Secondary Magazine Issue 80

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 80 of the Secondary Magazine.

Contents

Contributors to this issue include: Mary Pardoe, Richard Perring, Anne Price, Peter Ransom and Rosamund Sutherland.

From the editor

Welcome to another issue of the Secondary Magazine. Will you find reading it a ***hard and challenging*** activity? You might expect the editor to hope not. But suppose the editor hopes that there will be something to learn from what the contributors have provided?

In The Interview, Professor Rosamund Sutherland mentions heady days at the beginning of her career in mathematics education research – when she was meeting people such as Seymour Papert. In the June 1998 issue of *Game Developer* magazine, Seymour Papert wrote:

> *Two big lessons I have learned from computer games are opposites of the messages of the ads … . The first, … , is echoed by kids who talk about 'hard fun' and they don't mean it's fun in spite of being hard. They mean it's fun because it's hard. Listening to this and watching kids work at mastering games confirms what I know from my own experience: learning is essentially hard; it happens best when one is deeply engaged in* ***hard and challenging*** *activities. …The fact is that kids prefer things that are hard, as long as they are also interesting. The preoccupation in America with 'Making It Easy' is self-defeating and cause for serious worry about the deterioration of the learning environment.*

From: *Does Easy Do It? Children, Games, and Learning* by Seymour Papert

Four years later, Papert was continuing to try to communicate to the general public his observations about 'hard fun':

> *Once I was alerted to the concept of "hard fun" I began listening for it and heard it over and over. It is expressed in many different ways, all of which all boil down to the conclusion that everyone likes hard challenging things to do. But they have to be the right things matched to the individual and to the culture of the times.*

From: *Hard Fun* by Seymour Papert, Bangor Daily News, 2002

In Teaching for Learning Mathematics, published in 2006, Ros Sutherland addresses Papert's conclusions about 'hard fun'. In her opening remarks she writes:

> *What can we learn from mathematics lessons in which students are engaged and motivated to struggle with difficult mathematical ideas?*

Railway shunting problems are the subject of the Focus on… .Some students will find that shunting puzzles are 'hard fun', and even their teachers may be quite ***challenged*** if they try to make sense of patterns that can be detected in their solutions. And in Two birds – one stone the author sketches a picture of students learning through the ***hard*** but interesting ***challenge*** of endeavouring to engage younger pupils in doing some mathematics.

It may be helpful to ask ourselves whether we are regularly engaging our students in things that, because they are matched to individuals and really are hard and challenging for them, are also fun for them.

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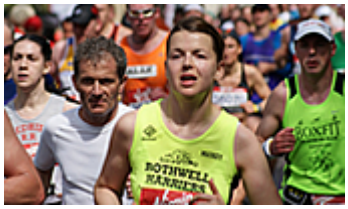

National Centre for Excellence in the Teaching of Mathematics

It's in the News!
A marathon a day…

The fortnightly *It's in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but as a framework which you can personalise to fit your classroom and your learners.

During 2010 and into 2011 Belgian Stefaan Engels completed an extraordinary feat of endurance, running 365 marathons in 365 days!

The races, in the UK, Spain, Portugal, Belgium, Canada, Mexico and the US, mean that Stefaan Engels ran more than 15 000 km in a year, averaging around four hours for each marathon.

This resource uses this extraordinary feat as a context for exploring notions of distance and allows the opportunity for students to practise proportional reasoning in a number of contexts.

Download this *It's in the News!* resource - in PowerPoint format

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The Interview

Name: Rosamund Sutherland

About you: I was born in Birmingham, and moved at the age of six to Cwmbran in South Wales. My father was a young physicist and he was taking up a new job at British Nylon Spinners in Pontypool, South Wales. When I was ten, we moved from the new town of Cwmbran to the small village of Raglan, and I passed the 11 Plus to go to Monmouth School for Girls. I was given huge freedom to play in the countryside, and often explored the ruins of Raglan Castle with friends, taking what I now think of as enormous risks as we climbed the ramparts. I now view playing and risk-taking as being important aspects of my working life.

