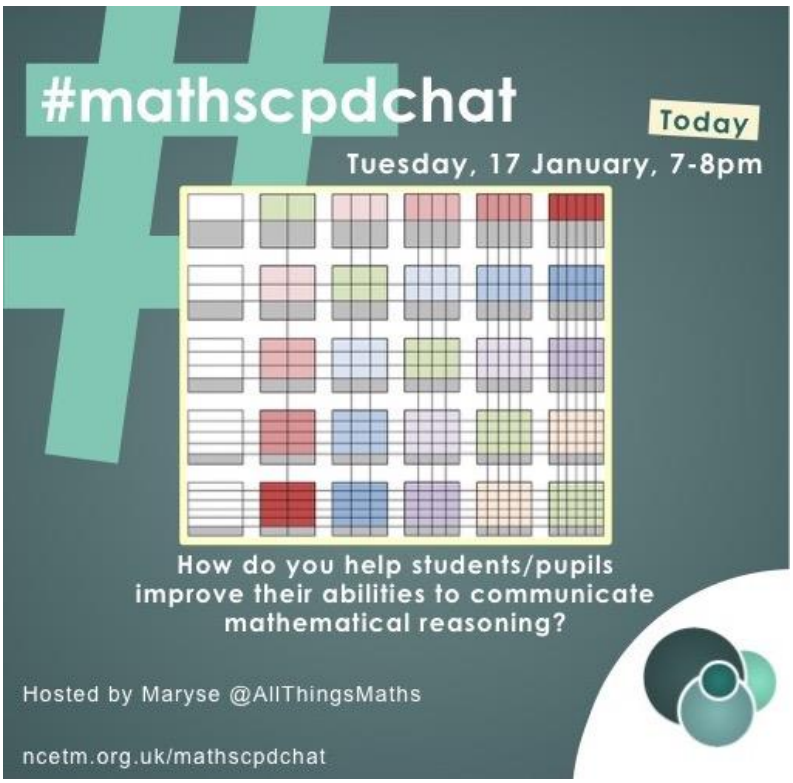
# #mathscpdchat 17 January 2023

**How do you help students/pupils improve their abilities to communicate mathematical reasoning?**
Hosted by Maryse

*This is a summary of the discussion – to see all the tweets, follow the hashtag* ***#mathscpdchat*** *in Twitter*




The links shared during this discussion were:

NCETM Checkpoints which are diagnostic maths activities to help teachers develop their assessment of students' prior learning for KS3. Wrapped around each activity, in each set of PowerPoint slides, is comprehensive guidance and advice for teachers. This guidance has been fully explained in professional development seminars. You can watch videos of these seminars on this page. The examples shared and discussed during this #mathsCPDchat were ***Checkpoint 5: True or False?*** from the collection ***Understanding multiplicative relationships: fractions and ratio***, for Y7, and ***Checkpoint 2: function machine pairs*** from the collection ***Properties of number***, for Y7. It was shared by Maryse

Using Mathematical Representations at KS3 which is guidance from the NCETM for some of the most useful representations for Key Stage 3 mathematics. It was shared by Maryse

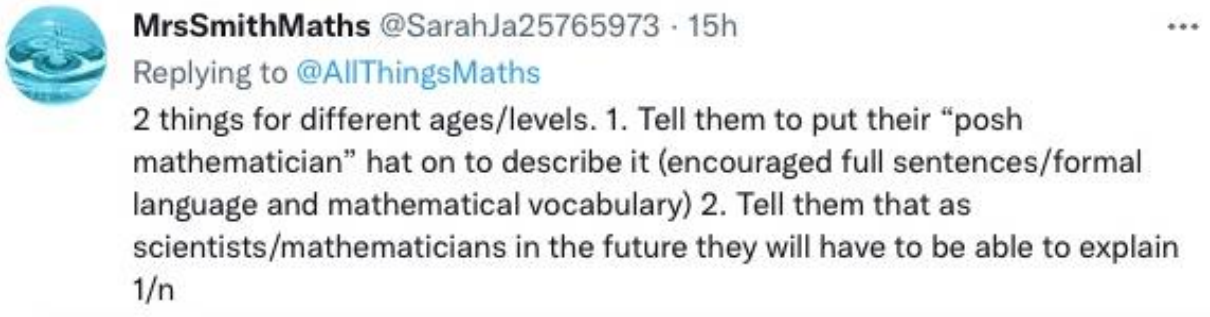
Secondary Magazine 144 which is an archived NCETM Secondary Magazine from 2017. This issue includes an article *Addressing the Reasoning and Problem-Solving demands of the new GCSE*, from which extracts were shared and discussed during this #mathsCPDchat. It was shared by Mary Pardoe

