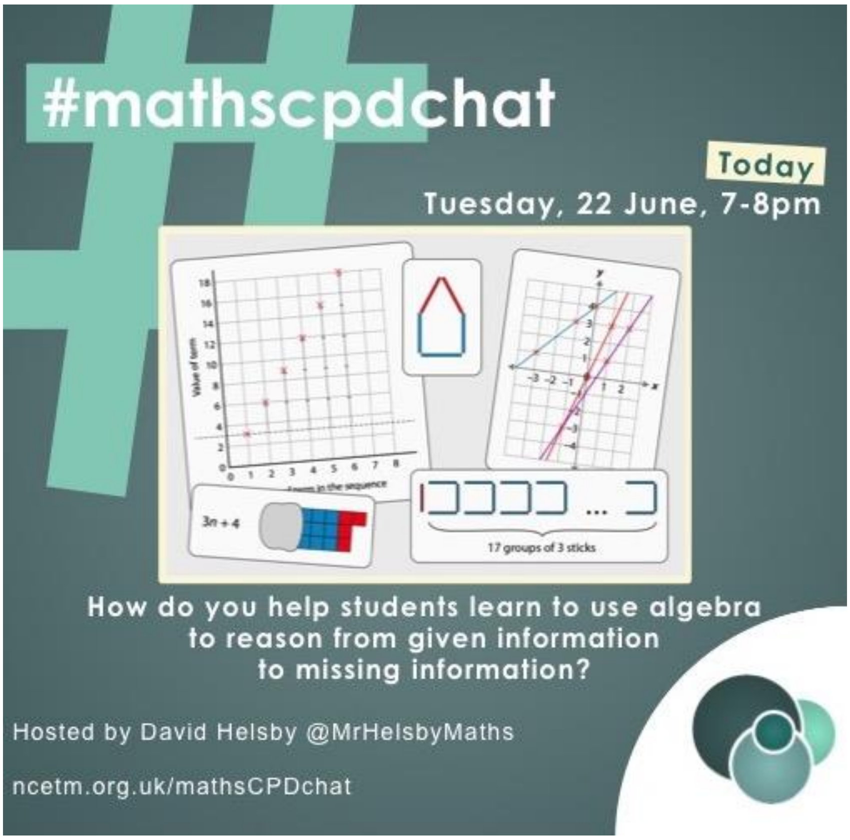
#mathscpdchat 22 June 2021

How do you help students learn to use algebra to reason from given information to missing information?

Hosted by [David Helsby]

*This is a brief summary of the discussion – to see all the tweets, follow the hashtag **#mathscpdchat** in Twitter*



Among the links shared during the discussion were:

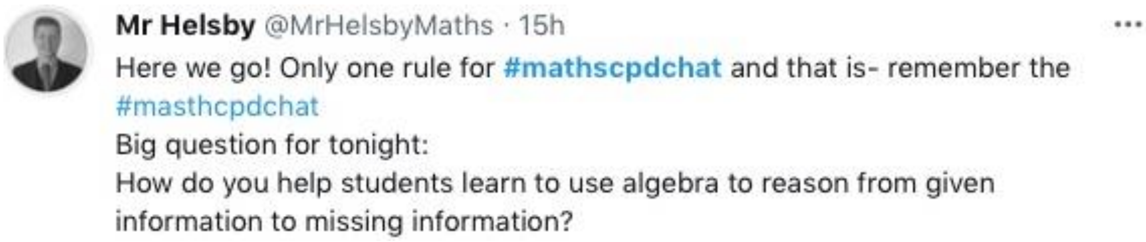
median: don steward: mathematics teaching 10-16 which is the well-known large collection of clearly-presented valuable tasks created by the late Don Steward. The tasks facilitate and enhance the mathematics learning and teaching of school mathematics. It was shared by David Helsby [both ways](#) which is one of the many tasks created and presented by the late Don Steward. This task is a simple introduction to algebraic expressions, with generalising and proof as a central feature. It was shared by [David Helsby](#)

[Collection of 'classroom ready' versions of resources from *Improving Learning in Mathematics*](#) which are free-to-download files on Craig Barton's website. Craig describes them as 'the pdfs and PowerPoints of the main activities which I have found really engage and challenge my students'. It was shared by [David Helsby](#)