I studied mathematics at Bristol University, and after a brief spell as a computer programmer for British Aerospace moved to the United Sates, where my daughter Joanna was born in 1972. My son Andrew was born two years later, in Hertfordshire.

My longest period of teaching mathematics was at a further education college in Borehamwood, where I also gained a teaching qualification by studying at what is now the University of Hertfordshire. At the same time, I worked as a mathematics tutor for the Open University and having met Celia Hoyles at an Open University Summer School, talked to her about the possibilities of doing research in mathematics education. Celia had become interested in Logo, and coincidentally Joanna and Andrew were learning Logo in their Hertfordshire primary school. We applied to the Leverhulme Trust for a research grant, and in 1983 the Logo Maths Project was born. This was the beginning of my research career at the Institute of Education in London. In 1995 I moved to the Graduate School of Education at Bristol University, where I have developed my research in both mathematics education and teaching and learning with ICT. I have been chair of the British Society for Research into Learning Mathematics (BSRLM) and chair of the Joint Mathematical Council of the UK (JMC). In 2006, I published Teaching for Learning Mathematics (Open University Press) which brings together my research in mathematics education. I am currently leading a JMC working party [scroll down to second afternoon discussion – Ed] on "The place of digital technology in school and college mathematics".

What got you interested in mathematics?
My father was very important in encouraging my love of mathematics. From a young age, I can remember him counting backwards to give me a deadline: 10,9, 7, …1, 0, but when he got to 1, he would give me more time by starting to introduce fractional numbers, ½, ¼, 1/8, 1/16… before reaching 0. I always enjoyed mathematics lessons, and when I reached secondary school my father encouraged my love of mathematics by giving me books by Martin Gardner and buying me copies of Scientific American, so that I could read Gardner's maths column.

All my close girl friends found mathematics difficult and I remember spending ages on the phone trying to explain mathematics homework to them (a skill which later became useful when I worked as an Open University tutor in the 1970s). I studied pure mathematics, applied mathematics and physics at A-level and also pure and applied mathematics at S-level. I can still remember the pleasure of tackling S-level problems, where it was always difficult to know how to get started and such a challenge to solve the problem. Years later, I helped my daughter with Further Mathematics A-level and she often talked about the pleasure of 'losing yourself' in a mathematics problem. I realised that as a teenager this had also been one of the allures of mathematics for me. I went on to study mathematics at the University of Bristol. Here I had my first experience of learning to program in Algol, punching cards and waiting for hours for the program to run, often over night (this is when I met my husband Ian and we have now been married for

42 years). There were very few female students on my mathematics degree course and most of the male students had learned more mathematics at school than I had, in order to apply for Oxford or Cambridge University. At the time, I thought that they were more able mathematically than I was, but now I suspect that they had just been taught more mathematics.

When did you start thinking about doing research in mathematics education?
After almost four years of teaching mathematics at a Further Education College and in my late 30s, I began to think about doing research. I had supported my husband to study for a PhD in the early years of my marriage, and my father had studied part-time for a PhD when I was a young girl (carrying out ultra-sound physics experiments in our spare bedroom) and so the idea of research was familiar to me. My children at primary school were learning Logo programming and I had started to teach Logo to a group of students at the FE college where I was working. With Celia Hoyles, I wrote a grant proposal to the Leverhulme Trust. We were successful with one year's funding, and later with two more years of funding.

This was the beginning of my research career and I started to study for a PhD part-time. My own focus was on Logo programming and learning algebra and I learned a tremendous amount from presenting my research at the annual international conference for the International Group for the Psychology of Mathematics Education (PME). These were heady days for me, when I began to meet people like Gerard Vergnaud, Seymour Papert and Hans Freudenthal. There was so much to learn about mathematics education research and I loved the intensity of those early research years. These were very exciting times, we really believed that Logo programming could change the learning of mathematics in schools.

Who has influenced your mathematics education research?
Celia Hoyles was very influential in the early days of my development as a researcher. I was privileged to start my research career at the Institute of Education in London, where without realising it I learned so much from being part of a strong research culture. I have also learned a tremendous amount from working collaboratively with colleagues in Mexico (Teresa Rojano) and France (Nicolas Balacheff).

