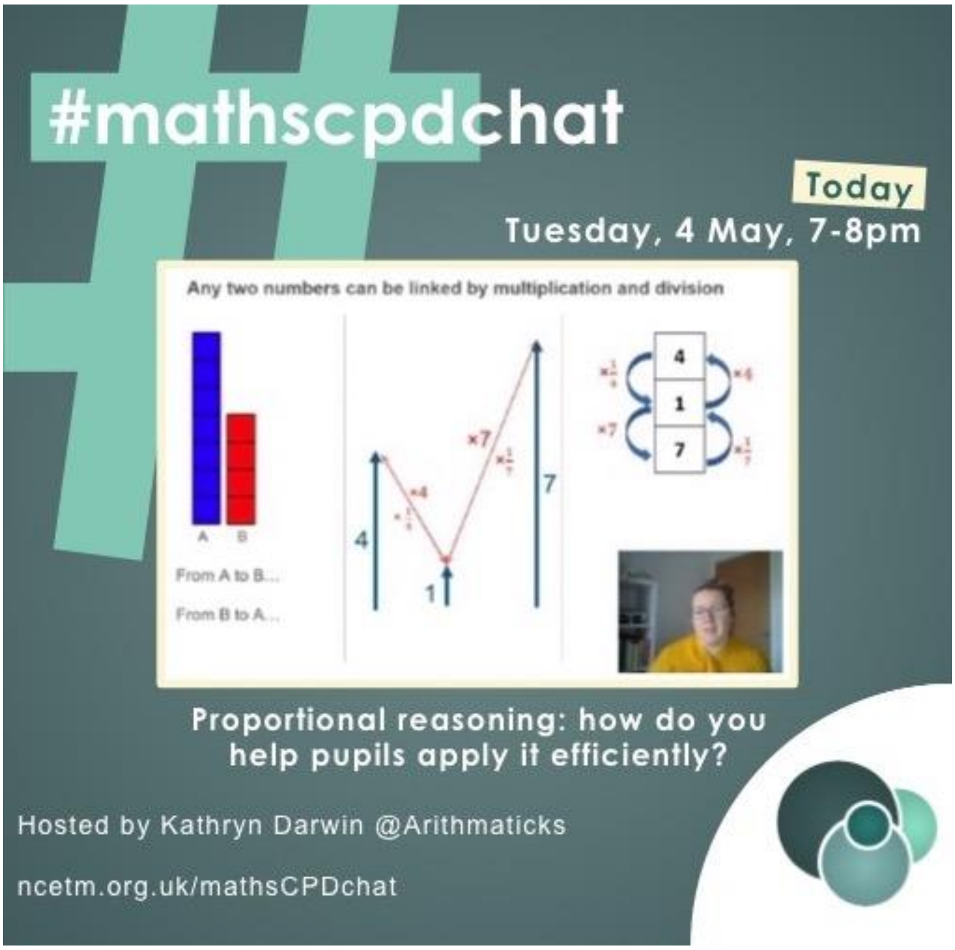
#mathscpdchat 4 May 2021

Proportional reasoning: how do you help pupils apply it efficiently?
Hosted by [Kathryn Darwin](#):

*This is a brief summary of the discussion – to see all the tweets, follow the hashtag **#mathscpdchat** in Twitter*



Among the links shared during the discussion were:

[Mastery Professional Development: 3.1 Understanding multiplicative relationships](#) which is an NCETM guidance document about teaching students in Key Stage 3 to understand and use multiplicative relationships. It was shared by [Kathryn Darwin](#)

[Secondary school-children's understanding of ratio and proportion] which is a report by Kath Hart of research on key concepts of ratio and proportion that appear in the secondary mathematics curriculum It was shared by Dawn Denyer

