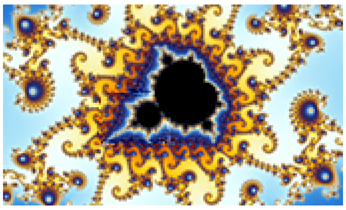
Secondary Magazine Issue 73



National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 73 of the Secondary Magazine.

Contents

Contributors to this issue include: Alison Clark-Wilson, Mary Pardoe, Richard Perring and Peter Ransom.

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Secondary Magazine Issue 73

National Centre for Excellence in the Teaching of Mathematics

From the editor

Welcome to Issue 73 of the Secondary Magazine – we hope that you will not find that it is either boring or difficult.

Research carried out during 2008 by Margaret Brown, Peter Brown and Tamara Bibby revealed five main reasons given by 16-year-old students for not opting to continue studying mathematics. The same reasons for 'rejecting' mathematics echo through research literature. In order of frequency, the students' reasons were:

- mathematics is too difficult
- I do not enjoy or like mathematics
- mathematics is boring
- I do not need mathematics for my future degree or career
- mathematics is not useful in life.

Furthermore, 42% of the students who were predicted to get the top grades said that they thought mathematics was too difficult to continue studying!

In a paper included in Issue 25 of the Philosophy of Mathematics Education Journal, published last month and featured in 5 things to do, Professor Hilary Povey takes that research as her starting point for an exploration of the connections between alienation from the study of mathematics and teaching mathematics for equity. She argues that failing to address equity issues in the teaching of mathematics in secondary schools is inter-connected with the reasons given by young people for their alienation from mathematics itself.

Benoît Mandelbrot, the creator of fractal geometry – on which we focus in Focus on introducing fractal ideas – had much of real interest to say about each of those five components of negative attitudes to mathematics. Professor Mandelbrot's extraordinary life's work led him to some clear conclusions about strategies for overcoming each of the five negative views of mathematics held by the students in the survey. We include a brief summary of some of those conclusions in Benoît Mandelbrot's thoughts about mathematics education.

Your students are not likely to find too difficult, or be bored by, the kind of activity described in An idea for using ICT in the classroom – using digital photos to reason geometrically.

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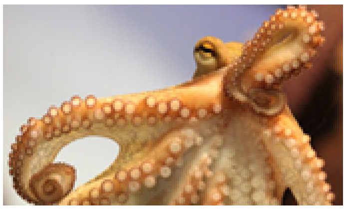

National Centre
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Teaching of Mathematics

It's in the News!
Psychic Octopus

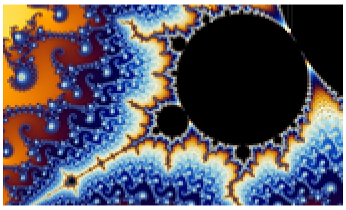
The fortnightly *It's in the News!* resources explore a range of mathematical themes in a topical context. The resource is not intended to be a set of instructions but as a framework which you can personalise to fit your classroom and your learners.

Do you remember Paul the Octopus? He achieved worldwide fame this summer during the football World Cup by correctly predicting the results of six of Germany's matches, as well as picking Spain to win the final, leading to claims that he was psychic.

Paul, who had been born in Dorset before moving to Germany, passed away peacefully in his tank at the end of October. Before he died, Paul had put his fame to good use, promoting the England bid to host the World Cup in 2018. The host will be announced by FIFA on the 2 December this year.

This resource uses the story of Paul as a context to generate discussion and debate around probability and chance.

Download this *It's in the News!* resource - in PowerPoint format



Focus on…introducing fractal ideas

"The Mandelbrot set covers a small space yet carries a large number of different implications. It is so simple that most children can program their home computer to produce the Mandelbrot set."

Benoît Mandelbrot, A fractal life – an interview with Benoît Mandelbrot, *New Scientist*, 13 November 2004

Some of the mathematics of fractal geometry, which was pioneered by Benoît Mandelbrot (1924 – 2010), is quite complex and involves sophisticated mathematical concepts. But students could explore some basic ideas such as those sketched below.

A good place at which to start refreshing your own understanding is the Yale University Fractal Geometry website created by Michael Frame, Benoît Mandelbrot, and Nial Neger.

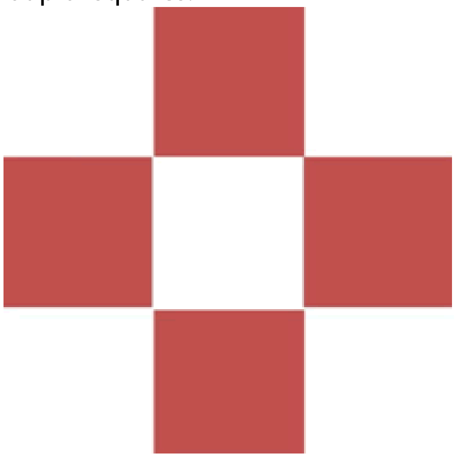
The simplest fractals are self-similar. They are made of smaller copies of themselves that are mathematically similar to a starting shape. Therefore creating and exploring self-similar fractals encourages learning about similarity and enlargement. And, in the process, students use naturally the intuitive idea of iteration.

The iterative process that produces a simple self-similar fractal can be summarised by the following steps:

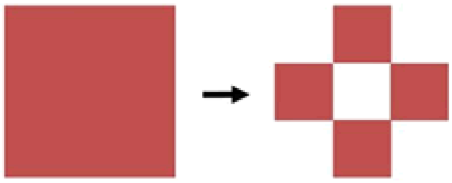
- choose a starting shape – the *initiator*
- design a collection of shapes that are mathematically similar to the initiator – the *generator*
- replace each copy of the initiator with a copy of the generator that is scaled appropriately – the *rule*.