What are you interested in?
I have always been interested in the ways in which computers (and now digital technologies more generally) can be used in schools to enhance the learning of mathematics. This has included Logo programming, spreadsheets and dynamic geometry. I realise that despite the research evidence, these technologies have had little impact on the day-to-day teaching of mathematics in schools. Nowadays, my research is focused on developing approaches to professional development that support mathematics teachers to be informed by relevant research on technology-enhanced learning. This was the main thrust of the ESRC InterActive Education project, published as Improving Learning with ICT.

What about outside mathematics education?
I am now a grandmother with twin grandsons of three and a half, a granddaughter who is almost one, and another grandson of eight months. There is nothing I enjoy more than playing with them, talking to them and taking them out and about. I can't wait to teach the twins Logo and hope that I am already, without knowing it, sharing with them my love of mathematics. I am a Governor of a new Academy in South Bristol, an area where less than five per cent continue to Higher Education. My involvement with this school is very important to me and will continue long after I have retired. Some years ago, I took up tango dancing and my fantasy is to spend hours learning to dance, but this will have to wait until after retirement.

The future?
I will be retiring in less than two years' time, but my identity is so tied up with my love of mathematics that I will continue to find ways of expressing this. It may be through work with local schools, through policy work at a national level, through playing with my grandchildren or through teaching some of the students

at the Academy in South Bristol. Digital technologies have not yet been exploited in mathematics classrooms and I hope that the JMC policy brief that I have coordinated will impact on changes to the mathematics curriculum – so that all young people are able to learn to use the power of digital software to produce and investigate mathematics.

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Focus on…shunting


Shunting wagons

Shunting problems, in which the challenge is to rearrange railway trucks with as few shunts as possible, provide situations in which a teacher can focus on some particular ways of acting mathematically.

The Shunting of Railway Wagons is an article published in 1994 in the IMA journal, Teaching Mathematics and its Applications. In it, Alan Jasper, a secondary school mathematics teacher, describes how he became fascinated by shunting problems when he first came across them in Hugh Burkhardt's book, The Real World and Mathematics - which is about the place of real problem solving in the mathematics curriculum.

> *It [the shunting of railway wagons] was used as an example to show how binary notation could be used for automatic decision-making. The article gave a worked example, outlining the method involved—and it worked. I couldn't see why (the method worked) but I wanted to try it out on some examples of my own. I was now hooked! I wanted to know more and more about it and also to give some of my pupils the opportunity of having a go!*

Alan Jasper begins his quest to try to find some 'principles' of efficient shunting by deciding to explore first a particular, relatively simple, shunting situation, about which he asks himself some questions:

- Can I rearrange the trucks in the order that I want by shunting them about on this track layout?
- If I achieve my aim, how can I know whether or not I have done it in the least number of shunts?
- What tactics should I use in this situation?

To help himself address these questions, Alan adopts a convention (about always moving the trucks in one siding before moving the trucks in another siding) in the hope that by keeping the process as orderly as possible some effective tactics will become apparent. He also develops a notation, which he uses to show actions and results in a way that he hopes will facilitate his thinking about them.

In doing these things, Alan is already acting mathematically by:

- first exploring a simple case
- setting himself a specific challenge, and asking questions about it
- adopting a convention which he makes explicit
- developing a notation in order to reveal structure.

Getting started

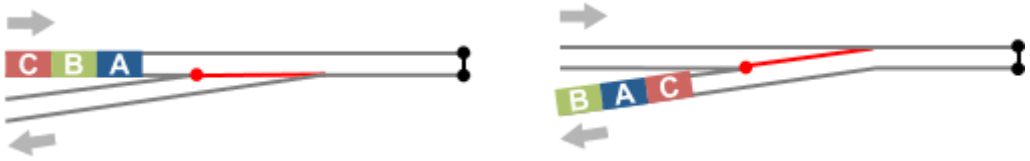
Alan Jasper describes how he introduced shunting to students in a Year 7 mixed ability class and in a Year 11 'Higher Level' class so that the students in both classes became fruitfully engaged in exploring shunting 'tactics'.

