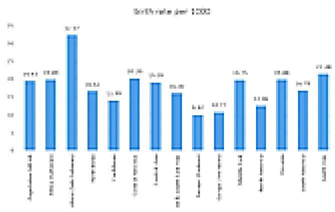
Secondary Magazine 147

National Centre for Excellence in the Teaching of Mathematics

With spring on the horizon, we bring you Issue 147 of our Secondary Magazine. In this issue, we hear a teacher's reflections on how use of the Large Data Set in the new A level can bring statistics to life for students, and we explore just what makes it onto the school maths curriculum – and what might have been left out.

Don't forget that all previous issues are available in the Archive.

This issue's featured articles



Stats that speak...making A level statistics lessons meaningful through use of the Large Data Set
Large Data Sets is new on the curriculum at A level. Bruce Hampton, an experienced maths teacher but new to teaching A level statistics, reflects on how he integrated the Large Data Set (LDS) into his lessons to encourage students to unpick the information in more detail. In this article, he explains how the LDS, rather than being a lesson 'bolt-on', is a tool for enabling investigation and discussion across all statistics topics.

Which is larger, $\frac{3}{7}$ or $\frac{1}{6}$?

School mathematics: who decides what maths is on the curriculum?
Are you teaching the same maths that you learned at school (Large Data Sets notwithstanding)? Here we look at what constitutes 'school mathematics' and consider what opportunities there are for teachers to take students outside and beyond the standard curriculum to deepen and enhance their understanding of mathematics.

And here are some other things for your attention:

- Girls can do anything they put their mind to – especially maths! In this interview from National Numeracy with Rachel Riley, co-presenter of *Countdown*, she busts some myths about gender and maths
- The DfE has announced details of the new Advanced Maths Premium which will be paid to schools who increase numbers of students taking Level 3 maths qualifications. Schools will receive £600 for every additional student taking A level Maths, Further Maths, Statistics or Core Maths. You can learn more about how it will work here
- Our latest podcast will get you heading back to your bookshelf! Five teachers describe the books that really made a difference to their maths teaching
- MEI have just published their March newsletter, full of news, views and updates from our partner organisation, as well as a link to practice papers for MEI AS and A levels
- Wednesday 16 May is National Numeracy Day. Schools and teachers are invited to get involved so their students can understand how numeracy is far more than simply what they experience in their maths lessons.

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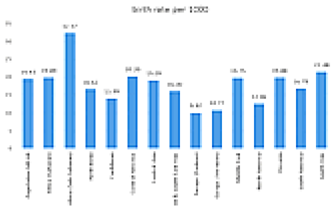


Stats that speak...making A level statistics lessons meaningful through use of the Large Data Set

Bruce Hampton is head of maths at Tudor Grange Academy in Redditch. This time last year, his department, as most other maths departments in the country, were involved in preparation for the new A level, to begin teaching in September 2017. The department worked with Simon Clay (Teaching Advanced Mathematics (TAM) Coordinator from MEI) who supported their preparation and encouraged them to write a blog, Thoughts on A Level Mathematics, to support others in the maths education community. Six months in, Bruce reflects for us on his experience of teaching using the Large Data Set.

One of the things that I wanted to achieve when teaching the new A-level in maths was that all aspects are linked together, and for both teachers to have ownership of the whole course. This was a big change for me, as previously I had not taught statistics, leaving it to other members of my departments.

When we initially started looking at which examination specification to follow one of the key driving points in our selection was the Large Data Set (LDS). As part of our drive to have a connected scheme of work, we wanted the LDS to be our vehicle for the statistics lessons, providing a context to show why we were covering different techniques. When asking others for their opinions on the large data set I encountered lots of negativity, with most people looking to 'teach' the large data set in a single lesson, as a bolt-on. I felt that this really missed the point of the specification change, the LDS should not be a separate topic; it is instead the tool that you use to teach all the data-handling parts of the course. By using the same data set frequently we allow students to take ownership of it, giving the topics more relevance.

I spent some time looking at the LDS's for each different specification and using this as part of the process for deciding which route we were going to follow. You can read about this process here. We chose to go with the OCR B (MEI) specification, largely because we could fit the LDS into a reasonably sized document (the MEI data set is available here).

We began teaching the statistics elements of the course in the first week of the year, jumping straight in at the deep end with working on the large data set. Students were provided with a paper copy of all the data on three A3 sheets of paper.

We started with measures of central tendency – introducing some new notation for the mean, and summation notation. The trick here was to give them a good enough explanation to be able answer the question, but to be vague enough that they had to use the LDS to help. Example 1 required students to find the total before dividing by five countries, finding the raw data from the LDS. Example 2 specifically required students to look at their copy of the LDS to work out the denominator (i.e. how many countries are there in Sub-Saharan Africa?), although the first part of the calculation (the total) had been worked out for them:

STATISTICS *Introduction to the Large Data Set*

$\bar{x} = \frac{\sum x}{n}$

Eg 1: Calculate the mean of the birth rate for Saharan Africa.

