Secondary Magazine
Issue 76

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 76 of the Secondary Magazine.

Contents

Contributors to this issue include: Steve Hewson, Mary Pardoe and Peter Ransom.

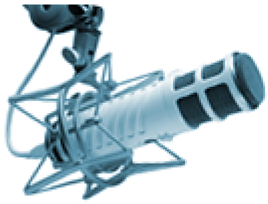
From the editor

Happy New Year! And welcome to Issue 76 of the Secondary Magazine. Whether or not you regard this as the first week of a new decade, it is an appropriate time to reflect on and discuss with colleagues how, and why, you are responding to recent changes to the secondary curriculum and assessment at Key Stage 4. In 5 things to do this fortnight we remind you that you may like to comment on observations and questions from Secondary Watch.

What has luck got to do with mathematics education? It has been widely reported in the past (for example in Who Succeeds in Science: The Gender Dimension, by Gerald Holton, 1995) that men were likely to view their successes in mathematics, science and engineering as consequences of their talent, while women were more likely to ascribe them to luck. And women were much more likely than men to regard bad luck as significant in their experiences. Is this true today? Are we endeavouring in our interactions with students to do what we can to ensure that it isn't?

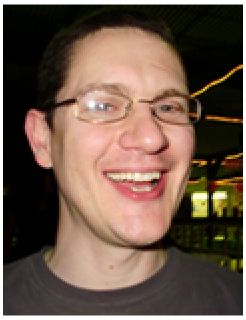
Apparently many gamblers believe that their success, or lack of success, when trying to win at Bingo is owing as much to factors such as the combinations and arrangements of the numbers on their cards as to the 'luck of the draw'! In Focus on…Bingo we briefly outline some ways in which explorations of a variety of forms of that game can provide opportunities for mathematical thinking, rather than merely recalling remembered facts – as is so often the case when Bingo is used in the classroom.

Wishing you very good luck as you face the many challenges that you will meet during 2011!



The Interview

Name: Steve Hewson



About you: I am a writer and mathematics educator, a state of being I arrived at via a research fellowship in string theory at the University of Cambridge, time as a quantitative analyst in the City, and secondary mathematics teaching. At present I feel privileged to be working on the NRICH mathematics project developing its post-16 areas, and leading the extension of the stemNRICH scientific mathematics resource down into Key Stages 3 and 4. On the writing side, the second edition of my university transitions book A Mathematical Bridge came out last year and I have just finished my first novel.

The most recent use of mathematics in your job was…
I am fortunate enough to have had a career where I have been able do and think about mathematics every day. At present I am grappling with the issue of the best way to grow enthusiastic young mathematicians and mathematically literate people. As a part of this I spend a lot of time crafting rich mathematics problems; this involves quite a bit of experimentation and investigation. This week I was mainly considering the ways in which mathematics enters into the Olympic Games – alongside doing some data analysis on a survey in which I am analysing key moments in mathematics education for gifted STEM undergraduates.

Why mathematics?
For different reasons over the years, but essentially because it is deductive, seductive, powerful, amazing and beautiful. And sometimes difficult. On a practical point, mathematical problem solving suits my hands-on approach to life and my poor memory!

Some mathematics that amazed you is…
Now this is a question I could talk about for weeks! I suppose that I will always remember the time I found out, in October 1990, that there are 2, 4 and 8 dimensional versions of the real numbers but no others (complex numbers, quaternions, and octonions) - I didn't expect that! I used the 2 and 4 dimensional numbers all the time in my string theory research but am still waiting for the time when the fundamental relationship between physics and the octonions is discovered - I'm sure that this will happen eventually!

A significant mathematics-related incident in your life was…
Starting university was incredibly significant to me as I could start to engage with real mathematics with like-minded peers. Had this opportunity not been available to me, my life would be radically different.

The best book you have ever read?
Quantifying the term 'best' is difficult: I suppose that the most *useful* mathematics book that I ever read was Futures, Options and Other Financial Derivatives, by John Hull; more generally I still recall What Do People Do All Day?, by Richard Scarry, with great fondness.