Teaching Mathematics at Secondary Level which is a book by Tony Gardiner. It focuses on central principles and concepts of school mathematics, and so provides food for thought for all mathematics teachers. It was shared by Mary Pardoe

**An illustrated summary of the discussions in this #mathsCPDchat follows.**

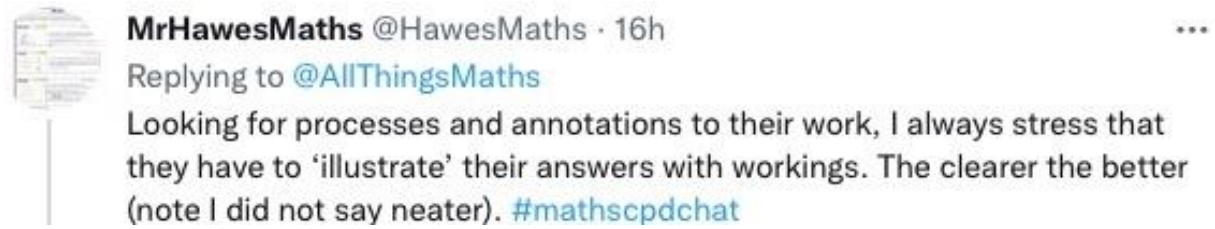
**The host's introductory tweet …**


**… prompted these comments …**





**… and this short conversation (later during the chat):**



**Maryse** @AllThingsMaths · 16h
Replying to @HawesMaths
Love the use of "illustrate" and I'm with you on clearer over neater.

#MathsCPDChat

**MrHawesMaths** @HawesMaths · 16h
Replying to @AllThingsMaths
I was looking at a bob ross meme and I thought that written communication is an art to be mastered and anyone can do it. So let's illustrate it
#mathscpdchat

**Maryse** @AllThingsMaths · 16h
Replying to @HawesMaths
This is one of the beauties for me. Mary shared an example earlier of a bunch of text reduced to a simple two way table. That simplicity of info is fab (IMO)

#mathsCPDChat

**This next main question from the host …**

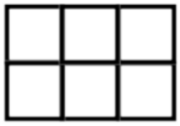
**Maryse** @AllThingsMaths · 16h
Are there are strategies you find useful when explaining to students, that they can then in turn use to explain their own thinking?

**#MathsCPDChat**

**… was followed by a sequence of replies from Maryse herself …**

**Maryse** @AllThingsMaths · 16h
Replying to @AllThingsMaths
One of my favourite in proof is representing odd and even numbers visually in proof.

**Maryse** @AllThingsMaths · 16h
An even number can represented:
#mathscpdchat

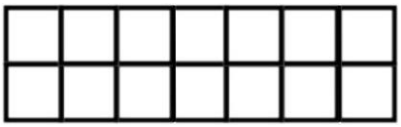


**Maryse** @AllThingsMaths · 16h

So an odd + odd = even

We then move forward to express algebraically

#MathsCPDChat



**… before this conversation developed:**

**Susan Whitehouse** @Whitehughes · 16h

Replying to @AllThingsMaths

It's not original but I like to try to make sure that I have an algebraic explanation, a graphical/pictorial illustration and a numerical example wherever I can #mathscpdchat

**Maryse** @AllThingsMaths · 16h

Replying to @Whitehughes

Love that! The different ways complement each other and show why/how each representation works

#mathscpdchat

**Maryse** @AllThingsMaths · 16h

Replying to @Whitehughes

Is there any particular topic you find this especially helpful with?

#mathscpdchat

**Susan Whitehouse** @Whitehughes · 16h

Replying to @AllThingsMaths

I try to do it across the board because I'm aware that the students all have different sticking points, and I want to help them make connections in their own individual mathematical networks. So I almost deliberately avoid thinking about where it is most useful! #mathscpdchat

**Maryse** @AllThingsMaths · 16h

Replying to @Whitehughes

Fair and valuable point.

Creating connections is also vital and will support their communication in turn.