> *I decided to give my Year 7 (mixed ability) class the first opportunity to have a go at Shunting. I was concerned as to how long it would take them to get to grips with it but I was surprised at how quickly they picked it up. I think this was partly due to the fact that I asked several pupils to act as wagons in order to demonstrate the shunting process and then allowed the class to solve simple problems—by*

moving pupils (who were carrying lettered cards). This was later reinforced by moving some magnetic letters about on the white board and using this to develop the notation I wanted them to use.

A simple introductory problem for students to tackle first – possibly as Alan describes in his article – might be, for example:

Shunt the trucks from... ...to...



The challenge is to shunt the trucks from their original order (blue, green, red – or ABC) on the inward line, so that eventually they are on the outward line in a new order (green, blue, red – or BAC).

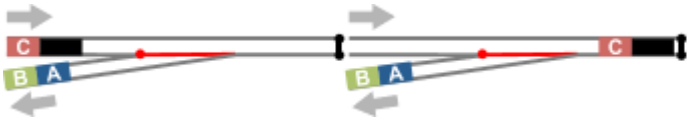
It could be done in the following way.

Engine comes in on the outward line, and shunts (pulls) blue truck and green truck into siding:

Points are changed, and engine shunts (pushes) blue truck and green truck onto the outward line:

Engine goes back to get red truck:

Engine shunts (pulls) red truck into siding:



Points are changed, and engine shunts (pushes) red truck onto the outward line:

Engine goes off (down the inward line) leaving the three trucks on the outward line in the new order:

Ten engine-movements, five point-changes, one un-coupling of trucks and one coupling of trucks are required to do this simple bit of shunting!

In Starting Points (Banwell, Saunders and Tahta, 1986) we are reminded that:

> *Discussion may reveal that there are two basic operations – bringing a truck (or pupil) into the siding and taking one out of the siding. These may be coded by two letters or symbols. One class chose 'I' and 'O' for 'in' and 'out' and then gradually changed to numerals 1 and 0.*

If we adopt this notation, then shunting from order ABC in direction of intended travel to BAC is represented by 110010, as was shown above.

Students could explore the different orders into which it is possible, on this particular track layout with one siding, to shunt three trucks, then four trucks, then five trucks… (They will need to make assumptions about the maximum number of trucks that the siding can hold at any one time).

Railway points photograph by S. Terfloth

They might also try to find conditions that a string of 0s and 1s necessarily satisfies if it represents the shunting of a train of trucks from one order to another.

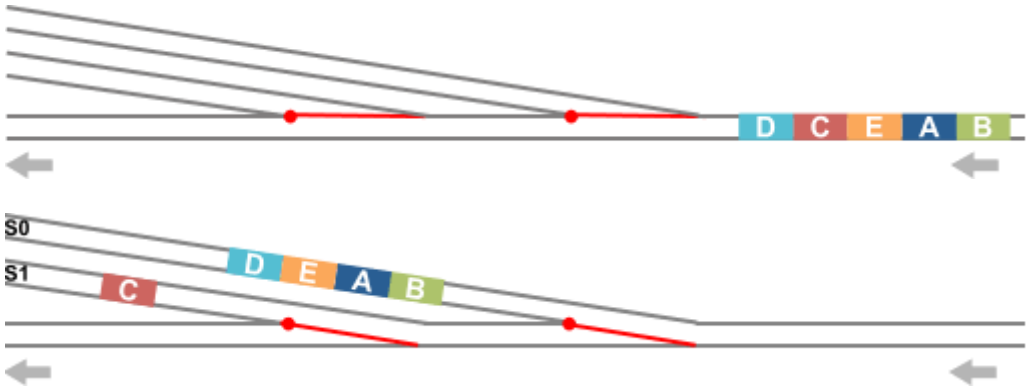
More complex situations

A natural next step might be to investigate track situations with more than one siding, extending, developing or re-inventing our notation in order to record more complex shunts.

