Core concept 6.1: Geometrical properties

This document is part of a set that forms the subject knowledge content audit for Key Stage 3 maths. The audit is based on the NCETM Secondary Professional Development materials and there is one document for each of the 17 core concepts. Each document contains audit questions with check boxes you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications and explanations, and further support links. At the end of each document there is space to type reflections, targets and notes. The document can then be saved for your records.

6.1.1 Understand and use angle properties

How confident are you that you can explain, and deduce the properties of angles between parallel lines?

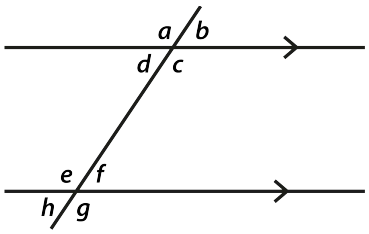
1 ☐ 2 ☐ 3 ☐ 4 ☐

How confident are you that you can prove the sum of the interior angles of a triangle, the sum of the interior angles of any given polygon and the sum of the exterior angles of any given polygon?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Alternate angles are formed when a pair of parallel lines is cut by a transversal, as in the diagram. Angles *d* and *f* (also *c* and *e*) are alternate angles and are equal.

Corresponding angles are formed when a pair of parallel lines is cut by a transversal, as in the diagram. Angles *a* and *e* are corresponding angles and are equal. Similarly, angles *b* and *f*, *c* and *g* and *d* and *h* are corresponding.



Two angles whose sum is 180° are **supplementary angles**. Two angles whose sum is 90° are **complementary angles**.

Proof that the angles in a triangle sum to 180°

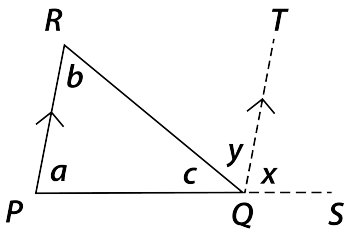
Students often confuse a demonstration for a proof. In Key Stage 2, students learn that the sum of the interior angles of a triangle is 180° and will have used this fact to calculate missing angles. In Key Stage 3, students will develop their understanding of what is meant by mathematical proof. This is likely to include understanding proof as a form of convincing argument based on logical deduction and an expression of generalization, as opposed to checking against a few specific cases. For example:

The diagram shows a triangle PQR. Extend the side PQ to S.

At Q draw a line QT parallel to PR.

Then angle *x* = angle *a* (corresponding angles) and angle *y* = angle *b* (alternate angles).

As *x*, *y* and *c* are angles on a straight line, their angle sum is 180°, that is $x + y + c = 180°$. So, $a + b + c = 180°$ and the statement *'The three angles of a triangle sum to 180°'* is proved.

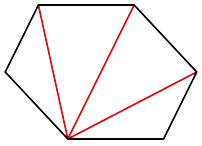


Proof that the angle sum of a n-sided polygon is (n – 2) × 180°

A polygon with n sides can be split into n – 2 triangles, each with an angle sum of 180°.

So, the interior angle sum is (n – 2) × 180°.

This will be 360° for a quadrilateral, 540° for a pentagon and 720° for a hexagon.



© Crown Copyright 2021

Proof that the sum of the exterior angles of a polygon is 360°

An exterior angle is on a straight line with an interior angle. Hence their sum is 180°.

In an *n*-sided polygon the sum of all the interior and exterior angles will be 180°*n*. The sum of the interior angles is (n – 2) × 180°.

Sum of the exterior angles = 180°*n* – (n – 2) × 180°.

= 360°

Further support links

- NCETM Secondary Professional Development materials: 6.1 Geometrical properties, pages 12–22

6.1.2 Understand and use similarity and congruence

How confident are you that you know the meanings of similar and congruent and the criteria by which triangles are congruent?

1 ☐ 2 ☐ 3 ☐ 4 ☐

Two shapes are **similar** if an enlargement of one will produce the other.

A geometric figure is **congruent** to another if it is the same size and shape. Congruent figures are the same in every way except for their position and orientation. In congruent figures, the corresponding angles and corresponding sides are equal.

To prove that two triangles are congruent, one of these four conditions must be proved for the two triangles:

- two sides and the included angle are equal (SAS)
- two angles and an included side are equal (ASA)
- three sides are equal (SSS)
- a right angle, the hypotenuse and one other side are equal (RHS)

Similar shapes have side lengths that are in proportion to each other and the angle size is preserved. In congruent shapes, both the length of the sides and size of the angles have been preserved.

Further support links

- NCETM Secondary Professional Development materials: 6.1 Geometrical properties, pages 24–27

6.1.3 Understand and use Pythagoras' theorem

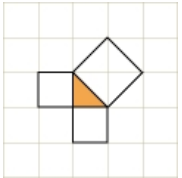
How confident are you that you know Pythagoras' theorem, and can use it to solve problems?

1 ☐ 2 ☐ 3 ☐ 4 ☐

The relationship described by Pythagoras' theorem offers another context for students to reason deductively and use known facts to generate other mathematical truths.

Pythagoras' theorem can be regarded as the following:

- A property of areas. In a right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.
- A property of lengths: $a^2 = b^2 + c^2$



© Crown Copyright 2021

It follows from Pythagoras' theorem that if $a^2 > b^2 + c^2$, then A is an obtuse angle; if $a^2 < b^2 + c^2$, then A is an acute angle.

This can be used to find missing lengths in right-angled triangles.

Further support links

- NCETM Secondary Professional Development materials: 6.1 Geometrical properties, page 28–31

Notes

© Crown Copyright 2021