For example, the initiator might be a square:

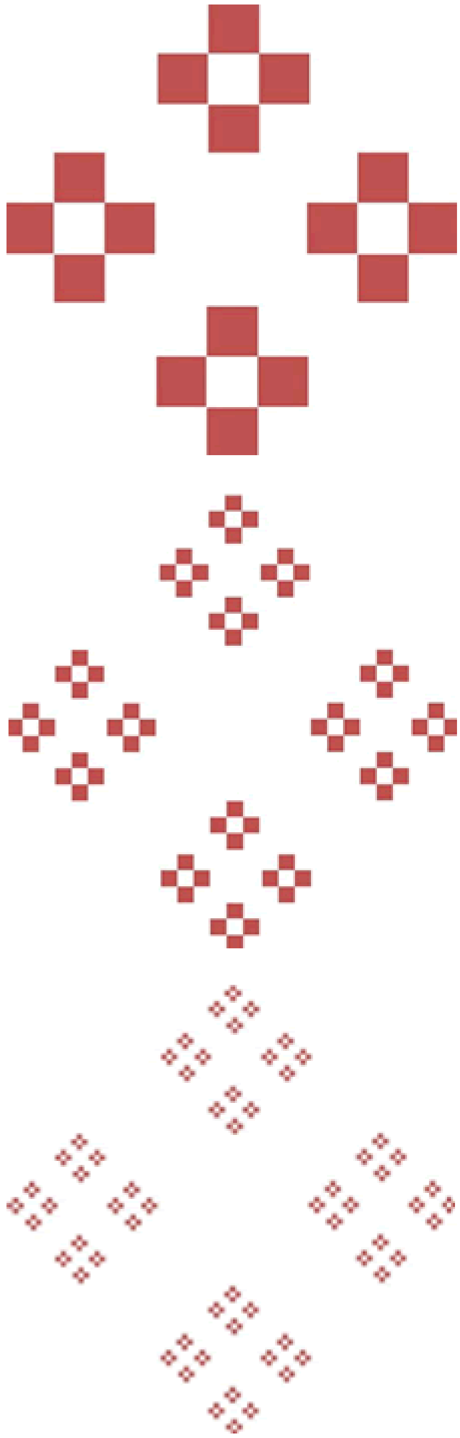
And the generator might be this group of squares:



Applying the rule once produces this change:



Therefore, these diagrams show the first four stages of the iterative process applied to this example:



Number sequences are associated with this fractal, which is made of:

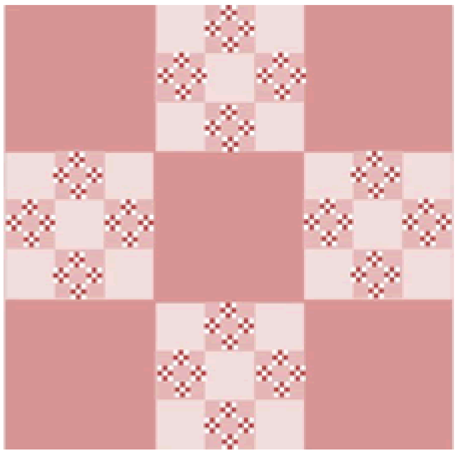
1 copy of the original shape
or
4 copies of the original shape, each of which is similar to the original with linear scale-factor 1/3
or
16 copies of the original shape, each of which is similar to the original with linear scale-factor 1/9
or
64 copies of the original shape, each of which is similar to the original with linear scale-factor 1/27

or

256 copies of the original shape, each of which is similar to the original with linear scale-factor 1/81

or

… … …



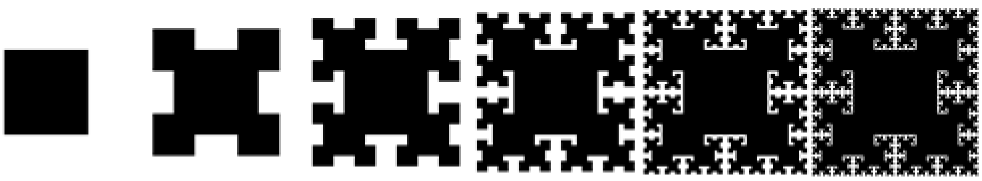
Students can imagine this process continuing forever – they can understand that the number of possible stages of the iterative process is infinite.

It is this aspect of a fractal structure that suggests the related ideas of ‘zooming in’ again and again forever, and that a fractal has no natural size.

Students who understand these simple ideas will probably enjoy creating their own fractal structures.

Some simple well-known fractal structures have special names. Students could be challenged to describe the particular iterative processes that produce them.