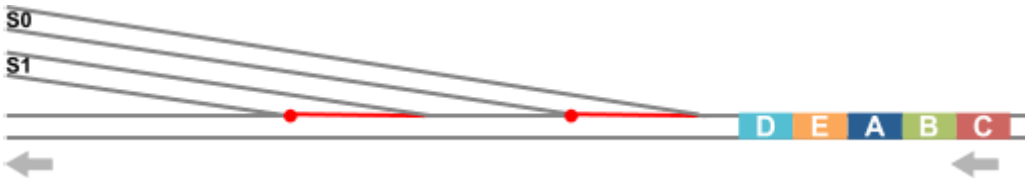
For example, Alan Jasper in *The Shunting of Railway Wagons* challenges himself to rearrange five trucks from the order DCEAB on a mainline to the order ABCDE, by shunting the trucks in and out of two sidings. He labels the sidings, S0 and S1, and solves the puzzle in two stages, each of which consists of several pushes and pulls, and which he denotes as *Shunt 1* and *Shunt 2*. (Students will realise that they need to know precisely to what they and others are referring when they use the word 'shunt'. Here, Alan Jasper is using 'complete shunt' to mean putting each truck from a queue into either S1 or S2, and then shunting

them back to produce a new sequence of trucks.) The following diagrams show the positions of the trucks at crucial times during the whole process, together with Alan Jasper's notation.

Shunt 1 changes the order from DCEAB to DEABC

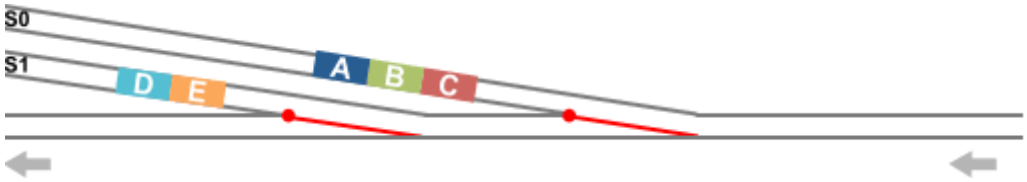


S0 : DEAB
S1 : C



Shunt back S1 : C
Shunt back S0: DEABC

Shunt 2 changes the order from DEABC to ABCDE



S0 : ABC
S1 : DE

Shunt back S1 : DE
Shunt back S0 : ABCDE

Alan Jasper's *Shunt 1* and *Shunt 2*, shown above, would both in reality involve a sequence of several actions.

It is possibly because, in reality, even relatively simple shunts require many pushes, pulls, point changes, un-couplings and couplings that shunting can appear complex, and the 'structure' of successive arrangements of trucks in different locations can therefore be obscured.

Shunting yard photograph by Rajithmohan

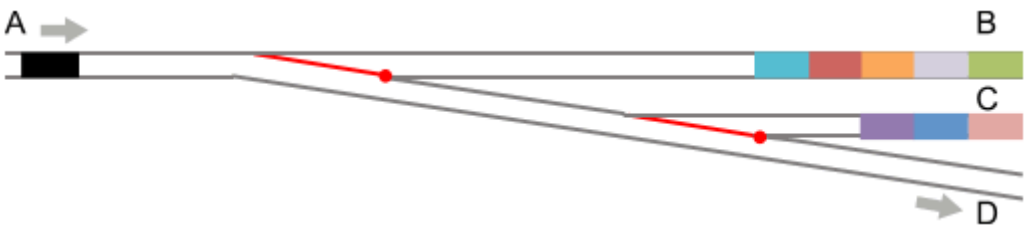
You may like to compare your students' effective approaches, and how they choose to record their findings, with the ways of displaying and recording combinations of shunts developed by Alan Jasper and his students. If you are feeling brave you can try to follow how 'a meaningful use of number bases … arose naturally'.

Classic model railways shunting puzzles

Model railway photograph by Benkid77

Model railway enthusiasts enjoy constructing various track layouts, and then shunting trucks on them! Two well-known layouts for model railway shunting puzzles are the classic British Inglenook Sidings and the American Timesaver.

Students can set up many different particular *Inglenook Sidings* puzzles. The challenge is to use an engine that is originally on an inward mainline, A, to assemble on an outward line, D, a train that is composed of five trucks which you have pre-selected from eight trucks that are originally in two sidings, B and C. Therefore, if you start with this ….