#MathsCPDChat

**Susan Whitehouse** @Whitehughes · 16h

Replying to @Whitehughes and @AllThingsMaths

I'm aware that sounds both pretentious and unhelpful! But I find it works better for me just to have it as my default mode #mathscpdchat

**Maryse** @AllThingsMaths · 16h

Replying to @Whitehughes

Not pretentious at all, and very helpful. So often the sticking points we come across are not those I would have predicted. So keeping the strategy running through all topics is important.
#MathsCPDChat

**The host's question about strategies students might use to help them articulate their reasoning also prompted these comments …**

**Mary Pardoe** @PardoeMary · 16h

Replying to @AllThingsMaths

There are some ideas for KS3/4 in the NCETM SecMag 144, here:
ncetm.org.uk/media/pkhh3bo1..
e.g. often very helpful to represent info given in words to yourself as a simple image …
#mathscpdchat (forgot hashtag!)

***Example 2: 2017, Edexcel, Higher Level, Paper 1, Question 14***

White shapes and black shapes are used in a game.
Some of the shapes are circles.
All the other shapes are squares.

The ratio of the number of white shapes to the number of black shapes is 3:7

The ratio of the number of white circles to the number of white squares is 4:5

The ratio of the number of black circles to the number of black squares is 2:5

Work out what fraction of all the shapes are circles.

| | White | Black |
|---|---|---|
| Circles | $\frac{4}{9}$ of $\frac{3}{10}$ | $\frac{2}{7}$ of $\frac{7}{10}$ |
| Squares | $\frac{5}{9}$ of $\frac{3}{10}$ | $\frac{5}{7}$ of $\frac{7}{10}$ |

**Maryse** @AllThingsMaths · 16h

Replying to @PardoeMary

I think this illustrates one of the challenges with the questions sometimes. There's a lot of information to break down and students need to take that first step into "rewriting it" mathematically.

It can sometimes be the other way round!
#mathscpdchat

**…this reply, which was quote-retweeted by Joanne Green with the comment shown …**

**Mary Pardoe** @PardoeMary · 16h

Replying to @AllThingsMaths

Practising writing complete chains of reasoning that they've first tried to 'talk through' …
(from here again: ncetm.org.uk/media/pkhh3bo1..) #mathscpdchat

In addition to practising …

- **representing** in a precise structural way, facts and relationships conveyed **at first** to you via word statements
- **identifying** mathematical knowledge and procedures that can be applied to a situation **to extend** what you know about it

… pupils should practise …

- **writing** complete **chains of reasoning**.

Writing complete chains of reasoning helps pupils to identify precisely where 'if … then …', 'because …' and 'therefore …' occur in their thinking, and so learn to apply similar thinking in new and different situations.

**Joanne Green** @MsJoanneGreen · 21h

@PardoeMary @AllThingsMaths **#mathscpdchat** I like that the page has the links on, and that those links lead to games. Games are sooo important to learning. People must be allowed to laugh and cheer when learning. I need the rules of games to be obvious - in my face!

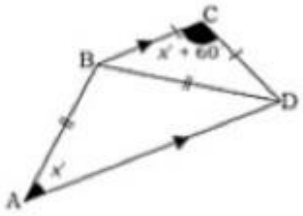
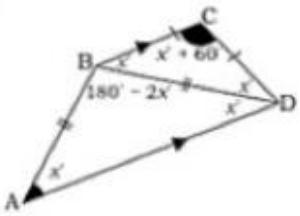
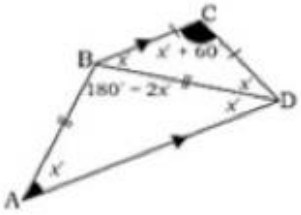
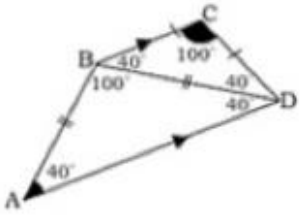
**… and these replies, which were also quote-retweeted by the same contributor (with comments as shown):**

**Mary Pardoe** @PardoeMary · 16h

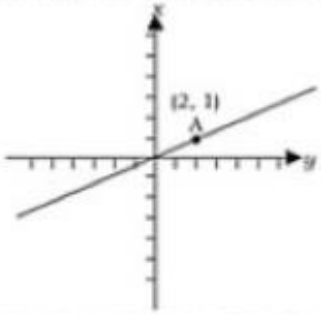
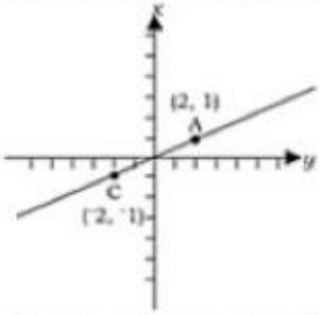
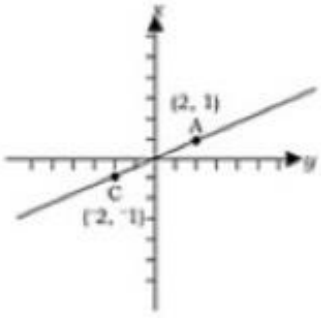
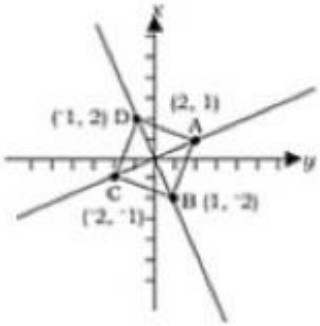
Replying to @AllThingsMaths