Eg 2: Calculate the mean of the birth rate for Sub-Saharan Africa, given that $\sum x = 1651.06$

no	Country	Sub region	population	birth rate per 1000	death rate per 1000	median age	life expectancy at birth	labor force	unemployment (%)	GDP per capita (US$)	physician density (physicians/1000 population)	Health expenditure (% of GDP)	Total area	Land area	Water area	Land borders	Dependency status
1	Algeria	Africa (Saharan)	39542166	23.67	4.31	27.5	76.59	11780000	12.4	14500	1.21	5.2	2381741	2381741	0	Yes	None
2	Egypt	Africa (Saharan)	88487396	22.9	4.77	25.4	73.7	31960000	13.1	11800	2.83	5.0	1001450	995450	6000	Yes	None
3	Libya	Africa (Saharan)	6411776	18.03	3.58	28	76.26	1153000	30.0	14600	1.90	3.9	1759540	1759540	0	Yes	None
4	Morocco	Africa (Saharan)	33322699	18.2	4.81	28.5	76.71	12230000	9.9	8200	0.62	6.4	446550	446300	250	Yes	None
5	Tunisia	Africa (Saharan)	11037225	16.64	5.98	31.9	75.89	4038000	15.4	11400	1.22	7.0	163610	155360	8250	Yes	None
6	Angola	Africa (Sub-Saharan)	19625353	38.78	11.49	18	55.63	10850000		7300	0.17	3.5	1246700	1246700	0	Yes	None
7	Benin	Africa (Sub-Saharan)	10448647	36.02	8.21	17.9	61.47	3662000		2100	0.06	4.5	112622	110622	2000	Yes	None
8	Botswana	Africa (Sub-Saharan)	2182719	20.96	13.39	23.1	54.18	1177000	20.0	16400	0.40	5.3	581730	566730	15000	Yes	None
9	Burkina Faso	Africa (Sub-Saharan)	18931686	42.03	11.72	17.1	55.12	7692000	77.0	1700	0.05	6.2	274200	273800	400	Yes	None
10	Burundi	Africa (Sub-Saharan)	10742276	42.01	9.27	17	60.09	5255000		800		8.1	27830	25680	2150	Yes	None
11	Cabo Verde	Africa (Sub-Saharan)	545993	20.33	6.11	24.5	71.85	196100	12.0	6500	0.31	3.9	4033	4033	0	No	None
12	Cameroon	Africa (Sub-Saharan)	23739218	36.17	10.11	18.4	57.93	9612000	30.0	3100	0.08	5.1	475440	472710	2730	Yes	None
13	Central African Republic	Africa (Sub-Saharan)	5391539	35.08	13.8	19.5	51.81	2421000	8.0	600	0.05	3.8	622984	622984	0	Yes	None
14	Chad	Africa (Sub-Saharan)	11631456	36.6	14.28	17.4	49.81	5457000		2600	0.04	3.5	1284000	1259200	24800	Yes	None
15	Comoros	Africa (Sub-Saharan)	780971	27.84	7.57	19.4	63.85	245200	6.5	1500		4.5	2235	2235	0	No	None
16	Congo, Democratic Republic of the	Africa (Sub-Saharan)	79375136	34.88	10.07	18.1	56.93	31080000		800		5.6	2344858	2267048	77810	Yes	None

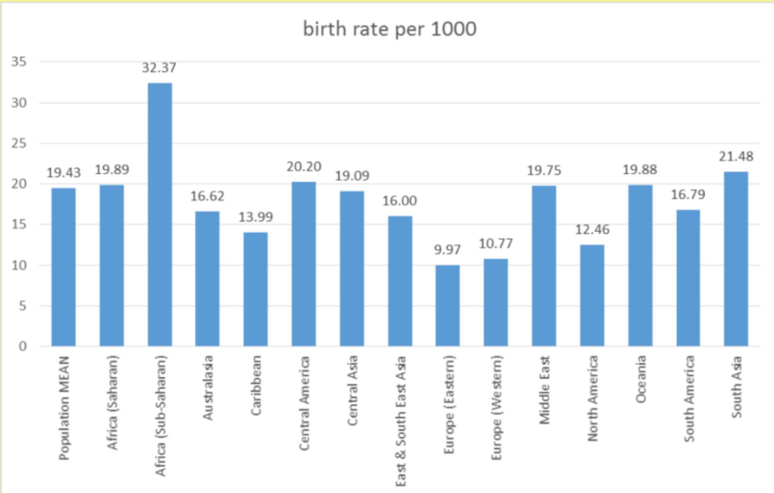
After I was satisfied that everyone was able to follow the examples I gave them their first task – calculating the means of all the other regions:

STATISTICS *Introduction to the Large Data Set*

$\bar{x} = \frac{\sum x}{n}$

Exercise 1: Calculate the mean for each of the regions (for birth rate per 1000).