… you can choose to try to achieve many different truck arrangements on the outward line.

On the more complex *Timesaver* layout, the challenge is to move trucks from starting positions that you set up yourself to selected *destinations* that you also set up yourself. Therefore, again, it is possible to explore very many different particular problems.

Students may enjoy shunting trucks virtually on interactive computer representations of the two model-railway shunting layouts just mentioned. They can explore a Riverside Yard puzzle, which is on the Inglenook Sidings layout, and the Skowhegan & Athens puzzle, which is posed in the *Timesaver* situation. The Dawson Station Railroad Shunting Puzzle is another interactive *Inglenook Sidings* puzzle.

Some other starting points

A problem which is simple to pose, but not so simple to solve, is the classic sequential movement railway shunting puzzle:

> How do you get two trains travelling in opposite directions on a single track to pass each other using just one siding that can hold only one truck?

Students may be interested in this discussion of a real shunting situation in Sweden.

At NRICH students will find a truck Shunting Puzzle and another Shunting Puzzle, in which they move, or visualise moving, counters on a board.

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Two birds – one stone

It's a June day; learners are spread across the field talking earnestly about mathematics. Some are engaged in solving the Six Frogs problem moving origami frogs into place, while others are trying to predict how long it will take to fill the white enamel bath propped against the classroom wall with water running into it from a hosepipe. It's the afternoon and although the learners have been working with their teachers on mathematical tasks all day, there has been no lapse of concentration.

Closer inspection reveals that the learners are Year 5 pupils and the 'teachers' are Year 10 students. There are a few grown-ups around and they too are doing mathematics, asking some good questions and moving the learning on.

Twenty able mathematicians in Year 10 had planned this enrichment day specifically for approximately 45 Year 5 pupils from several different local schools. In taking on much of the planning and all of the delivery, the Year 10 students developed their personal organisational and communication skills and, perhaps surprisingly, were seen to be wrestling with some of the challenges they set for Year 5. It is well recognised that teaching others deepens one's own understanding!

The Year 10 students decided to begin the day with a non-threatening task involving Pascal's triangle because they thought that the younger pupils were likely to be a bit apprehensive on their first visit to the secondary school. Three 45-minute slots followed during the morning, allowing time for each task to be explored in some depth. One theme was networks, which built on activities such as Path to the Stars and Home Time. Starting points for the tasks were presented in attractive colours on laminated sheets on which pupils could scribble without restraint, and which could be re-used. Pupils worked in groups of three or four throughout the day.

For the after-lunch session the Year 10 students, with support from their class teachers, had devised a treasure hunt – pupils had to solve mathematical clues that took them from task to task each manned by helpful and enthusiastic students. Some pupils became engrossed in tasks that particularly interested them and spent extended time in discussion with their 'teachers'. Others were eager to complete the treasure hunt in record time. At the end of the day, groups of Year 5 pupils presented their findings to their parents, proudly sharing the mathematics they had learnt.

I have run similar days with students in other years – for example with Year 10 students 'teaching' students in Year 7 – all of which have been successful. The stages in the process that achieved this were:

- a planning meeting with school staff to fix the date, to decide on numbers to be involved and agree time off from the normal timetable, and so on
- issuing invitations to those pupils to be involved, and to the parents of the younger pupils
- a planning meeting with the older students allowing plenty of time (approximately half a day) to shape the enrichment day, explore possible tasks, and reach decisions about what to include – students may be hesitant at first but this stage does not last long
- the older students taking responsibility for producing materials, but with some support from teachers
- where appropriate, the older students appointing one of their group to take on the role of a 'journalist' to take photographs, and to collect comments from participants, for the school website(s).

If you decide to try something similar, have fun!

5 things to do this fortnight

- On 24 March, the Institute of Mathematics and its Applications (IMA) will hold a one-day conference, IMA Mathematics 2011, to consider current issues in mathematics and its applications. The conference will address research topics in mathematics, public understanding of mathematics, and industrial applications of mathematics. The speakers include Celia Hoyles, Director of the NCETM, and Steve Humble, NCETM Senior Regional Coordinator.