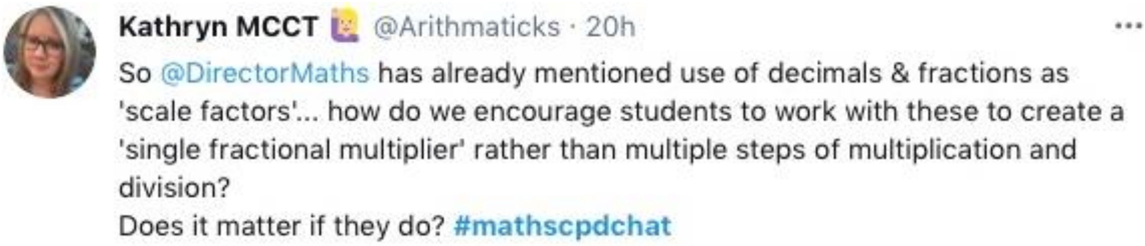
ICCAMS Maths: Publications which is a website page providing the links to all ICCAMS (Increasing Competence and Confidence in Algebra and Multiplicative Structures) papers that are currently available for free download. It was shared by Mary Pardoe

Understanding properties of division with function machines which is a blog by Ashton Coward. He discusses and provides freely-downloadable sets of tasks that he has designed with the aim of attempting to deepen students' understanding of relationships between combinations of division and multiplication operations, including general facts such as that A ÷ B ÷ C is equivalent to A ÷ (B × C). It was shared by Ashton Coward

You've never seen the GCSE Maths curriculum like this before... which is a *Great Maths Teaching Ideas* article by William Emeny. It focusses on a network diagram with 164 colour-coded nodes in which each node is intended to represent a topic in the GCSE Mathematics curriculum. It was shared by Ellejay

The screenshots below, of chains of tweets posted during the chat, show parts of conversations about developing an understanding of, and the abiity to use, fractional multipliers when quantities are linked by a constant of proportionality. It includes tweets about enabling/encouraging this understanding to develop alongside understanding of why some situations require additive reasoning and others require multiplicative reasoning. **Click on any of these screenshots-of-a-tweet to go to that actual tweet on Twitter.**

The conversations were generated by this tweet from Kathryn Darwin:

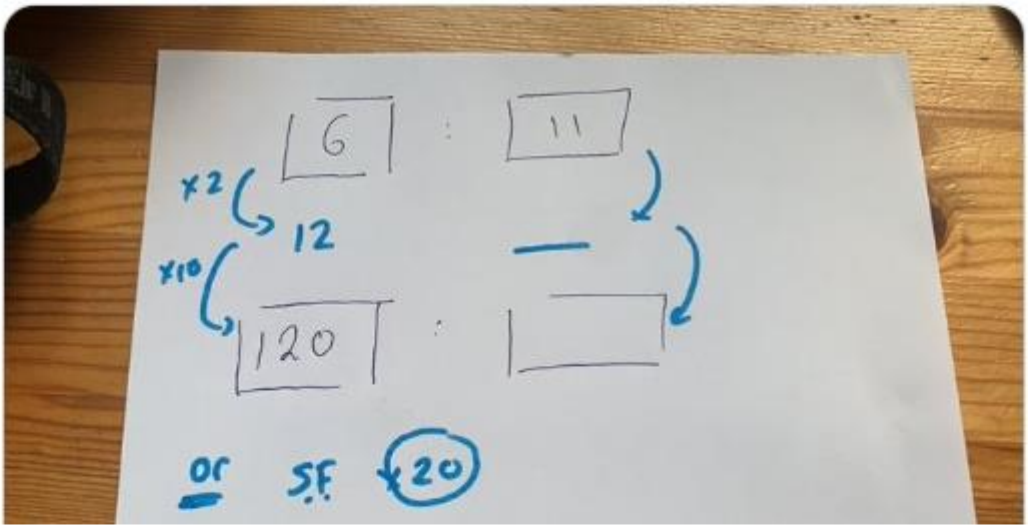


and included these from Mr Neasham, Kathryn Darwin and Professor Smudge:

MrN @neasham_a · 19h

Replying to @Arithmaticks and @DirectorMaths

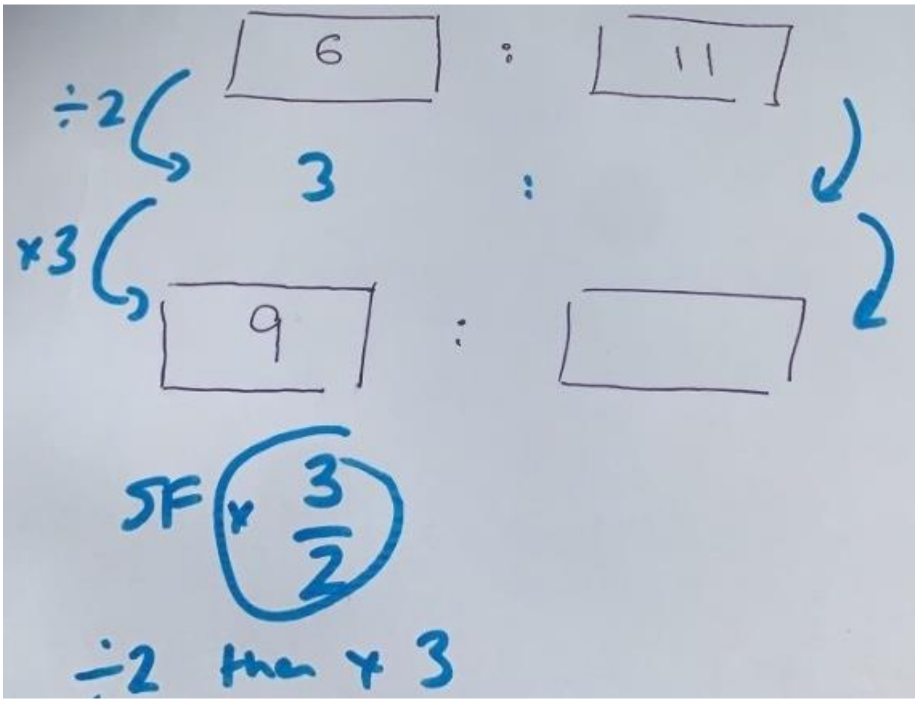
I go for "stepping stone ideas ". For most multiplicative reasoning questions. Good for factors and stuff. Something like. And then show it could have been just scale factor 20.



MrN @neasham_a · 19h

Replying to @Arithmaticks and @DirectorMaths

2. Then generalising stuff like this. Divide by 2 x by 3 and referring to commutative.



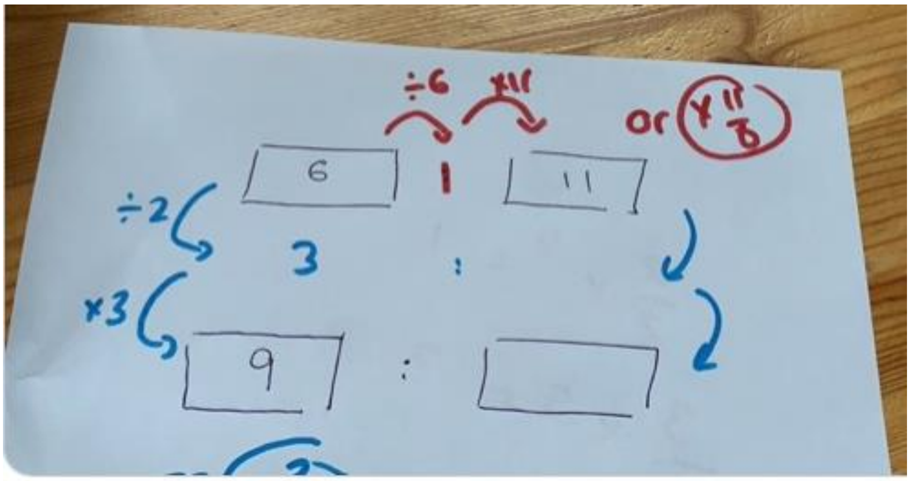
Kathryn MCCT 🙋 @Arithmaticks · 19h

Out of curiosity.... how would you show going horizontally not vertically? #mathscpdchat

MrN @neasham_a · May 4

Replying to @Arithmaticks and @DirectorMaths

Same way. Tend to use vertically a lot tho. Obvs talk about factors. Eg 1 could always be used as stepping stone. But best to use HCF. More efficient and students recognise easier (less work)



Professor Smudge @ProfSmudge · May 4

Replying to @Arithmaticks @neasham_a and @DirectorMaths

- also depends on the context (when there is one); more 'natural' to work within measure spaces than across

these from Gemma Scott, Kathryn Darwin, Charlotte Hawthorne, Ashton Coward and Mary Pardoe:

Director of Maths @DirectorMaths · 19h

Replying to @Arithmaticks

I'd say start with what they instinctively know/ do, if that's multiple steps then ok but use questioning and modelling to show how we can get there more efficiently #mathcpdchat

Kathryn MCCT 🙋 @Arithmaticks · 19h

What would that modelling look like? #mathscpdchat

Charlotte ✏️📐🧮🤓 @mrshawthorne7 · 19h

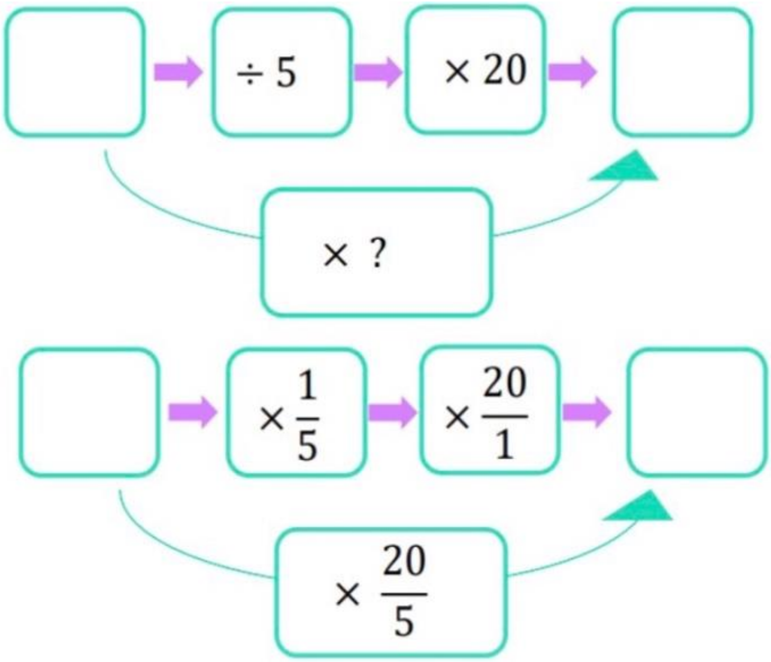
I think modelling division as multiplication of the the reciprocal is so important here. #mathscpdchat Makes it obvious IMO

Kathryn MCCT 🙋 @Arithmaticks · 19h

Absolutley made this easier with Year 8 this year! #mathscpdchat

Charlotte ✏️📐🧮🤓 @mrshawthorne7 · 19h

Things like this maybe? #mathscpdchat



Director of Maths @DirectorMaths · 19h

Love this! 😍 #mathscpdchat

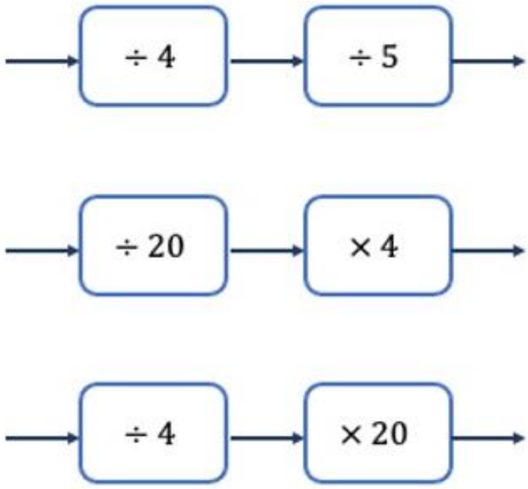
Charlotte ✏️📐🧮🤓 @mrshawthorne7 · 19h

Thanks. Made this quick after a discussion with @ashtonC94 think he has a task around these ideas?

Ashton Coward @ashtonC94 · 19h

I certainly do :D

Could you write these as one multiplication or division?



Mary Pardoe @PardoeMary · May 4

Replying to @Arithmaticks @neasham_a and @DirectorMaths

This document is very helpful (another free ICCAMS download): iccams-maths.org/wp-content/upl..

