in which to perform the operations. The numbers involved are of varying sizes, so students will need to practise rounding accordingly.

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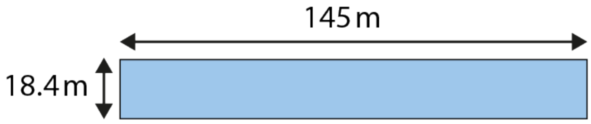
Understand how to apply estimation to problems in a given context.

Example 5:

A rectangle has dimensions 145 m by 18.4 m.



Which calculation would give a suitable estimate for the area of this rectangle?

a) 150×18

b) 150×20

c) 100×20

d) 18×145

V In *Example 5*, the estimation is for finding the area of a rectangle, a context that students should already be familiar with from Key Stage 2. Students must select an appropriate calculation, with options designed to identify common misconceptions.

In part a), students may have rounded to two significant figures. In part b), they have rounded to the nearest 10. Numbers in part c) have been rounded to one significant figure. In part d), they have been rounded to the nearest integer.

PD It could be argued that all of the options are suitable estimations for this question. How could you facilitate discussion in your classroom, so that students present their opinions and understand that the nature of estimation means there is often more than one correct answer?

As a department, how have you decided what you deem to be an appropriate level of accuracy when estimating? How do you communicate this with your students?

What other contexts would your students find useful? How can you make connections to other areas of mathematics?

Example 6:

A school puts on a show and charges £4.50 per seat. The school hall has capacity for 27 rows of chairs with 48 chairs in each row.

a) Estimate the amount of money the school will take if the show is sold out.

b) Calculate the amount of money the school will take if the show is sold out.

c) Compare your answers to parts a) and b) above. Explain why they are different.

d) Olivia calculates the total income for the school as £583 200. Explain how Olivia could use estimation to help check her answer.

D Students will inevitably come up with different estimates for part a), so it will be important to discuss these and encourage students to justify different methods and judge their relative merit.

For example:

- $30 \times 50 \times 4.5$ for a factor of 100 to be removed, giving 15×450
- $30 \times 50 \times 5$ might be too much rounding up, resulting in an excessive overestimate
- $25 \times 50 \times 4.5$ might be a better estimate as one number has been rounded up and the other rounded down.

It is often through the discussion after students have worked on a set of examples that the real learning takes place, so it will be

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	important to give enough time to this element of the classroom activity. **PD** This example may lead to discussions related to future work on underestimates and overestimates. What questions might you ask to prepare the ground for this?
Understand how to use and apply estimation in problem-solving situations. *Example 7:* *Use the digits 1–9 once only to create a calculation that would produce the following estimate:* $\dfrac{\square\square+\square\square.\square}{\square.\square\square} \approx \square 0$	**D** *Example 7* is an empty box problem which has more than one solution. Questions like these encourage students to consider the overall structure of the calculation. Asking students to explain the process they went through to find a solution will also help to refine their mathematical thinking. **PD** How would you manage a question like this with your classes? How long would you give them before you intervene and support? What prompt could you offer? How can you support students who are struggling to access the task?

Weblinks

1. NCETM primary mastery professional development materials https://www.ncetm.org.uk/resources/50639
2. NCETM primary assessment materials https://www.ncetm.org.uk/resources/46689

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