- During Mathematics teaching in change, an evening at the Institute of Education, University of London, on 22 March, you could join in a discussion following input by a panel of experts from the Institute – which, for many years, has been at the leading edge of research in mathematics education. Professor Celia Hoyles, Dr Jenny Houssart, Professor Richard Noss, and Professor Dave Pratt will respond to the current increased scrutiny of mathematics teaching and learning by presenting new ideas based on research evidence. In order to secure your place just send an email RSVP to Melanie Harradine by **11 March**.

- Professor Marcus du Sautoy has accepted an invitation to be President of the Mathematical Association in 2012-13. You might investigate the possibility of inviting Marcus' Marvellous Mathemagicians to engage your students in an inspirational mathematical workshop.

- Have you visited Gresham College's new website? Gresham College runs a series of free public lectures, many of which are available to listen to or to watch. You can now immediately see all upcoming and past mathematics lectures.

- René Descartes would have been 415 on **31 March**. During 1637, he developed the Cartesian coordinate system, which, apparently, is so called after his Latin name, Renatus Cartesius. Why not allow Descartes and Tom Lehrer to entertain your students for a few moments?

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Subject Leadership Diary

Back to school this week after a refreshing half term and I find that things move swiftly in education! So much has happened so quickly this year that it is hard to keep track of everything that affects us all. For example, The Education Bill was introduced into the House of Commons on Wednesday 26 January 2011. The Department for Education states that "The Bill is an important step in implementing the Government's education reform programme and helping to create an education system that delivers ever higher standards for all children. The Education Bill takes forward the legislative proposals in the Schools White Paper, The Importance of Teaching and measures from the Department for Business, Innovation and Skills to improve skills, including two elements of the reforms to higher education funding." On 2 February it received its second reading.

In the summary, I noticed that it mentioned the demise of the GTC ("So what's the problem there?" I hear you ask). That reminded me about the fact that all the research published on the Becta website is to go somewhere else since Becta ceased to exist at the end of January.

Fortunately, we still have the NCETM website where we can access different areas of research. Since you are reading this diary you know about the National Centre, but what about all those mathematics teachers who do not know about it? How many mathematics teachers have YOU told about the NCETM? How often do you use the website? I must admit that I feel the whole faculty and I would benefit by using it more often!

I had the good fortune to be able to attend a conference in London in January run by Nuffield and Bowland that outlined their latest work. GAIM (Graded Assessment in Mathematics) materials were produced under the leadership of Margaret Brown in the 1980s and these have now been updated by the Nuffield Foundation to AMP (Applying Mathematical Processes). There are so many free applications on this AMP microsite that you MUST find time to look at them – not only with your faculty but also with your students. In my faculty, we will be looking at the materials in pairs, deciding where they fit into our schemes and the best way to use them with our students. Now is the time to abandon solitaire and become a team player! We are very grateful to the Nuffield Foundation for providing these investigations and practical activities.

The other part of the conference was concerned with Bowland Maths. They have recently added some new Professional Development materials on assessing the key processes, and the video clips will be most useful. We will probably show one each time we have a faculty meeting, since we have assimilated the case studies into our schemes of work. There are some new case studies soon to be put on the Bowland website, so why not ask someone to keep a watching brief on the website. Thanks to the Clothworkers' Foundation for hosting this event and to all who helped plan it and contribute to it.

It is surprising how we have turned our faculty meetings around over the past two years. They used to be occasions for just dealing with the administrative side of things – like sorting out lists and dealing with paperwork that senior managers required – but now we always start with faculty members 'sharing good practice'. This is when one or two members of the team talk about something they have used which went well, or would be 'even better if'. Our PGCE students have contributed to this over the past two years and we all gain from everyone's experience. Then, we had a spell when we would do some reading around abridged research reports on mathematical education, each member summarising what they had read. The Ofsted report Mathematics: understanding the score was very useful in persuading senior management that we needed time to engage with mathematical pedagogy!

It's been a busy half term – I've just got back from a mathematical conference in San Antonio, but you'll have to wait for a later diary to hear about what happened there. I managed to get some real data from the plane on altitude and outside temperature, so I'm interested in finding out a bit more about temperature gradient!