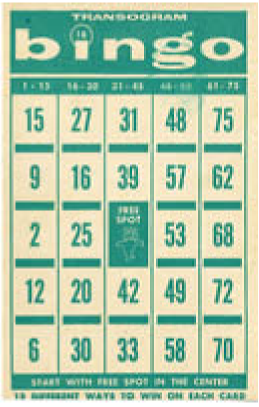
Who inspired you?
Einstein. I was amazed by the concepts of general and special relativity as a boy. I wanted to know more – so I set my sights on black holes and curved spacetime geometries. I also liked the fact that Einstein came up with these amazing mathematical theories from a few basic physical observations simply by thinking hard.

If you weren't doing this job you would…
probably be running my own consultancy business and devoting a lot of effort to writing novels. Alternatively, I'd be a piano player.

Focus on…Bingo

"Wait till you try Bingo – you'll hyperventilate!"

(Maggie Jones as Blanche Hunt in *Blanche and the alcoholics' support group meeting* – voted as the greatest scene of the first 50 years of Coronation Street).



Bingo card photograph by Abbey Hendrickson

Rovers Return photograph by Paul Walker

It seems that the popular game called Bingo originated in Italy during the 16th century as a game called *Lotto*, which was brought to France as *Le Lotto* in the eighteenth century. Apparently, during the 19th century, the Germans used it to help students learn mathematics, spellings and history! When the game arrived in North America in 1929 it was at first called *Beano*, but then renamed and publicised by a New York toy salesman, Edwin S. Lowe, as *Bingo*. By then it was being played by huge groups of people in vast venues.

Bingo players by Kees Jonker

Bingo players 'cross off' entries – which are usually numbers – in cells on a rectangular grid, which is the Bingo card, as the numbers (or their 'equivalents') are called out. The winner is the first player to cross off all the entries in a group of cells that satisfy a particular condition – such as forming a row, column, or diagonal of the grid. The most common game, in which the winner is the first player to get five numbers in a row on a 5-by-5 grid, is illustrated in this free interactive resource, Bingo Card Generator from The Wolfram Demonstrations Project.

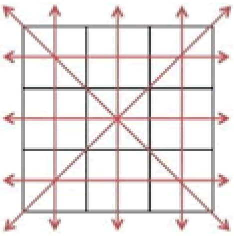
Bingo games of various kinds are described or provided in mathematics textbooks, and can easily be found in collections of teaching resources. The idea is that by playing these games, usually as a whole class with the teacher calling out numbers, or number expressions, which students try to find or match on their Bingo cards, students will 'consolidate' their knowledge of some mathematical facts.

However, it is also possible to use Bingo in the classroom in ways that provide opportunities for mathematical thinking rather than the mere recall of remembered facts. A few starting points are suggested below.

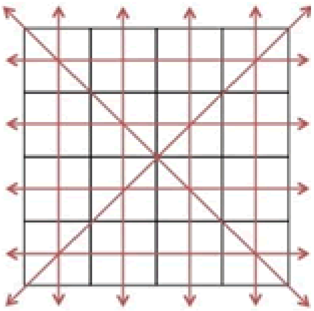
Winning arrangements

Students might explore the 'winning arrangements' in Bingo games played on rectangular grids of various sizes for various 'winning arrangement' criteria.

For example, on a 3-by-3 board for a Bingo game in which a winning arrangement is any complete row, column or diagonal, there are eight winning arrangements…

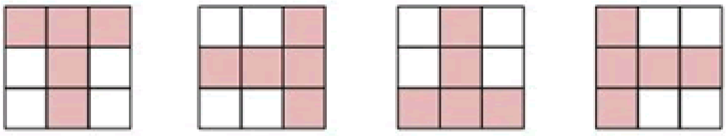


…and on a 4-by-4 grid there are ten:

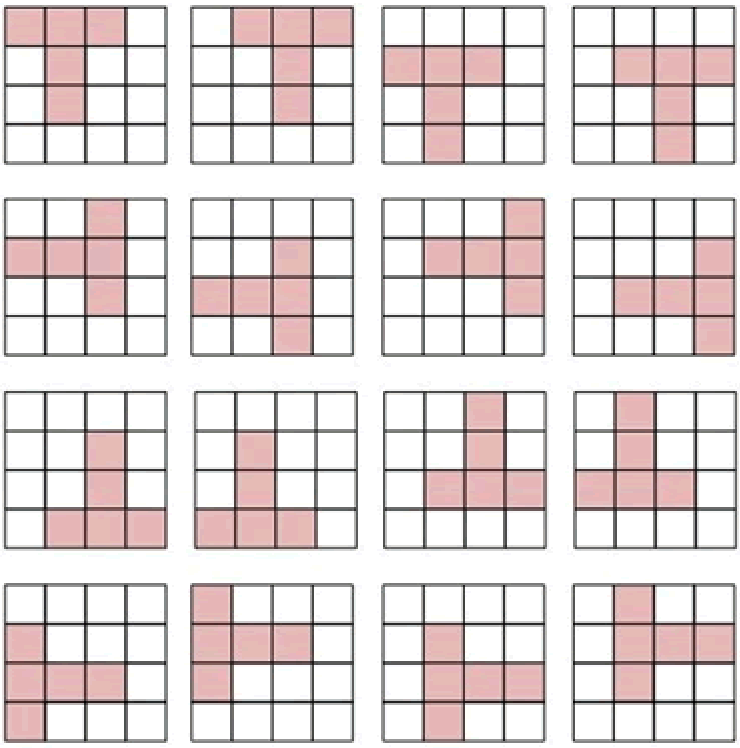


The number of winning arrangements on an n-by-n board is simply $2(n + 1)$. Can students find a general expression in *n* and *m* for rectangular *n* x *m* boards?

If a winning arrangement is a 5-cell 'T-shape', on a 3-by-3 board there are four winning arrangements…



…and on a 4-by-4 there are 16:



Students might challenge themselves to find for this winning arrangement a general expression for the number of winning arrangements on an n-by-n board.

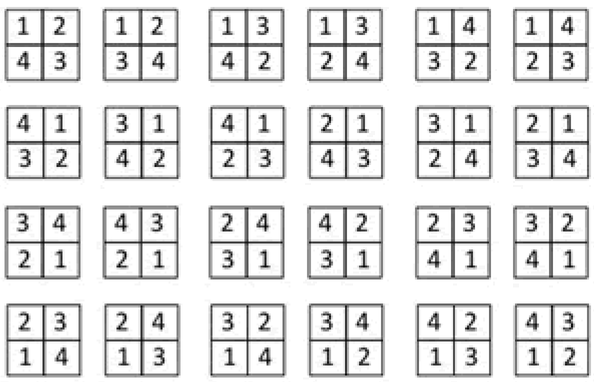
Is it perhaps $4^{(n-2)}$?

What happens for a different winning arrangement – such as a block of four cells forming a square?

Number arrangements

Students may be surprised by the large number of possible arrangements of the whole numbers from 1 to n^2 on an n-by-n board – even when n is small.

Students might try to get an initial 'feel for' this phenomenon by following a system to sketch all possible 2-by-2 boards containing the numbers 1, 2, 3 and 4…



…or, instead of listing every possibility, students might reason that if they are to enter one of the numbers 1, 2, 3 or 4 in each of the four cells, there are:

- 4 possibilities for the first cell that they choose
- 3 possibilities for the second cell for each of the 4 possibilities for the first cell, making 12 possibilities
- 2 possibilities for the third cell for each of the 12 possibilities for the first two cells, making 24 possibilities
- 1 possibility for the fourth cell for each of the 24 possibilities for the first three cells, making 24 possibilities in all.

This might be a good opportunity to introduce the conventional factorial notation, $4! = 4 \times 3 \times 2 \times 1 = 24$.

What if the numbers 1, 2, 3, 4 and 5 are available?

What if only the numbers 1 and 2 are available, and either number may appear on the board any number of times?

What are the possibilities for boards of other dimensions?

Probabilities

Because Bingo is a gambling game it is natural to be interested in the probabilities of winning in various situations.