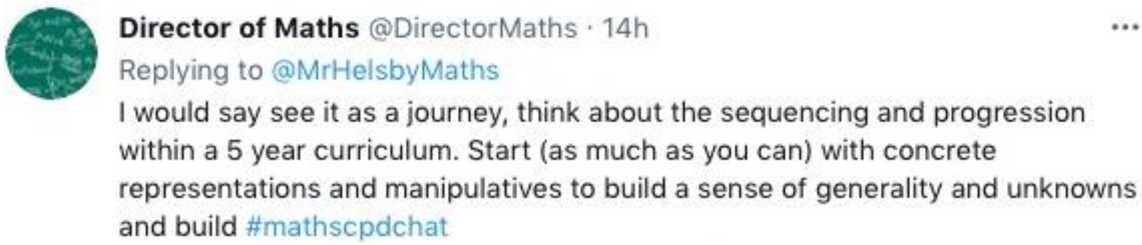
[Chapter 8: Algebra ... in *Children's understanding of mathematics: 11-16*](#) which is a paper by Dietmar Küchemann. The author explains what can be deduced from the responses of students to test items (from extended research) that sample a wide range of typical secondary school algebra. The detailed analysis reveals important considerations for planning the teaching of algebra to students of any age. It was shared by [Mary Pardoe](#)

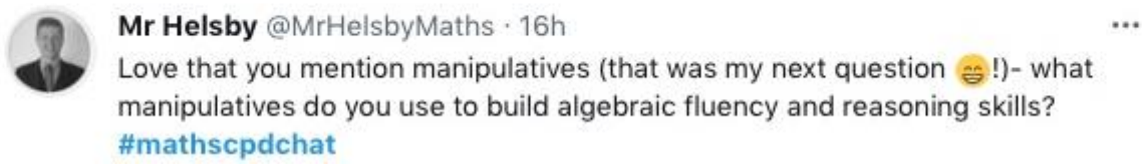
The screenshots below, of chains of tweets posted during the chat, show parts of three conversations; about teaching KS3/4 students to understand and use algebra, about a primary teacher's 'way in' to teaching algebra explicitly, and about tasks intended to help students use algebra in their reasoning. **Click on any of these screenshots-of-a-tweet to go to that actual tweet on Twitter.**

The conversations were generated by this tweet from [David Helsby](#):



and included these from [Gemma Scott](#) and [David Helsby](#):





Director of Maths @DirectorMaths · 16h

Replying to @MrHelsbyMaths

This moves away from manipulatives but to pick up on the development of "algebraic" reasoning we have had a big push this year on numerical reasoning with KS3 this year which is starting to feed through into their algebraic reasoning #mathscpdchat

Mr Helsby @MrHelsbyMaths · 15h

That is really positive to see that change further on the line, strong number work has been mentioned a few times this evening. Have you got any resources which develop numerical reasoning which you have/ could revisit to develop algebraic reasoning further on? #mathscpdchat

Director of Maths @DirectorMaths · 15h

Our KS3 SoL is based around "skill", "reason", "apply" and at the end of each series (2-3) lessons students answer a question in each category. Nothing ground breaking but for reason it's usually something along the lines of "Bob says.... do you agree? Explain why" #mathscpdchat

Mr Helsby @MrHelsbyMaths · 15h

That's a really interesting structure, we can see a lot more of these types o question on the GCSE, do you tend to go for misconceptions, correct ideas which need explaining or a bit of both? #mathscpdchat

Director of Maths @DirectorMaths · 15h

Bit of both 😊 don't want things to become too predictable so they can bypass the reasoning stage #mathscpdchat

these from David Helsby, Martyn Yeo and Mary Pardoe:

Mr Helsby @MrHelsbyMaths · 15h

Question for all key stages:
What does YOUR 'first' lesson including algebra look like? What activities do you choose to use? **#mathscpdchat**

Martyn (He/Him) @martynyeouk · 15h

Replying to @MrHelsbyMaths

Best one I ever started with in reception was the function machine. I put one apple inside a cardboard box and out came 2!

Great start to algebra!
#mathscpdchat

Mr Helsby @MrHelsbyMaths · 15h

Great to get some primary input (pun intended 😃!) here! Intrigued to know what happened next, how did you develop this idea further? #mathscpdchat

Martyn (He/Him) @martynyeouk · 16h
Replying to @MrHelsbyMaths
Keep using "the magic box" that could change things!

It could double things, halve things, add 2 more!

Then bought in some numbers too! Kids loved it al could work out what special thing the box was doing next!