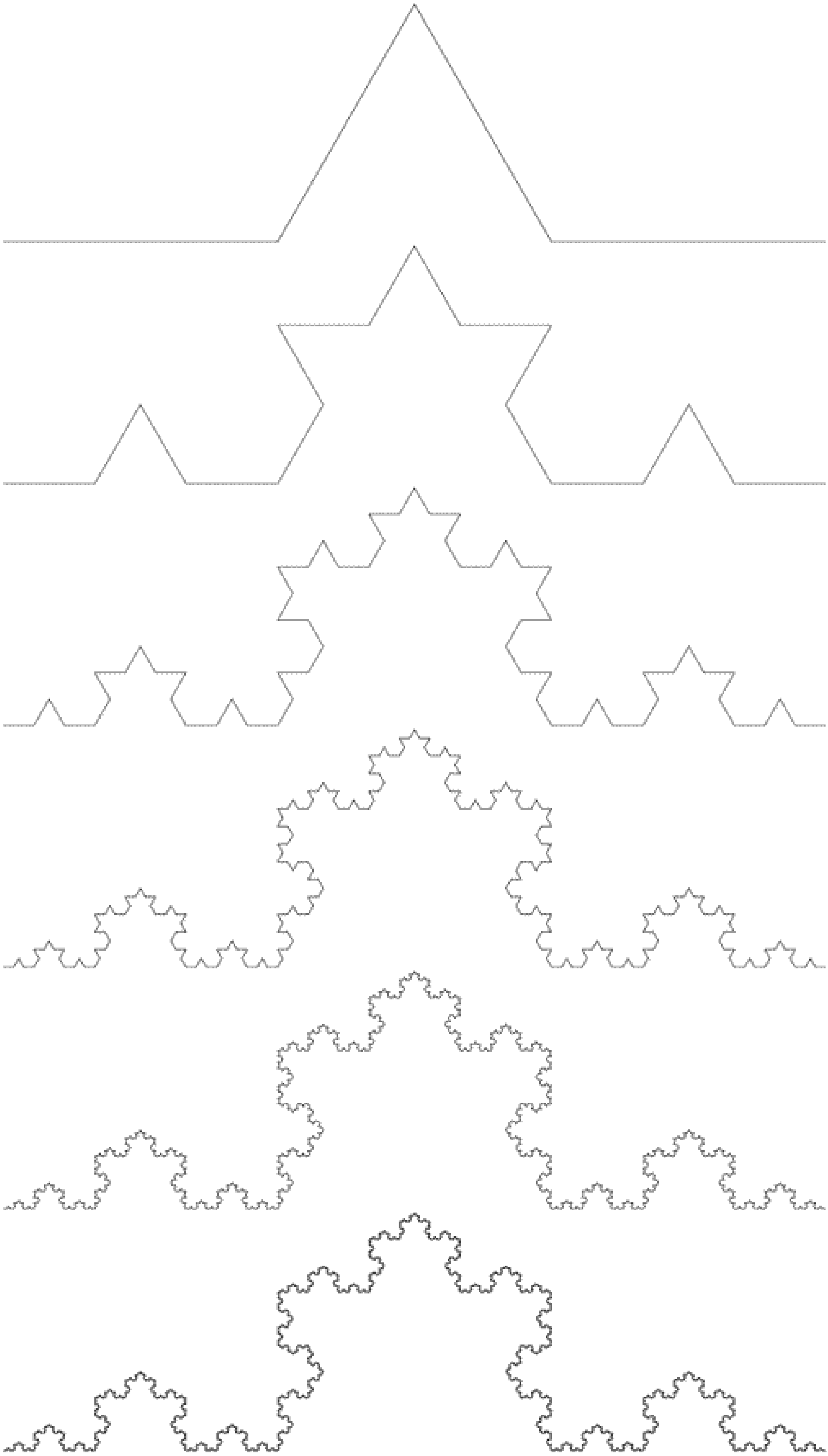
For example, what combination of initiator, generator and rule is generating this *T-square* fractal?

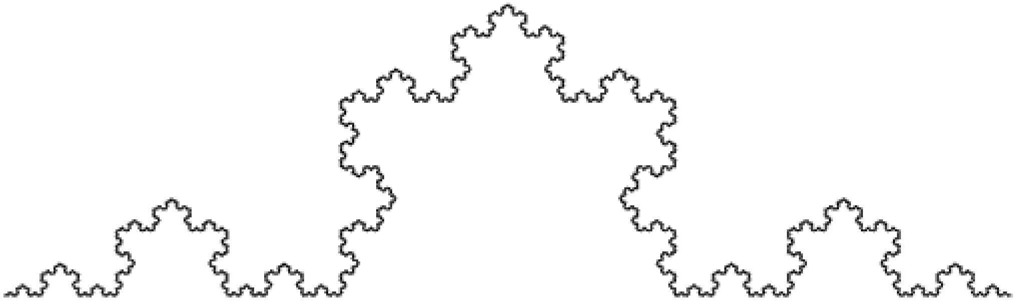


T-Square fractal by [Solkoll]

Can students themselves generate the four sequences of shapes at the top of [this page]?

What initiator and generator together generate this sequence of shapes that converges to the fractal known as the *Koch curve* – which is the limit approached as the iterative steps are followed over and over again?

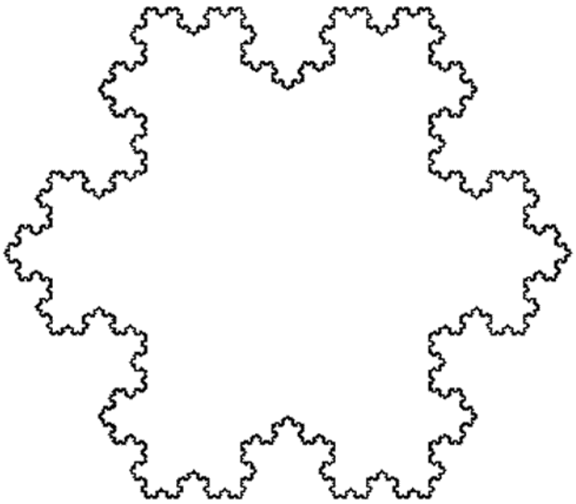




Koch curves by Christophe Dang Ngoc Chan

The Koch curve can be seen as four copies of itself, or as two copies of itself. Challenge students to explain how!

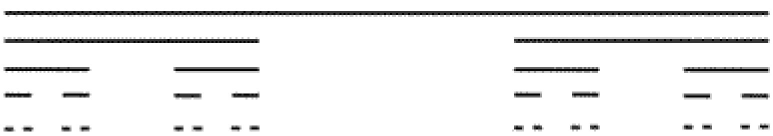
By starting with an equilateral triangle, rather than a single line segment, students could create the first few stages towards a limit called the *Koch snowflake*.