Region	Total
Africa (Saharan)	99.44
Africa (Sub-Saharan)	1651.06
Australasia	$\boxed{}$
Caribbean	307.80
Central America	141.43
Central Asia	$\boxed{}$
East & South East Asia	304.03
Europe (Eastern)	159.59
Europe (Western)	344.52
Middle East	375.24
North America	$\boxed{}$
Oceania	357.81
South America	201.48
South Asia	171.86
POPULATION	4353.41



Once I'd unveiled the graph featuring all the means, I asked for someone to tell me something about the birth rates. One of our more confident students was quick off the mark:

> *"Sub-Saharan Africa has a significantly higher birth rate than any other region"*

Brilliant! I was hoping for a response like that, but I wanted a reason... and I wasn't disappointed:

> *"Sub-Saharan Africa possibly has less access to contraception that other regions"*

I then asked why both European regions might have the lowest birth rate, someone suggested that

"It is cultural, that in Europe families have fewer children"

It is those types of thoughts, thoughts which show understanding that data can tell us something interesting about life, that I want to foster as we continue to dig through the data set.

The more the students immerse themselves in the LDS the more familiar they become with it. Homework can be to do some calculations or create some charts (and email them to use in advance where appropriate) then as a group we can discuss them next lesson.

Initially we planned to have a show-me/tell-me starter (regardless of whether the lesson was on statistics or not) each week, although in practice this has been less frequent. In this, students are encouraged to do a little investigation themselves, then get the class to discuss the potential causes (e.g. our outliers). An example that we saw early on, was that it was noticed that the outliers for birth rates in the African countries were the island nations, where birth rates were much lower than on the mainland. In this way we have built a bank of interesting features that students can use when explaining features of the LDS in examination questions.

We have also been trying to make our students read more about mathematics. To do this we provide an article from a magazine or newspaper which students are expected to read and answer questions about. Early in the year, we deliberately picked an article on global population growth, so that we could link back to the LDS for questions.

In summary I feel that the use of the LDS has benefited us hugely in the teaching of the statistics elements of the course. We are working towards our goal of students having a greater understanding of the data they are using. They are familiar with the data and are able to draw conclusions from summary statistics which are taken from it, without having the raw data in front of them. We have finished the new material needed for statistics in the AS course, but will continue to look regularly at the LDS so that familiarity is still there for the AS exams, and of course for the new content that will be present in the full A-level as we progress to next year. I have also very much enjoyed being part of the statistics teaching and have a much better view of where students are across the whole course, rather than just one module, as in our previous structure.

Teachers looking for professional development around teaching using the Large Data Set, could investigate these possibilities from the Further Mathematics Support Programme (FMSP):

- Teaching Statistics 1 (TS1) – a sustained professional development course which devotes nearly a whole day to the use of the LDS
- Local events through the FMSP: searching for either 'Technology' or 'Statistics with Technology' days will lead to the most relevant courses. An example would be Using Technology in the Statistics Classroom
- Also, there is a series of excellent professional development videos.

In addition, if a school registers with the FMSP, the registration provides teacher access to the Integral resources about the LDS. This includes 'How to' videos designed for teacher development.

Exam boards are also a likely source of professional development with a focus on LDS.

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Which is larger, $\frac{3}{7}$ or $\frac{1}{6}$?

National Centre for Excellence in the Teaching of Mathematics

School mathematics: who decides what maths is on the curriculum?

Gwen Tresidder is the NCETM Communications Manager, previously a secondary maths teacher

A couple of incidents last year have caused me to reflect on the content of the school maths curriculum.

'School mathematics' is a body of mathematics that remains strikingly constant, despite huge and frequent changes in emphasis, delivery style, learning opportunities, and examination requirements. Notwithstanding a few tweaks around the edges – the appearance of box plots and moving averages, or the requirement to memorise trig ratios and imperial/metric conversions, for example - what we are teaching now is, more or less, what I learned at school 30 years ago. Small wonder that it is hard for teachers of mathematics not to replicate the teaching style that they experienced at school!

Because the body of knowledge that is 'school mathematics' remains remarkably consistent, I have developed the (largely unquestioned) impression that this is all the maths that is accessible to most school aged children, that this is maths at this level. But two recent incidents have prompted me to think about this more deeply…

Common numerators

First of all, whilst working on an article about intelligent practice for pupils learning about comparing the size of fractions, I came across this question:

> Which is larger, $^{3}/_{7}$ or $^{1}/_{6}$?