A crucial question that often generates heated discussion among students is:

> *Are your chances of winning a game of Bingo affected by the particular numbers, or by the way that the numbers are arranged, on the Bingo card that you choose?*

Many Bingo websites quote excerpts from the book How To Win At Bingo, by Joseph E. Granville. They make misleading statements based on false conclusions, such as:

> *"Naturally, the heart of any winning Bingo system is card selection. Granville isolated crucial relationships between winning Bingo numbers and the master board. He demonstrated how to use these simple and proven truths to select a greater number of winning cards. Granville found that most methods players use to select their cards are completely backwards. Players are working against themselves without even realizing it!"*
> (From Bingo Strategies To Win Bingo)

> *"Extensive study of thousands of games has led Granville to the inescapable conclusion that every Bingo game follows definite patterns…patterns the average player is completely unaware of. By utilizing these patterns, Granville had discovered how to beat the odds at Bingo. Now you can too!"*
> (From Bingo Strategy)

You could challenge students to explain why, in opposition to Granville's apparent conclusions, the answers to all three of the following questions is 'No'!

> *Are there good cards or bad Bingo cards?*
> *Are there good or bad Bingo numbers?*

Is there such a thing as good or bad symmetry formed by the numbers on the card?

In How To Win At Bingo? Jim Loy presents three proofs that every Bingo card, whatever numbers are printed on it, and in whatever arrangement, has the same exact chance of being a 'winning' card, as any other card. This is an excellent opportunity to generate classroom discussion about the Gambler's Fallacy - and possibly a Gambler's Ruin situation.

What's Luck Got to Do with It? The History, Mathematics, and Psychology of the Gambler's Illusion by Joseph Mazur was published in June. It might help you appreciate the illusions of some Bingo players in a wider context!

Another question about probabilities that interests Bingo players is:

What is the probability of a single player – with no other players playing – getting a Bingo (winning) after a certain number of calls?

This question might be the starting point for rich investigation by students. In deciding how to approach the problem there is opportunity for much decision-making.

A student might start with a very simple situation in which she chooses to make and state certain assumptions, identifies what can be varied, and then explores what happens when those aspects of the situation are varied one at a time.

For example, suppose a student starts by considering a 2-by-2 card with the numbers 1, 2, 3 and 4 arranged randomly, the winning arrangement being any straight line across the card, and with 1, 2, 3 and 4 being the only numbers that might be called out.

Assuming that no number will be called out more than once, the probability of a single player getting a Bingo is 0 after 1 call, and 1 after 2 calls. But, if the set of numbers that might be called is expanded to include the number 5, the probability of getting a Bingo after two calls is now 12/20 = 3/5, although the probability after 1 call is still 0.

The student can explore what happens to the probability of getting a Bingo after two calls as the set of numbers that might be called is enlarged to include 5, and then 6, and then 7, and then 8, …, while no other assumptions about the situation are changed. The probability decreases.

The student will find, however, that, for a fixed set of numbers that might be called, as the number of calls increases the probability of getting a Bingo also increases. For example, if the numbers that might be called are the whole numbers from 1 to 10, the probabilities of getting a Bingo after 1, 2 and 3 calls are 0, 2/15 (8/60) and 11/60 respectively.

The probabilities of some situations in Bingo games are discussed in Math of Gambling and Durango Bill's Bingo Probabilities. But beware – in most discussions on the Internet of probabilities in Bingo, assumptions about situations are not clearly stated!

Creating Bingo 'matching' games

Activities in which students design collaboratively their own 'matching' Bingo games can provide opportunities for learning through discussion about possibilities; and in these kinds of activity, misconceptions are sometimes exposed.

In these activities the challenge for students is to think of numbers or expressions to be called out that 'fit' – usually that are 'equivalent to' in a clearly defined way – numbers, expressions or diagrams that are in the cells of Bingo cards that the students also design themselves.

Examples of some kinds of expressions and numbers that might be on students' Bingo cards can be seen at maths-bingo.com - but, of course, many other mathematical 'objects' might be represented on the cards. You and your students could use the free print-bingo.com resource to design, and then print out, your own sets of Bingo cards. However, normal drawing and presentation software allows more flexibility.

Instead of expecting students to think of the numbers or expressions to be called out AND design the Bingo cards, you might prefer to give students random Bingo cards, and challenge them to think of expressions to be called out (or displayed) in a Bingo game using those cards.

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5 things to do this fortnight

- Every New Year, let alone every new decade, brings opportunities to meet new challenges. This is an excellent time to reflect on, and respond to, some questions posed by Secondary Watch. Do you believe that you are helping your students' to become functional in mathematics? How? These are real, not rhetorical, questions. Why not respond by posting a comment – a genuine thought, observation, story or question – on the Blog, about the issues raised?