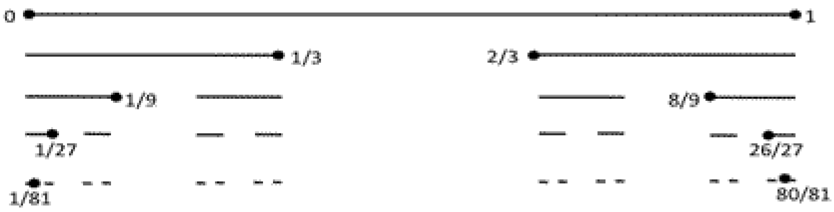
Koch snowflake by Saperaud

You could show students the first five stages of a sequence of patterns, the limit of which is a fractal named the Moore curve. Can they describe the iterative process, and then produce their own versions?

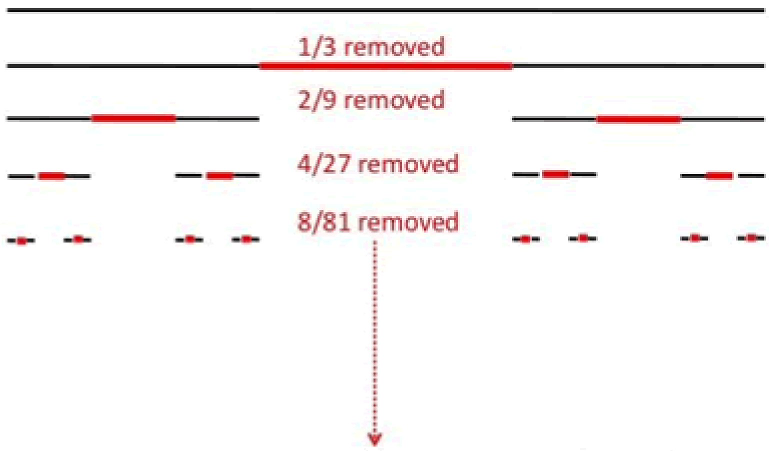
The *Cantor set* – or *Cantor's Dust* – is a very simple self-similar fractal which is, nevertheless, extremely interesting. It contradicts intuitive ideas about space!



The starting point of the endless journey towards the Cantor set is a line segment representing the interval on the real number line from 0 to 1 inclusive. The first step is to remove the middle third of the line segment, between 1/3 and 2/3, leaving behind the points representing 1/3 and 2/3. In the second step the middle third of each of the remaining two line segments are removed, again leaving behind the end-points. This process is repeated indefinitely.



More advanced students can start to see how strange is the Cantor set by thinking about the sum of the segments that are removed.



$$\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\frac{8}{81}+\cdots\cdots = \frac{1}{3}\sum_{n=0}^{\infty}\frac{2^n}{3^n} = \frac{1}{3}\left\{\frac{1}{1-\frac{2}{3}}\right\} = \frac{1}{3}\times 3 = 1$$

This seems to be telling us that by the end of eternity the WHOLE line segment – from 0 to 1 – will have been removed. But it is fairly easy to show that there will in fact be as many points remaining as there were to start with!

For more elucidation go to mathacademy.com or to Cut The Knot.

For a good introduction to the **Mandelbrot set** go to the Yale University Fractal Geometry website. And Mad Teddy's Cantor and Mandelbrot web page is helpful, with excellent links.

The Mandelbrot set is a wonderful phenomenon that can greatly enhance students' explorations of complex numbers and the Argand plane.
It is created using the quadratic recurrence equation $z_{n+1} = z_n^2 + c$.

Following a convention that has become established, points whose distance from the origin stays 'forever' less than or equal to 2 are black, and other points in the Argand plane are coloured according to the number of iterations before they 'escape' – before their distance from the origin is greater than 2. This diagram shows approximately how the Mandelbrot set relates to the circle of radius 2 centred on the origin. The Mandelbrot set consists of the black points inside the circle:

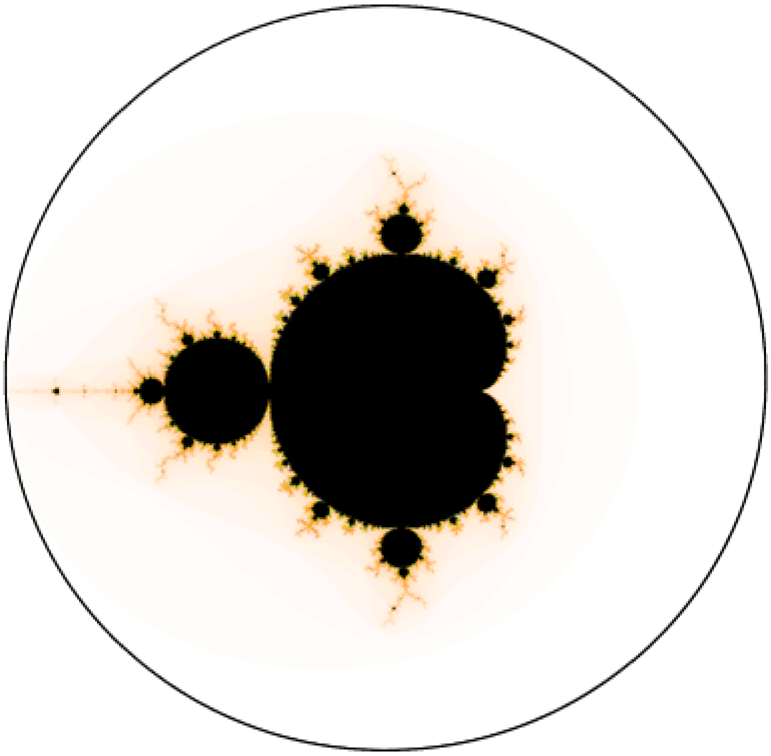


Illustration based upon fractal image by Yami89

A sequence of pictures that zoom into the Mandelbrot set can be found towards the bottom of this page. Because its shape forever and ever repeats as you look closer and closer at it, Benoît Mandelbrot called it a 'fractal'!

Issue 28 of the Secondary Magazine also focussed on fractals.

But the best person to help us understand and appreciate fractal geometry is its great originator, Benoit Mandelbrot himself – who was the one and only full-time fractalist! Watch and listen to him, and read what he says, in this fascinating Big Think Mandelbrot video with transcript.