Challenging students to extract as much more information as they can from information given, using their existing knowledge. This isn't about getting 'an answer'. #mathscpdchat
For example …

**List A**

| Adding this mathematical knowledge / applying these mathematical procedures ... | ... transforms this representation ... | ... to this representation. |
|---|---|---|
| • base angles of isosceles triangle are equal<br>• alternate angles (parallel lines) are equal<br>• angle sum of triangle is 180° |  |  |
| • form and solve an equation:<br>$3x + 60 = 180$<br>$3x = 120$<br>$x = 40$ |  |  |

**List B**

| Adding this mathematical knowledge / applying these mathematical procedures ... | ... transforms this representation ... | ... to this representation. |
|---|---|---|
| • the centre of a square is at the mid-point of its diagonals |  |  |
| • the diagonals of a square bisect each-other at right angles |  |  |

**Joanne Green** @MsJoanneGreen · 21h

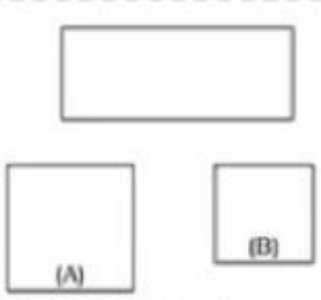
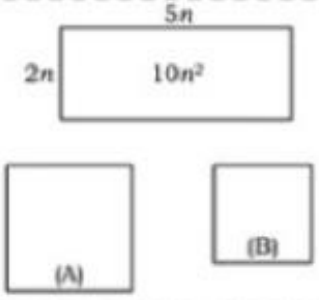
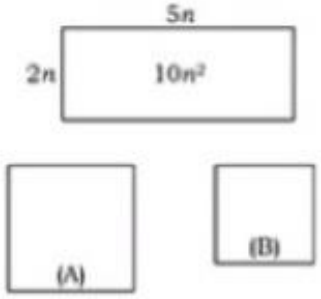
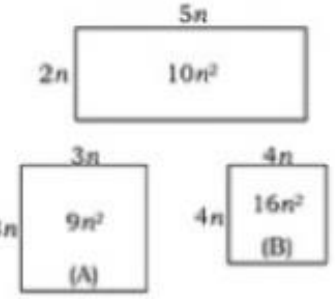
@PardoeMary @AllThingsMaths **#mathscpdchat** Yes, I compile lists of information I find, or lists of things I need to find - and in all subjects. Lists enables you to structure answers: some things can be grouped. Lists help to focus rather than be over-faced with lines.

**Mary Pardoe** @PardoeMary · 16h

Replying to @PardoeMary and @AllThingsMaths

… and another … #mathscpdchat

**List C**

| Adding this mathematical knowledge / applying these mathematical procedures … | … transforms this representation … | … to this representation. |
|---|---|---|
| • because the ratio of the side-lengths of the rectangle is 2 : 5, if one side-length is $2n$, then the other is $5n$, for any value of $n$<br>• the area of a rectangle is length × width |   |   |
| • '90% of' is equivalent to '9 tenths of'<br>• the side-length of a square is the square-root of its area<br>• if $3n = \frac{3}{4}$ of $x$, then $x = 4/3$ of $3n$<br>• the area of a square is its side-length squared (side-length)² |   |   |

**Joanne Green** @MsJoanneGreen · 20h

@AllThingsMaths #mathscpdchat @PardoeMary

, not seen 3n=3/4 of x is 4/3 of 3n before.
When n = 1 then 3(1) = 3. so, when 3/4 of x = 0.75 as x = 1.
3n+1 = 4. This is shown by:
4/3 of 3n when n=1 by 4/3 x 3(1) = 4.
Girlies, I like 🥰
I keep forgetting to tag!