- Promoting and Assessing Mathematical Thinking is the focus of the next meeting of the Birmingham Branch of the Association of Teachers of Mathematics. John Mason will invite you to engage in mathematical thinking for yourself, prompting you to reflect on how to promote it in others and assess its development in learners. Although tasks will be chosen to interest, challenge and engage you, you are likely to be able to modify them to use with your students. Dave Hewitt is the Birmingham ATM Branch contact person. If you would like to be part of this all too rare kind of event on Saturday 29 January from 9.30am to 1pm in Room G39, School of Education, University of Birmingham please email Dave.

- What do you know about Realistic Mathematics Education (RME), which is an approach to teaching mathematics that has been used successfully over many years in other countries? Mathematics in Education and Industry (MEI) are producing new online GCSE resources based on Realistic Mathematics Education. These teaching materials enable students to develop problem solving and functional skills, and cover everything necessary for the Mathematics GCSE. Why not find out more about these new resources that are designed to help students really make sense of topics?

- Have you and your students seen the six short illustrated descriptions of topics of modern mathematics research, Maths Matters, which can now be downloaded as PDF documents from Maths Careers?

- Wednesday 23 January is the birth date of David Hilbert. You might celebrate by enjoying Beyond Infinity?, an effectively-illustrated talk by Dr Joel Feinstein of the School of Mathematical Sciences in the University of Nottingham.

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Subject Leadership Diary

Well merry Christmaths has gone now, everybody's had fun and decked the halls with boughs of holly. When I was at school there was a lad in the class called Derek Hall, so one year we covered him with holly!

So what's been on the agenda recently? Faculty meetings have been going on as usual with our sharing of good practice. We heard about how in another school the mathematics faculty meet over lunch on a Friday and discuss informally a specific topic that has been agreed in advance. We intend to give that a go this term. Part of the problem in meeting at lunchtimes is the commitment many of the faculty have to clubs and duties, but we'll get round that one way or another!

The second in faculty, or assistant subject leader – or whatever title is used – is an invaluable person. Of course every faculty member is greatly valued because we all play a part. The assistant advises the subject leader on all matter of things, and picks up all manner of jobs. It's hard to imagine how I would cope without mine – my assistant knows just when I'm frazzled, and when to ease the pressure on me by taking over tasks that need doing. We set faculty meeting agendas together, and have recently been working to time-specified agendas so that certain items do not overrun. That way we get through our agendas in the time allocated, and don't impose on people's undirected time. It also helps focus the mind and stop irrelevancies creeping in – a good chair makes sure that doesn't happen anyway, but who chairs the chair?

Our Christmas Carol Service came and went – for me that service marks the start of Christmas. It is held in the local abbey and that adds an atmosphere second to none. It makes up for all the lessons the musical students have managed to avoid! The mince pies served afterwards were superb – this year we surveyed staff members – who had 5 different varieties of pies to taste and rate. Results were collated and analysed, and the overall winners were the pies made by Year 7 students in food technology lessons. They beat the commercial ones hands down - or 'hands in' since that was the way they were made!

GCSE mocks are done and marked now. No great surprises, so we're busy collating results and preparing appropriate letters to be posted to parents. They get one of three types of letter. The BUTB (boot up the backside) letter spells out in no uncertain terms that the student is not working to the standard we expect, and exam results suggest they will not achieve their target grade. The TLC (tender loving care) letter is sent to those students who haven't met their targets, but is more sensitive than the BUTB letter since we don't want to tip them over the edge. The other letter mentions that the student is on line to meet or exceed their target. All letters contain a timetable of revision sessions this coming term, stating topics that will be covered, and a tear-off slip to return – with an email address to use in case the tear-off slip is not likely to make it back to school!

We lost two days for snow at the start of December which was a blessing in disguise since it gave us a chance to catch up with some preparation – and marking if one had students' books at home! Incidents like this can also help focus students' minds because although they enjoy the time out of school they are also made acutely conscious that they have missed two days of schooling – we use our VLE to communicate with students about what they should be doing. No hiding place these days!

Well, I hope that all readers have had a restful break. Season's greetings to all engaged in furthering the mathematical education of young people everywhere.