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Mandelbrot fractal set - images by Wolfgang Beyer some rights reserved

Secondary Magazine Issue 73

National Centre for Excellence in the Teaching of Mathematics

An idea for using ICT in the classroom – using digital photos to reason geometrically

Issue 71 of the Secondary Magazine included an article that explored how digital images might be imported into a range of different software to provide a motivating starting point for students exploring transformations.

In this issue, this approach is extended to consider how we can use mathematics software to support students to reason geometrically about the shapes and images within carefully selected photographs...

So look at the photograph of The Northgate, Chester...

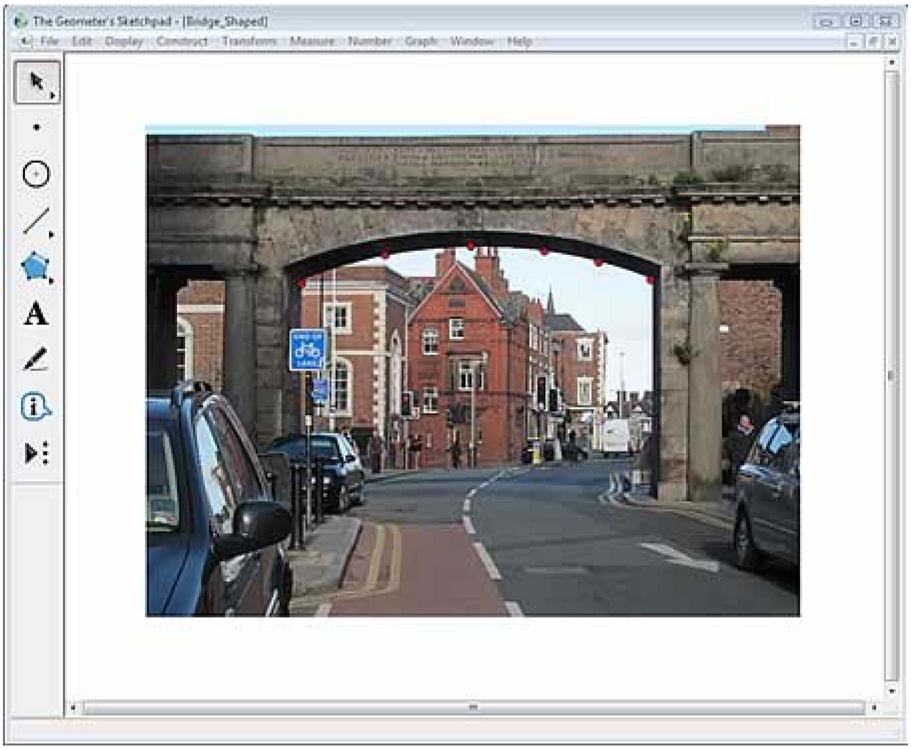
What sort of curve do you think is formed by the arch?

How could you prove or disprove your conjecture?

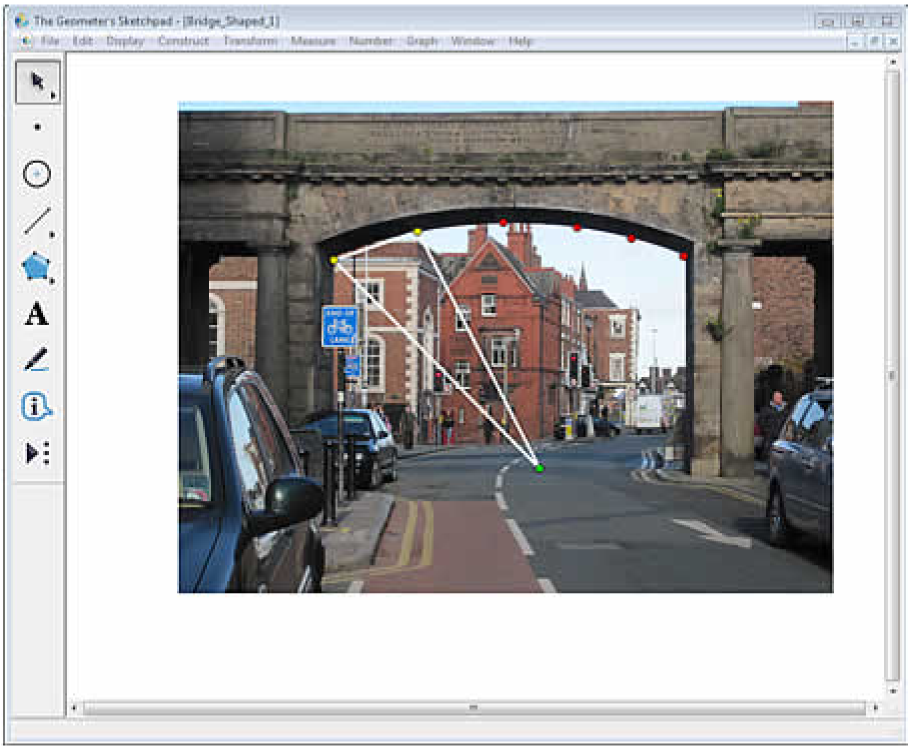
Importing the image into a dynamic geometry package will support you to reason mathematically about your conjecture....

A first instinct might be that the arch forms an arc of a circle...

One approach would be to place some points on the curve...