**There was also the following short discussion …**

**Simon Ball** @ballyzero · 16h

Replying to @AllThingsMaths

I find myself thinking about the consequences of a choice and how it will play out, which I sometimes ask them to do (most particularly during integration problems - "What will happen if I choose that substitution?"). #mathscpdchat

**Maryse** @AllThingsMaths · Jan 17

Replying to @ballyzero

Letting them play with a suggestion (either from you or them) and seeing where it takes you?

#MathsCPDChat

**Simon Ball** @ballyzero · Jan 17

Replying to @AllThingsMaths

Both, but more often from me, to be honest!

Apologies for the delayed reply - I've not been home for long.
#mathscpdchat

**Maryse** @AllThingsMaths · Jan 17

Replying to @ballyzero

I guess we often know some of the interesting wrong paths to take, or sometimes they're the ones that lead to an interesting discussion.

#mathscpdchat

… and this one:

**Joanne Green** @MsJoanneGreen · 17h

A mix of strategies are needed in high school due to the numerous strategies used in primary schools.

**Maryse** @AllThingsMaths · 17h

Replying to @MsJoanneGreen

Do you find there's a big range in strategies coming from Primary these days?

#MathsCPDChat

**Joanne Green** @MsJoanneGreen · 17h

Interesting. No, the range seems to have stopped growing, and could possibly be shrinking as less and less pupils seem to be able to know what a method is. I think that's because the software programmes test pupils in bespoke automated ways.

When the host, Maryse, shared the following link and example …

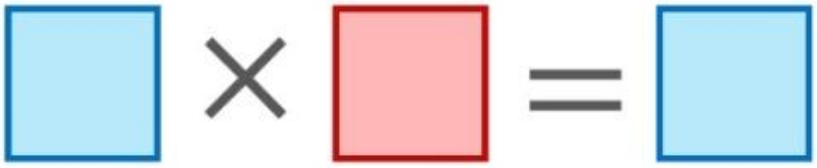
**Maryse** @AllThingsMaths · 17h

I was looking at this question recently as a prompt for explaining "why"
#MathsCPDChat

Source: ncetm.org.uk/media/y3kpmyrc...

**Checkpoint 5: True or false?**

Najma says, 'Write any numbers you like in the blue boxes, and I'll be able to write a number in the red box so that the multiplication is correct.'



Is Najma correct? Give some examples to support your answer.

… they prompted this response …

**Richard Perring** @LearningMaths · Jan 17

Replying to @AllThingsMaths

I started using "how do you know" instead of "why".
I found that students were maybe more open with that prompt? 'Why' seemed to be perceived as asking for a 'mathematically correct' answer while 'how do you know' was more personal? I felt I got better replies!

**Maryse** @AllThingsMaths · Jan 17

Replying to @LearningMaths

Good tip. I like the reason as well. "Why" can feel interrogative. How do you know" is bringing out their own thoughts and reasoning.