…with the suggestion that pupils should be encouraged to notice that for this question, finding a common numerator (by converting $^{1}/_{6}$ to $^{3}/_{18}$, and so comparing $^{3}/_{7}$ and $^{3}/_{18}$, is more efficient than the routine that most of us will have learned at school of converting both fractions to give a common denominator (in this case rather unwieldy 42nds) before comparison.

Although stunningly simple, and apparently obvious, this was a total revelation to me! Common numerators – who was ever taught to find them? Or, perhaps more to the point, how many of us have gained sufficient easy familiarity with number generally, and fractions specifically, to be able to make these obvious efficiency savings instinctively?

My first reaction was rather an embarrassed 'how did I never see this?', after which I sheepishly tested it out on a couple of the best mathematicians I know, who also said that they would have automatically found 42nds before comparing, and had never considered finding common numerators. How thoughtlessly we apply algorithms that work for us, regardless of the situation.

I am not suggesting that children should be taught to find common numerators in preference to finding common denominators. Finding a common denominator is a more useful process – it can then be used, in a way that common numerators can't, to add and subtract fractions.

What I am suggesting is that a focus on learning the best and most transferable algorithm, even for those that understand how they work, may be less effective in teaching for deep understanding, than exposure to a variety of methods for solving such problems. As a teacher, I have often sought the 'best' method for children to learn to do a calculation. To some extent this has been governed by time constraints. Now that maths teachers are being encouraged to spend longer exploring concepts with students, and are being released from the pressure of moving them on to the next concept once they can 'do', for example, comparing fractions, the hope is that they are able to spend more time looking at different methods for solving the same calculation.

How many divisors?

In a separate incident, I was running some professional development for a small group of Turkish teachers. In a session on Rich Tasks, I was encouraging them to use the NRICH website. I chose the task 14 Divisors for them to get their teeth into. The initial questions posed are:

What is the smallest number with exactly 12 divisors?

What is the smallest number with exactly fourteen divisors?

Having used this before, and having found it to be a rich and rewarding problem with lots of extensions and different strands to follow, I was therefore a bit disappointed when one of the teachers said quickly that he had 'the answer'. I wondered if I had missed something, having begun a lengthy process of listing factors and pattern spotting. Indeed I had – an algorithm for finding how many factors a number has, that this teacher taught regularly to his upper secondary school pupils. The algorithm goes:

1. Find the prime factorization of a number (each one of the number's prime factors raised to the appropriate power).
2. List all of the exponents.
3. Add one to each of the exponents. ...
4. Multiply the resulting numbers.

Source: http://www.gmathacks.com/gmat-math/number-of-factors-of-a-large-integer.html

For example, for finding the number of factors that 72 has:

$72 = 3^2 \times 2^3$

Exponents: 2,3

Add 1 to each of them: 3,4

Multiply them: $3 \times 4 = 12$

So: 72 has 12 factors (1,2,3,4,6,8,9,12,18,24,36,72)

This algorithm can be fairly easily applied in reverse, to find out the smallest number with 14 divisors since 14 has only two pairs of factors (finding the smallest number with 12 divisors is slightly harder since 12 has more factors):

$14 = 7 \times 2$ (using $14 = 14 \times 1$, also gives a solution, but a bigger one)

Subtract 1 from each of the factors of 14: 6,1

So the smallest number with 14 factors must be $2^6 \times 3^1$ (using smallest possible non-trivial bases) = 192

More complete solutions are available from NRICH.

Again I found myself reflecting about 'school maths'. In Turkey, this is part of their national curriculum, part of their 'school maths'. Perhaps some teachers in the UK explore this when they are teaching prime factorisation. Maybe some have discovered it with their pupils when engaging with NRICH's problem. But the fact that it is not explicitly on the National Curriculum means that many, possibly the majority of teachers (and pupils) will remain as ignorant of it as me. It is a useful, elegant and easy to follow algorithm, potentially enabling many bridges of understanding to be built in the world of number theory – so why is it not on the curriculum?

I suspect it is the tip of the iceberg, that there are many such bits of maths that are accessible to school students that are just not taught because there is not time to teach everything. The question of what is included in the National Curriculum, what becomes part of that accepted body of 'maths that everyone should meet at school', doesn't seem to be one that is often considered, despite a seemingly constant evaluation of how the curriculum is taught. There is also a strong emphasis, at the moment, on the importance of consistency in maths teaching. While it is important that all pupils consistently receive good maths teaching, in our quest for consistency, do we risk reducing opportunities for teachers to step outside the curriculum and draw pupils' attention to bits of maths like this? Perhaps there's a place for different teachers knowing different things and having different opportunities to offer students.

Additionally, however good teachers' professional development opportunities are, if these opportunities are always focused on pedagogy and never on engaging with maths that is new to us, it is hard to see how we can keep the subject alive in our classrooms, and not just keep on teaching the same maths that we learnt at school, falling like autopilots, into the same pedagogy used when we were at school.