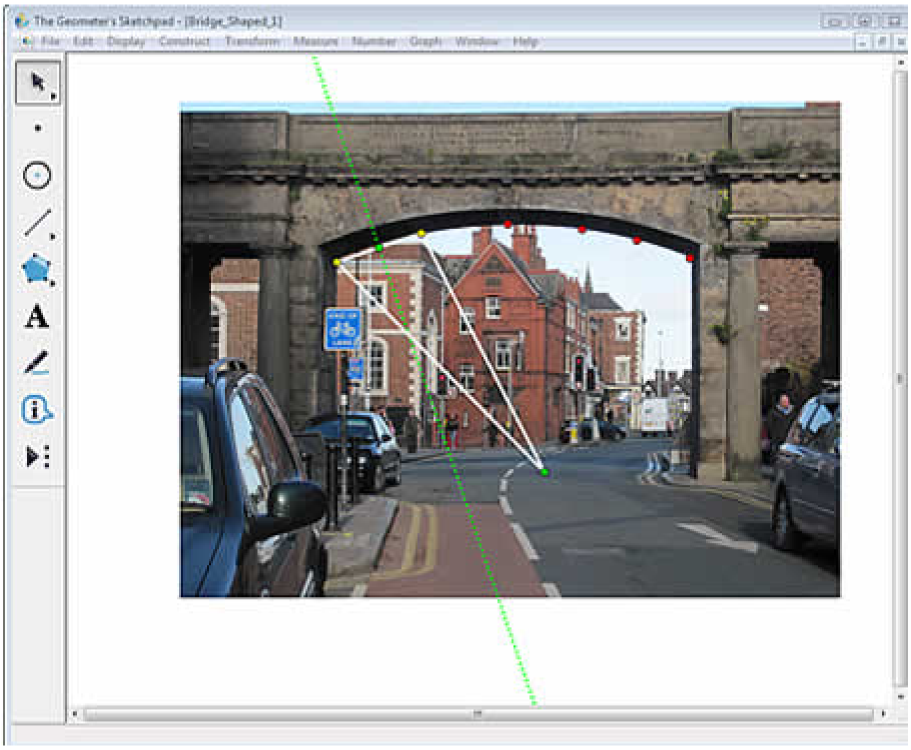
If the conjecture is true, then any of the red points, when joined to the 'centre' of the circle will form line segments of equal lengths.



So if we pick out a triangle formed by the two yellow points and 'guess' the centre of the circle, we can make a triangle as shown above...

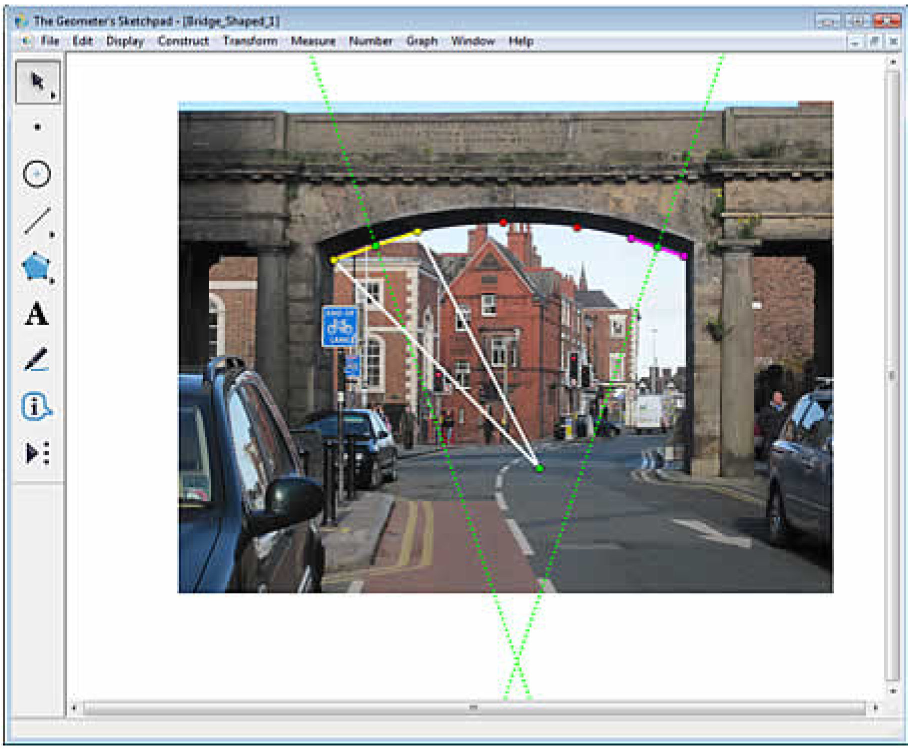
But surely, if the arch is an arc of a circle, the white triangle should be isosceles!

Using the software we can construct a perpendicular bisector and hopefully show where the centre of the circle should be!



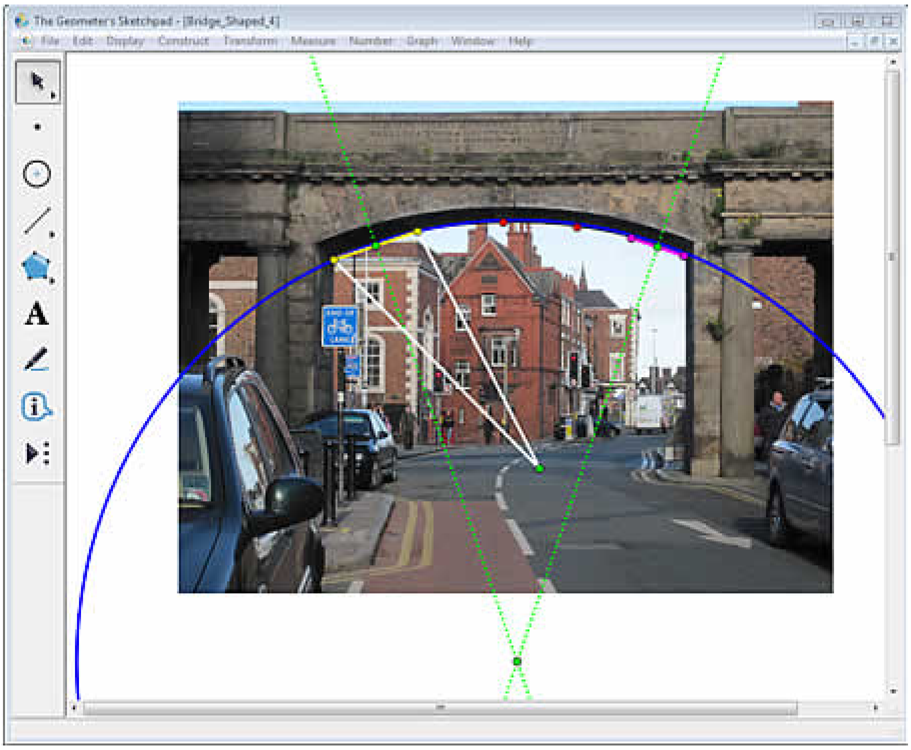
But how do we know where the centre of the circle should be positioned along the green dotted line?