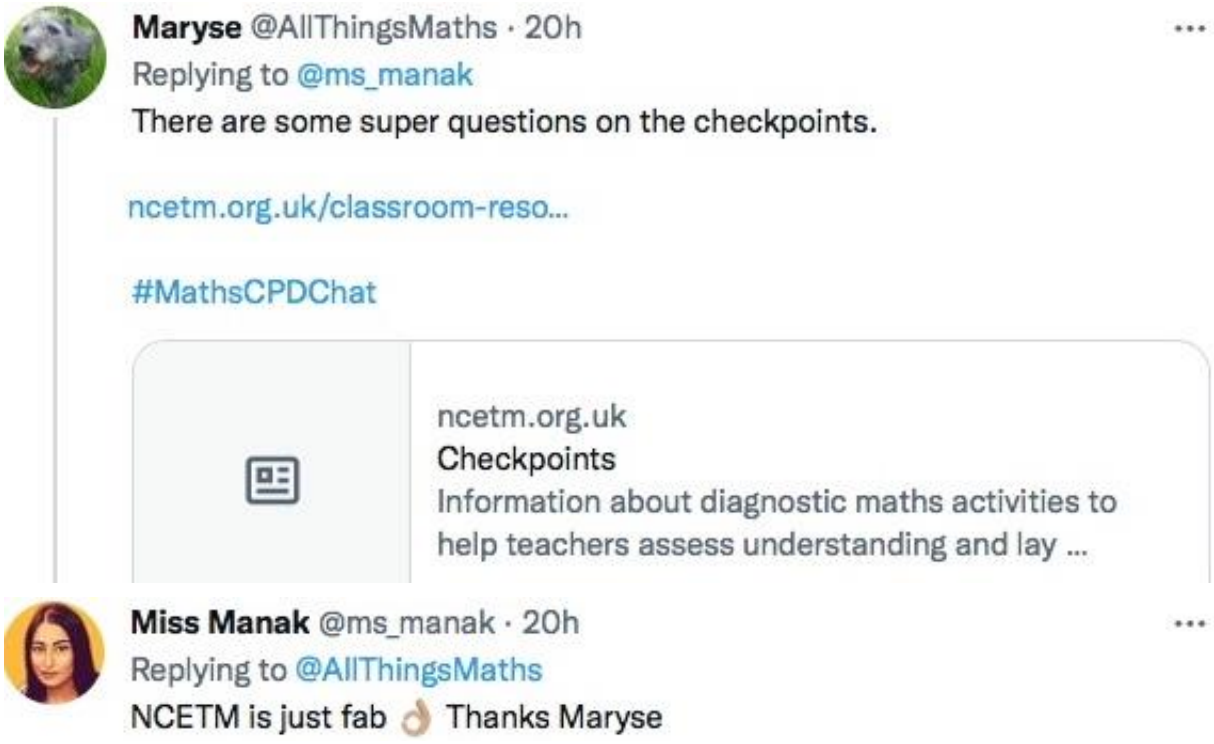
#MathsCPDChat

… and this comment:

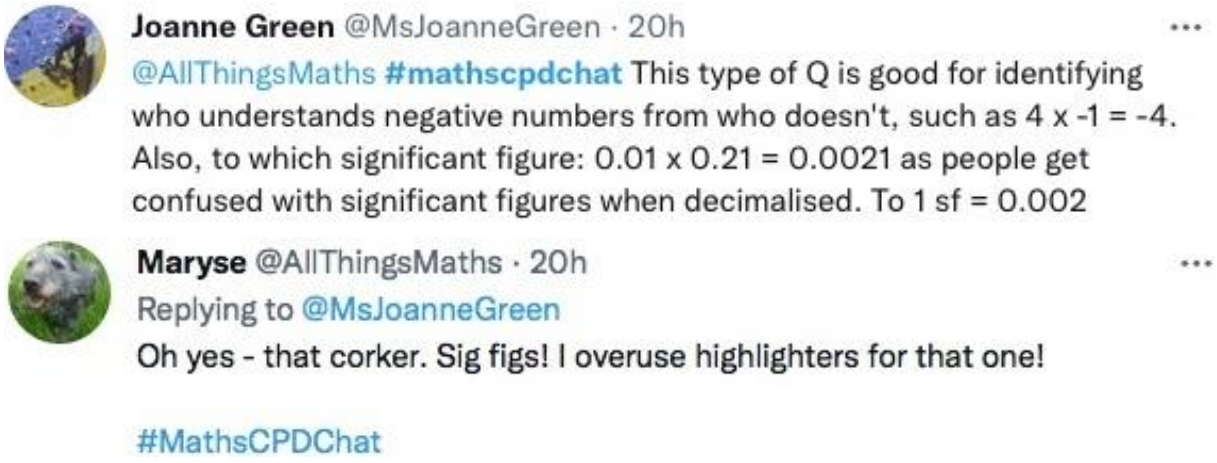
**Miss Manak** @ms_manak · 20h

Replying to @AllThingsMaths

I really like this!




**And the Checkpoints example above also prompted this comment and reply:**




**Independently of any of her main questions, the host contributed this link …**

**Maryse** @AllThingsMaths · 17h

There's examples of manipulatives on the @NCETM website here:
ncetm.org.uk/classroom-reso..

#MathsCPDChat

ncetm.org.uk
Using mathematical representations at KS3
Guidance for some of the most useful representations for Key Stage 3 mathematics.

**… and another Checkpoints example:**

**Maryse** @AllThingsMaths · 19h

Another example of a question to prompt discussion (and so practise communicating ideas):
#mathscpdchat

Source: view.officeapps.live.com/op/view.aspx?s..

**Checkpoint 2: Function machine pairs**

Whole numbers are put into these pairs of function machines.

× 2 → × 5 — Explain why **every** number that comes out of this pair of function machines is a multiple of 10.

+ 2 → × 10 — Explain why **every** number that comes out of this pair of function machines is a multiple of 10.

× 10 → + 2 — Explain why **no** number that comes out of this pair of function machines is a multiple of 10.

? Design your own pairs of function machines to create multiples of 11 and 12. Is there more than one way to do this?

NCETM

**The host's next main question …**

**Maryse** @AllThingsMaths · 19h

What (if any) are the barriers for students communicating mathematically?