#mathscpdchat

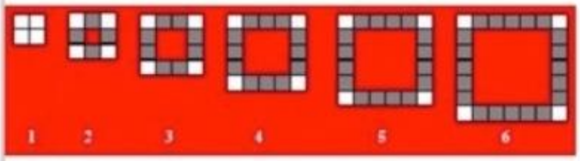
Mary Pardoe @PardoeMary · 16h
I love this! So pupils might unprompted SAY 'it's doing that (the same thing) to ANY number'! #mathscpdchat

Martyn (He/Him) @martynyeouk · 16h
Yep - stealth algebra!
I cant take full credit as I learnt it from somewhere else (but canr remember where!)
#mathscpdchat

and these from [David Helsby]() and [Mary Pardoe]():

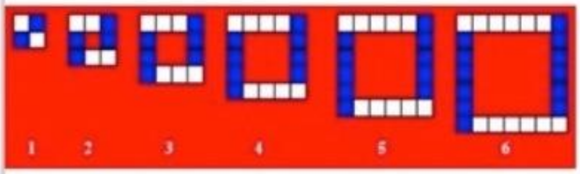
Mr Helsby @MrHelsbyMaths · 16h
What activities do you use that allow students to use their algebraic skills to reason mathematically? **#mathscpdchat**

Mary Pardoe @PardoeMary · 16h
Replying to @MrHelsbyMaths
I have found that it helps to get students talking about how they see the structure of sequences of visual images ... e.g. ...
#mathscpdchat

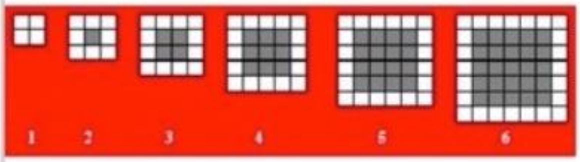


So the number of squares in a ring is four, plus four times one-less-than-the-position-of-the-ring.

B: You can split a 'ring' into four equal bits. It's just four times its position!



D: They are proper whole squares, with smaller proper whole squares removed from the middle.



Mary Pardoe @PardoeMary · 16h

... depends on 'where they are', but when they say what they see in ordinary language first, then algebraic representation of what they are saying must mean something to them? #mathscpdchat

e.g. ...

T: Choose a letter to be the number that gives the position of the ring in the sequence.
What is your letter?

D: p

T: What is the number of squares in the pth ring?

D: p plus 1 squared, minus p minus one squared.

T: Can you write that without words?

D eventually writes: $(p + 1)^2 - (p - 1)^2$

T: So the number of squares in the pth ring is that (pointing to $(p + 1)^2 - (p - 1)^2$)?

A: But my way it's not that complicated! It's four, plus four times one less than p.

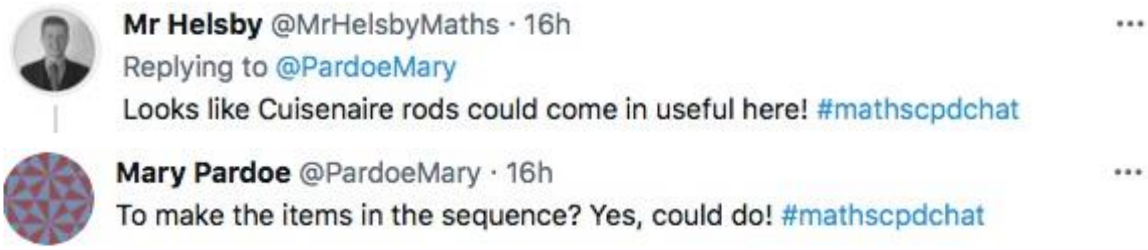
T: Can you write that without words?

A eventually writes: $4 + 4(p - 1)$

T: So the number of squares in the pth ring is that (pointing to $4 + 4(p - 1)$)?

B: But it's just 4p! That was what I said.

E: But it's - they're all the same! How can it be $(p + 1)^2 - (p - 1)^2$ and also $4 + 4(p - 1)$ and also just $4p$?