Picking another couple of red points and repeating the construction might help!



So it is beginning to look convincing – but is a third point really needed?

Finally, the software itself will allow you to satisfy yourself that the conjecture is true...



This was just one conjecture and one possible approach...
What alternative approaches could you take?

And how did the lack of actual measurements support the geometrical reasoning?

What about different images with underlying geometric features?

Benoît Mandelbrot's thoughts about mathematics

> *There is a supply of unsolved, elementary problems that give students the opportunity to learn how mathematics can be done by enabling them to do new (if not necessarily earth-shaking) mathematics; there is a continuing flow of new results in unexpected directions.*

From Chapter One of *Fractals, Graphics, and Mathematics Education* by M.L. Frame and B.B. Mandelbrot

In this book, Benoît Mandelbrot and Michael Frame remind readers of why students came away from fractal geometry courses 'feeling they had understood some little bit of how the world works' – they were given opportunities to learn to do certain things themselves, such as grow fractal trees and synthesise their own fractal mountains. Mathematics had been revealed to them as 'an enterprise as full of guesses, mistakes, and luck as any other creative activity', with amazing surprises waiting around almost every corner.

When students experience mathematics as a closed, finished subject they are put off. Mandelbrot saw that students were engaged by the 'unfinished' appeal of fractal geometry – they understood problems in it that are still unsolved today. He believed, as do other mathematics educators, that seeing mathematics as a 'lively, growing field' to which it is possible that they can contribute, motivates students. Few things bring home to students the accessibility of mathematics so much as seeing and understanding something new in mathematics that has been done by another student. And it's very exciting to be able to show other students what you have discovered. Benoît Mandelbrot understood how vital these observations about 'ownership' are to any thinking about the teaching of mathematics.

He believed that students eagerly explore fractal geometry because it keeps alive, or re-awakens, natural, youthful curiosity. He knew that to teach mathematics so that students are curious and eager to contribute demands faith in students' capabilities. While acknowledging that students cannot learn all mathematics by 'reconstructing it from the ground up', Mandelbrot and Frame observed:

> "Generally, giving a student an open-ended project and the responsibility for formulating at least some of the questions, and being interested in what the student has to say about these questions, is a wonderful way to extract hard work."

Benoît Mandelbrot also felt that the fascinating visual appeal of fractal structures motivated students – living in a world in which much communication is visual (television, video and the internet), the more explicitly visual are the ways in which we present and communicate mathematical ideas the more readily students may engage with them. In an interview in 1984, Mandelbrot said:

"Before, people would run a mile from my papers, but they could not run from my pictures. In the beginning, I used the graphics purely for this reason - to illustrate my ideas and to force people to accept them. But I soon realized that this method enabled me to go further and integrate into a single theory a collection of things that otherwise would have seemed unrelated. Now, very complex geometric shapes could be compared with one another and with reality. The equations behind the shapes were abstract, but the shapes themselves looked alive."

In Chapter Three of *Fractals, Graphics, and Mathematics Education*, Mandelbrot observed 'that fractals – together with chaos, easy graphics, and the computer – enchant many young people and make them excited about learning mathematics and physics. In part, this is because an element of instant gratification happens to be strongly present in this piece of mathematics called fractal geometry. The belief is that this excitement can help make these subjects easier to teach to teenagers. This is true even of those students who do not feel they will need mathematics and physics in their professions."

Benoît Mandelbrot goes on to express his strongly-felt belief (and hope) that the unusual interest shown by the general public in fractal geometry might result in more people than do presently, regarding a good understanding of some mathematics as an essential outcome of everybody's education. He comments that "vital decisions about science and technology policy are all too often taken either by people so closely concerned that they have strong vested interests, or by people who went through the schools with no math or science."

He explains why he does not believe that the need for everyone to be better educated mathematically ought to be justified by purely utilitarian considerations – because we live in an increasingly technological society. "The lesson for the educator is obvious. Motivate the students by that which is fascinating, and hope that the resulting enthusiasm will create sufficient momentum to move them through material that must be studied but is less widely viewed as fun."

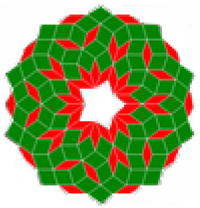
Mathematics educators need to know, Benoît Mandelbrot fervently believed, how research mathematicians view their craft, and how that view has continually changed throughout history, particularly during the last century. In Chapter Four, *Mathematics and Society in the 20th Century*, Mandelbrot compares the 'conservative' view of mathematics – as up on a high hill, understood only by the few professional mathematicians, 'looking down on' ordinary people who do not aspire to try to comprehend it – with his own view in which mathematics is attractive to people who are not professional mathematicians, including school mathematics teachers and students.

The use of computers had a profound effect on the life and mind of Benoît Mandelbrot. In another 1984 interview he mentioned implications of computers for mathematics teaching:

> "We have entered a period of intense change in the mood of mathematics. Increasingly many research mathematicians use computer graphics to enhance their geometric intuition, others cease to hide (from outsiders, or even from themselves) the fact that they had been practicing geometry. This return of geometry to the frontiers of mathematics and of physics should have an effect on the teaching of geometry in colleges, high schools, and even in elementary schools, because so much geometry which had been quite impractical can now be easily done with the help of computers."