#MathsCPDChat

… was followed by another …

**Maryse** @AllThingsMaths · 19h

Replying to @AllThingsMaths

Do you find there's a correlation between reading ages and communicating mathematically?

#MathsCPDChat

… which prompted the following short conversation:

**MrHawesMaths** @HawesMaths · 19h

Replying to @AllThingsMaths

SEND. If they can't get it on paper, then maybe a think pair share idea would help. Getting students to verbalise what processes are happening and their reasoning. Pairing up to write down a good representation of work. Evaluating each other's answers and marks #mathscpdchat

**Maryse** @AllThingsMaths · 19h

Replying to @HawesMaths

Think, Pair, Share is one area I've dropped over the last few years. Seeing my trainee using it has been a good reminder of its use in the classroom.

#mathsCPDChat

**Maryse** @AllThingsMaths · 19h

Replying to @HawesMaths

I also like the idea of evaluating each other's work. It's about what the other person gets from their communication.

I've been toying with the idea of "drafting" answers using the same routine as the English dept.

The question about possible 'barriers for students communicating mathematically' prompted this reply …


**theperfectlanguage** @theperfectlang1 · 18h

Replying to @AllThingsMaths

I think it's a lack of confidence in using vocabulary. There is a tendency to ask question of the teacher to get them to fill in the gaps. Students often KNOW the language but are not familiar in using it so shy away from expressly using an unfamiliar term.

… this interchange, mentioning 'Ukrainian maths' …

**Dr Jenny Argyle** @madaboutmaths · 19h

Replying to @AllThingsMaths

English not first language - mechanics & statistics more challenging to access

**Maryse** @AllThingsMaths · 19h

Replying to @madaboutmaths

Great point. We did a quick read around Ukrainian maths to pick up some pointers for their transition as an example. It isn't just a vocab issue but also use of symbols.

#MathsCPDChat

**… these comments and report …**

**Susan Whitehouse** @Whitehughes · 19h

Replying to @AllThingsMaths

The (widespread) idea that the point of a maths question is to get to the correct answer as quickly as possible! There's often pride taken in showing as little working as possible #mathscpdchat

**Maryse** @AllThingsMaths · 19h

Replying to @Whitehughes

You and Mary tweeted the same idea simultaneously!

One of the joys is in the playing around with things in maths. Sometimes it's the tussle.

Y11 did parallel lines and angles today. We annotated the diagrams with little arrows and colour coded it all.

#mathscpdchat

**Maryse** @AllThingsMaths · 19h

Replying to @AllThingsMaths and @Whitehughes

We just played with the diagrams and then moved onto the formal exam questions. Diagrams were exactly the same and they'd already worked out the answer and the reasons.

#MathsCPDChat

**… and the following suggestions:**

**Mary Pardoe** @PardoeMary · 19h

Replying to @AllThingsMaths

These are some possible strategies to try to overcome those barriers ... #mathscpdchat

Some questions are particularly difficult because pupils must find a starting-point for themselves. Also the construction of an adequate response depends heavily on the pupil's ability to reason mathematically, to argue 'if … then …', to provide justifications for facts that they state. In the 2017 GCSE papers, questions of this kind can be sorted into three broad categories:

1. 'Work out …' problems where something particular has to be found
2. algebraic proofs about number
3. geometric proofs.

When selecting or designing tasks to help pupils develop the skills needed to succeed with these questions it is helpful to be guided by some general principles that apply to all three categories:

- Challenge pupils to find as many mathematical relationships as possible between the constituent parts of situations about which it is possible to reason mathematically (see Task 1 below)
- Provide opportunities for pupils to see things for themselves. Then challenge them to try to convince other people that what they see *must be true because* … .
- Given a problem, challenge pupils to construct their own 'similar' problems by varying some aspect, or aspects, of it.
- Design tasks that will help pupils develop the habit of laying out calculations and deductions line-by-line so that a sequence of successive steps can be seen as a single chain of reasoning.
- Challenge pupils to think of different chains of reasoning to a particular result.
- Design tasks that will help pupils develop the habit of simplifying calculations and expressions wherever possible.

**Maryse** @AllThingsMaths · 19h

Replying to @PardoeMary

A couple of things jump out at me with this.

I always build in questions to explain. May be an always true/never true/sometimes true, or a "who is right and why" (also weeds out some misconceptions).

#MathsCPDChat

**Maryse** @AllThingsMaths · 19h

Replying to @AllThingsMaths and @PardoeMary

Another is practice is so important for students to build strategies.