(to read the discussion sequence generated by any tweet look at the 'replies' to that tweet)

Other areas where discussion focused were:

the host first reminded contributors of the main question of the chat, which was 'how do you help students learn to use algebra to reason from given information to missing information?' … some of the resulting discussion is represented in the sequence of screenshots of tweets reproduced above:

- there was also a discussion about using **algebra discs** 'because they (the students) are already familiar with them in Year 7 from adding with negative numbers' … a teacher tweeted some images showing some ways in which he uses algebra tiles, such as this image, as an example of how he uses them to represent linear expressions …

To obtain the negative of $3x$, i.e. $-(3x)$, we flip the three (x) discs as shown:

(x) (x) (x) —flip→ (−x) (−x) (−x) we write $-(3x) = -3x$

To obtain the negative of $-3x$, i.e. $-(-3x)$, we flip the three (−x) discs as shown:

(−x) (−x) (−x) —flip→ (x) (x) (x) we write $-(-3x) = 3x$

What happens if we put three (x) discs and three (−x) discs together?

(x) (−x)
(x) (−x) we write $3x + (-3x) = 0$
(x) (−x)

We will get zero pairs.

We can also use algebra discs to represent linear expressions.

Example: $4x + 2$

(x) (x) (x) (x) (1) (1) $4x + 2 = x + x + x + x + 1 + 1$

Example: $3x - 1$

(x) (x) (x) (−1) $3x - 1 = x + x + x + (-1)$

Example: $-2x + 3$

(−x) (−x) (1) (1) (1) $-2x + 3 = (-x) + (-x) + 1 + 1 + 1$

Example: $x + 2y$

(x) (y) (y) $x + 2y = x + y + y$

What is the linear expression represented by (x) (x) (−y) (1) (1)?

… and added this comment:

Lee Overy @Lwdajo · 16h

It's also used in solving linear equations, expanding linear expressions, factorising quadratic expressions...#mathscpdchat

the host tweeted this poll, in order 'to get people talking' …

Mr Helsby @MrHelsbyMaths · 15h

#mathscpdchat What comes first?
(Comments below!)

Algebraic fluency	**56.5%**
Algebraic reasoning	34.8%
Other	8.7%

23 votes · Final results

… to which there were only two response-tweets during the chat …

Lee Overy @Lwdajo · 14h
Replying to @MrHelsbyMaths
Algebraic representation. #mathscpdchat

Mary Pardoe @PardoeMary · 15h
Replying to @MrHelsbyMaths
This may be relevant!

From here: researchgate.net/profile/Dietma…

#mathsCPDchat

Implications for teaching

It is hoped that this research will help teachers see more clearly the diverse conceptual demands of seemingly commonplace activities in school algebra. The research has identified a number of different meanings that can be given to the letters in generalised arithmetic, the choice of which may depend to a large degree on children's cognitive levels. More generally, in algebra and in the other topics investigated, the research has found that children frequently tackle mathematics problems with methods that have little or nothing to do with what has been taught. This may be because mathematics teaching is often seen as an initiation into rules and procedures which, though very powerful (and therefore attractive to teachers), are often seen by children as meaningless. It follows that children's methods and their levels of understanding need to be taken far more into account, however difficult this may be in practice.

the host asked teachers what their 'first' lesson that included algebra typically looks like; one resulting discussion is shown in the sequence of screenshots of tweets above, but there were other replies too:

- a teacher commented that in a 'first lesson involving algebra' she would 'explain that we are **looking at the language of maths**' and 'introduce the concept of touching signifying multiplication' … 'and so a + a + a = 3a and talk about why that's useful';
- another teacher would **start with 'algebraic notation**, a comparison with arithmetic, showing the similarity of language, the meaning of 3(a + b), for example, and the link to the distributive law' … the host commented that 'a really strong understanding of number (and therefore arithmetic) seems to be key';
- a substantial discussion was generated by a tweet in which a teacher mentioned that another teacher had once told his pupils that **'he had 3 apples and 2 bananas, and wished there was a quicker way to write it, and the children suggested 3a and 2b**' … in reply a teacher posted this tweet …

Lee Overy @Lwdajo · 16h
Replying to @martynyeouk @MrHelsbyMaths and @AJMagicMessage
But what does the variable "a" represent in this representation? It's not number of apples! #mathscpdchat

… and later commented that 'I would avoid 'a' and 'b' when discussing number of apples and bananas' to make it less likely for pupils to interpret '3a' as 'three apples' (rather than 'three times the number of apples') and '2b' as 'two bananas' (rather than 'two times the number of bananas') … a contributor to the chat then tweeted these two images of extracts from Chapter 8 (written by Dietmar Küchemann) in *Children's Understanding of Mathematics 11-1*6 (2005) edited by Kath Hart (link provided above), in which the author reports and discusses implications of findings from substantial research …

Letter as object

This category has already been discussed in the context of the perimeters in Question 9, where, for the first three figures, the letters can be regarded as denoting the sides of the figures rather than their unknown lengths.