Perhaps students can be inspired by Benoît Mandelbrot's reflections on the beginning of his own lifelong learning journey:

> "During the last term at Polytechnique, I looked for ways to apply my gift for shape, and a growing knowledge of various fields, to real, concrete, and complex problems. I wanted to keep far from organized physics and mathematics and instead find a degree of order in some area – significant or not – where everyone else saw a lawless mess."



5 things to do this fortnight

- Until 15 December all young people born between the years 1992–1995 can apply to take part in the 2011 International Millennium Youth Camp. The targets are to find young people interested in mathematics, science and technology, and help them start up a career in these fields, raise awareness of the educational and working opportunities available in Finland, and promote the Millennium Technology Prize. The main themes of the 2011 camp are climate change, renewable natural resources and energy, water, ICT and applied mathematics.

- Have a look at the Geometry Garret. This very extensive, informative and beautiful single page from Alan Schoen is a 'pot-pourri' of people, places, Penrose patterns, polyhedra, polyominoes, posters, posies, and puzzles. Benoît Mandelbrot appears in a photograph at the top of the page, which was taken in 2006 just after he delivered the AMS Einstein Public Lecture in Mathematics.

- A special Critical Mathematics Education issue of the Philosophy of Mathematics Education Journal was published during October. The many varied papers include Teaching for Equity, Teaching for Mathematical Engagement by Professor Hilary Povey, who argues that failing to address equity issues in the teaching of mathematics in secondary schools is interconnected with the reasons given by young people for their alienation from mathematics itself. And if you're in Sheffield on 30 November, Professor Povey will be giving a lecture about this at 6.30pm - more details on the Sheffield Hallam University website.

- Have you seen the Bowland Maths Assessment Tasks? Each task includes a task worksheet, teachers' guide and an optional presentation for projection. Level suitabilities are provided to give you an idea of the pupils for whom the task is suitable, but they are not precise limits on the level of possible outcomes from the task. When used in the type of classroom activities described in the Professional Development, the tasks are suitable for a wide range of ability.

- Wednesday, 1 December, is the birth date of Nikolai Ivanovich Lobachevsky, who was born in 1792, and lived to develop non-Euclidean geometry independently of and at about the same time as János Bolyai. Students might like to interact with a spherical geometry demonstration applet.

Image Credits

Page header - Penrose tiles in a star- image by Tomruen in the public domain.

Subject Leadership Diary

What joy! What bliss! Half-term is here and the weather is good, the foliage is that brilliant array of oranges and browns and it is a delight to walk in the cool crisp air. During the week before half-term we have had the faculty few down with colds, as our batteries get a bit low and are in need of recharging. However, it has not stopped us moving on with our ‘financial well-being’ week. I have had the pleasure and privilege of addressing a number of teachers about this important aspect of Every Child Matters - students achieving economic well-being, which is part of the Big Picture. We try as hard as possible to keep this in our sights whenever we revise our schemes of work. A couple of years ago we built into all our schemes a week during which we focus on financial awareness. This time, Year 7 students have concentrated on mobile phones and the costs involved in the contracts they use. It was rather frightening to discover that a lot of them are unaware of how much their phones are costing their parents because they do not have to pay the bills themselves. Hopefully, they will now have more idea, and therefore think more carefully about how they use them.

We looked at one of the National Centre's Teacher Enquiry Funded Projects, SMART MONEY - using personal finance education to promote engagement with mathematics, which was led by Margaret Ballantyne. From the final report, we learned that: “The project began in April and ended in December 2009. The research was funded by the NCETM and was supported by the personal finance education group (pfeg). The research centred on able but less motivated/engaged students in year 10 recruited from both of the secondary schools in Bognor Regis – Felpham Community College and Bognor Regis Community College. Intervention took the form of a series of after school workshops, which continued into the autumn term of year 11. For each workshop, existing materials were drawn together and new resources were developed on personal finance issues which would directly impact on the students in the future.”

While this gave us some excellent advice, we did not want to do the work after school, but to incorporate it into the normal school day. We debated the usual questions about how we can spend a week on personal finance education while still having enough time for everything else that we normally do. The answer is that we use the financial work to ‘cover’ some of what is usually done. For example, working with money (four rules with money) does not need to be addressed in Year 7 as a special topic in addition to the work on mobile phones – there are plenty of opportunities within the ‘mobile phones’ work to practise money calculations. Also, students find it more motivating to work with real data that they themselves have produced. Noting the number of calls and their durations, and texts, that they make or send at various times of the day, provides them with their own data. Discovering, as a result of their own reasoning and calculating, the total cost over a month, and also annually, gives some of them quite a shock!

Year 8 students plan and cost a holiday. We trawl travel agents to collect the necessary up-to-date literature, and then students consider planning a holiday for their family, given a budget. This has to include travel to the airport or ferry (if they decide to go abroad), so not only do they have to cope with finance, but also with timetables.

Students in Year 9 explore savings and bank accounts. We use resources on the NatWest MoneySense For Schools website to help here – recreating something like this ourselves from scratch would involve too much time. Year 10 students deal with wages, tax and national insurance, giving them some idea of how little money they might be left with after all the deductions have been taken and they have had to pay for rent, food and utility bills! Not quite what they expected.

What about Year 11? Well, they have worked through some of our 'finance' activities in previous years, and with mock GCSEs looming in December we decided that discretion was needed with them, so we currently do not have anything specifically financial in place for them.

Well, time to get out and about before the next half-term starts – the apples from the garden have been collected and stored (and we've eaten quite a lot of them already) and the days are shorter. With the clocks going back there's an extra hour of this half-term break – yippee!