And a third is having the opportunity to go down the wrong path, and start again... and knowing that's absolutely fine.

#MathsCPDChat

**Maryse's next main question …**

**Maryse** @AllThingsMaths · 21h

Does it matter if students are always rigorous in using certain notations? E.g. using <ABC to describe an angle?

#mathscpdchat

**… prompted these thoughts …**

**Susan Whitehouse** @Whitehughes · 21h

Replying to @AllThingsMaths

I think that sometimes we have to separate out clear communication of mathematical thinking from using correct mathematical language. Both are important but I think we can muddy the waters for students by not distinguishing between them. #mathscpdchat

**Mary Pardoe** @PardoeMary · 21h

Replying to @Whitehughes and @AllThingsMaths

Yes! They need to be aware of what kind of 'doing-maths mode' or stage in their mathematical thinking they are in. Possibly obvious, but Tony Gardiner wrote a note about that here: wob.com/en-gb/books/to. #mathscpdchat

This should be interpreted as part of the (unstated) requirement that pupils should at all times expect the methods of elementary mathematics to *make sense*. But there are different kinds of "sense making": some involve *inference*; some involve plausibility arguments; and some are rooted in *deduction*. The requirement for pupils to "reason deductively" means that they need to be clear

- when they are experimenting or conjecturing, and when they are working "deductively";

and also

- when they are working in rough, and when they are writing for others to read.

That is, they need some way of demonstrating (to themselves and to others) which mode they are in at any given time. For pupils who are ready for the formal pro-

**The following interesting comment came 'out of the blue', not in response to any particular question from Maryse:**

**MrHawesMaths** @HawesMaths · 21h

I also think that the expectation and pressure sometimes to deliver an answer quickly during class discussion translates to the need to deliver answers quickly on paper. Along with the culture of on demand. Really trying to slow the 'questioning' #mathscpdchat

**Maryse** @AllThingsMaths · 21h

Replying to @HawesMaths

LOVE this. It fits in with the ideas Susan and Mary shared earlier too. Maths is not about "WHAT'S THE ANSWER" but sometimes a slow and leisurely exploration before the end point #MathsCPDChat

**Although there were no direct replies to this question from the host …**

**Maryse** @AllThingsMaths · 21h

Does anyone have any particular questions (in addition to resources shared) that you've found particularly useful to develop communication in maths?

#MathsCPDChat

**… Maryse shared this useful observation:**

**Maryse** @AllThingsMaths · 21h

One thing I saw my trainee do last week was mirroring back with students. It was clear the student understood. The answer was acknowledged and then reframed in reply. Development. Didn't feel like correction. Built confidence for students to speak up.

#MathsCPDChat

**The host's final main question …**

**Maryse** @AllThingsMaths · Jan 17

We're heading into the last 5 minutes of #MathsCPDChat.

Are there any areas of development (either for you or maths ed in general) you can identify around communication in maths? Any other gems to share?

**… prompted two replies: this …**

**Mary Pardoe** @PardoeMary · Jan 17

Replying to @AllThingsMaths

I think one of the most important needs is to establish a learning environment in which pupils/students feel safe to talk about their thinking even when unsure if it's 'right'. More talk helps facilitate more writing! #mathscpdchat

**Maryse** @AllThingsMaths · Jan 17

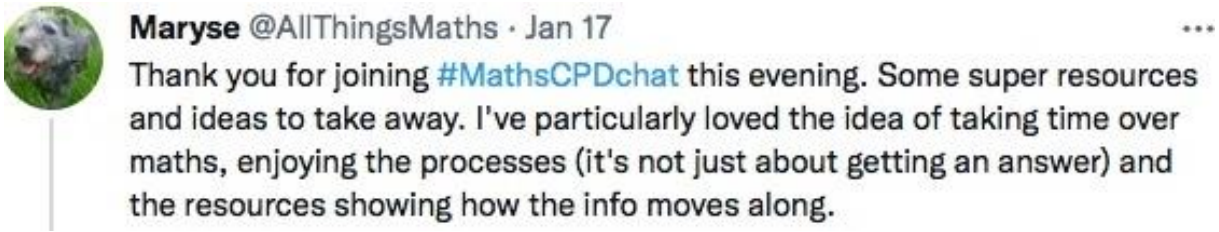
Replying to @PardoeMary

Pure gold. Yes.

**… and this:**


**Just before the hour of this #mathsCPDchat came to an end, Maryse (the host) shared the following observation:**


**Her closing message …**




**… was followed by this interchange:**