This interpretation of the letters can also be used successfully in some, but not all, of the parts of Question 13 illustrated below, in which children were asked to simplify various algebraic expressions.

Table 8.4 Correct responses (14 year olds)

13(i) (Level 1)		13(iv) (Level 2)		13(viii) (Level 3)		13(v) (Level 4)	
2a + 5a =		2a + 5b + a =		3a − b + a =		(a − b) + b =	
7a	86%	3a + 5b	60%	4a − b	47%	a	23%

Using a letter as an object, which amounts to reducing the letter's meaning from something quite abstract to something far more concrete and 'real', allowed many children to answer certain items successfully which they would not have coped with if they had had to use the intended meaning of the letter. However this reduction in meaning frequently occurred when it was *not* appropriate. This happened particularly with items that involved 'objects' (cabbages, pencils, wages, hours, etc.) but where it was essential to distinguish between the objects themselves and their *number*. This distinction can sometimes be very hard to grasp. A classic example is (or used to be) the translation of the relationship 'one shilling equal 12 pence' into 's = 12d', (letter as object) instead of 'd = 12s'.

This confusion arose in Question 22 (Table 8.5), even with children who did well on the test as a whole. To solve this item, the letters have to be regarded, at a

Table 8.5 Children's responses (14 year olds)

Question 22 (Level 4)

Blue pencils cost 5 pence each and red pencils cost 6 pence each. I buy some blue and some red pencils and altogether it costs me 90 pence.

If b is the number of blue pencils bought and if r is the number of red pencils bought, what can you write down about b and r?

5b + 6r = 90	10%
Two correct pairs, of (6, 10), (12, 5), (18, 0), (0, 15).	1%
b + r = 90	17%
6b + 10r = 90 or 12b + 5r = 90	6%

… which prompted this short discussion:

Lee Overy @Lwdajo · 16h
This was in part the point I was making about the classic apples and bananas use of "a" and "b". Perhaps it's better to use letters unconnected with the objects, so to help minmise the misconception that "a" represents the number of apples. #mathscpdchat

1 2

Mr Helsby @MrHelsbyMaths · 16h
Is there not something to be said for students making connections (and suggestions) to make maths meaningful for them? For them the initial notation of 3a+2b is their 'shorthand' for the amount of fruit... How do we bring these two ideas together? #mathscpdchat

1 2

Lee Overy @Lwdajo · 15h
I think it may plant the seeds of misconception as to what the letter represents in this model, and add to extrinsic load. #mathscpdchat

2

Tom Oakley @ThatMathsMan · 3h
Sorry to join in late. Is 'a stands for the price of an apple' and 'b stands for the price of a banana' (where prices might vary) an ok way in to substitution? E.g. Calculate the price of 2 apples and 3 bananas when apples cost a pence and bananas cost b pence each. #mathscpdchat

- there was a very brief discussion about the meaning of 'variable', during which an image of this extract from the Chapter 8 referred to above was tweeted:

The added meaning that relationships of this kind give to 5b + 6r = 90 is a genuine advance over interpreting the letters as specific unknowns or generalised numbers, and it was decided to regard the letters used in this way as variables. An important feature of these relationships is that their elements are themselves relationships, so they can be called '*second-order* relationships' (' "12 is greater than 6" by more than "5 is less than 10" '). This characteristic provides a useful operational definition of variables, in effect 'letters are used as variables when a second (or higher)- order relationship is established between them'.

Having found a way of defining the concept of a variable the problem remained that items involving variables could often be solved in a simpler way, and it proved very difficult to devise items where it could reasonably be assumed that variables were required to solve them. The best item in this respect was Question 3 (see Table 8.8), which was deliberately worded in such a way as to minimise this problem.

Table 8.8 Various responses to Question 3 (14 year olds)

Which is larger, 2n or n + 2? Explain.	
Correct, conditional response (eg 2n, when n > 2)	6%
2n	71%
n + 2 or 'the same'	16%

The point of this question was to see whether children would recognise that the relative size of the two expressions (2n and n + 2) was dependent on the value of n. As can be seen from the Table, most children wrote that 2n was the larger, and

the host asked contributors about tasks that their students have worked on in which the students have opportunities to use their algebraic skills to reason mathematically … as this was asked near the end of the hour-long chat there was only one response, which is shown in the sequence of screenshots-of-tweets-linked-to-the-actual-tweets provided above.