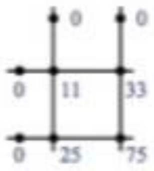
e.g. ... #mathscpdchat

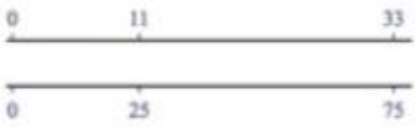
The use of the DNL to solve or analyse ratio tasks is not as straightforward as many curriculum materials seem to suggest. It is often possible to create *two* DNLs for a given task, and they can represent the situation in subtly different ways, or in ways that are hard to interpret.

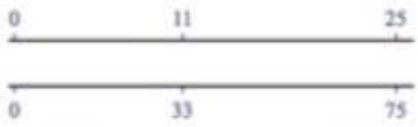
Imagine we have a table of numbers (right) where there is a ratio relation between the rows, ie 11/25 = 33/75 (and hence between the columns, ie 11/33=25/75). We can extend the rows with other numbers fitting the 11/25 relation, and we can extend the columns with other numbers fitting the 11/33 relation, eg like this (near right). And we can express this in a more general and coherent way using a horizontal DNL and a vertical DNL (far right). [The DNLs are drawn again (below), in the usual format.]

11	33
25	75

	1	3					
5.5	11	33	22	66	1.1	67.1	0.11
12.5	25	75	50	150	2.5	152.5	0.25
	33	99					
	8	24					







these from [Tom Francome](#) and [Kathryn Darwin](#):

Tom Francome @TFrancome · May 4
Replying to @Arithmaticks @neasham_a and @DirectorMaths
What do you multiply 6 by to get 11?
#mathscpdchat

Kathryn MCCT 🙋 @Arithmaticks · May 4
"You can't, Sir!" #mathscpdchat

Tom Francome @TFrancome · May 4
"Oh?!?!

...

I think you can?"

Let's see if anyone can do it!
#mathscpdchat

Tom Francome @TFrancome · May 4
Can anyone find two ways?
A way using two operations?
Three ways with three operations?
...
#mathscpdchat

and these from [Tom Francome](), [Kathryn Darwin]() and [Professor Smudge]():

Tom Francome @TFrancome · May 4
As is often the case, I defer to @ProfSmudge #mathscpdchat

Kathryn MCCT 🙋 @Arithmaticks · May 4
The ICCAMS hero we all need #mathscpdchat

Professor Smudge @ProfSmudge · May 4
In ICCAMS our focus was on teasing out students' thinking and developing their understanding of proportion; am I right in thinking your interest seems to be on more developing efficient procedures?

Kathryn MCCT 🙋 @Arithmaticks · May 4
Not at all... just considering it as an option! If we have a means to consider a single fractional multiplier, why not? But equally multiple steps do the same thing.

Professor Smudge @ProfSmudge · 10h
Developing an understanding and use of fractional multipliers is important, but I think I would try to let this happen hand in hand with developing an understanding of why some some situations are additive and some are not.

(to read the discussion-sequence generated by any tweet look at the 'replies' to that tweet)

Some of the other areas where discussion focussed were:

the host asked what multiplicative reasoning is, and how it develops as the maths learning of pupils progresses:

- some teachers' tweets implied that they (those teachers) regard a student as being **capable of multiplicative reasoning if the student demonstrates that they understand the effect of multiplying any number by a fraction or a decimal between 0 and 1** … some people commented that it is important to know how students build on their 'by heart' knowledge of multiples and factors of whole numbers to gain that understanding;
- some teachers prefer 'to **have a whole unit for proportional reasoning by itself**' … others 'want to spend most of KS3 doing multiplicative relationships in different guises' … there was a comment that proportional reasoning is **'key to so many topics at GCSE'**;
- at least two teachers are planning to 'create some resources around scaling up and down using ratio tables' including tasks in which students '**explore different problems using the same reasoning**';

what students need to know before they can begin to think multiplicatively:

- at least one teacher is 'keen to **use unitary methods initially** and build up around that' … for example, using recipe proportion problems;
- several teachers mentioned that they want students to have '**a good understanding of what "lots of" means**' … they want students to see multiplication as repeated addition;
- there was a brief discussion about the 'layout' of students' written responses to tasks … teachers want students to **be able to 'structure their answers so that it follows logical reasoning'**;

how multiplicative reasoning differs from additive thinking:

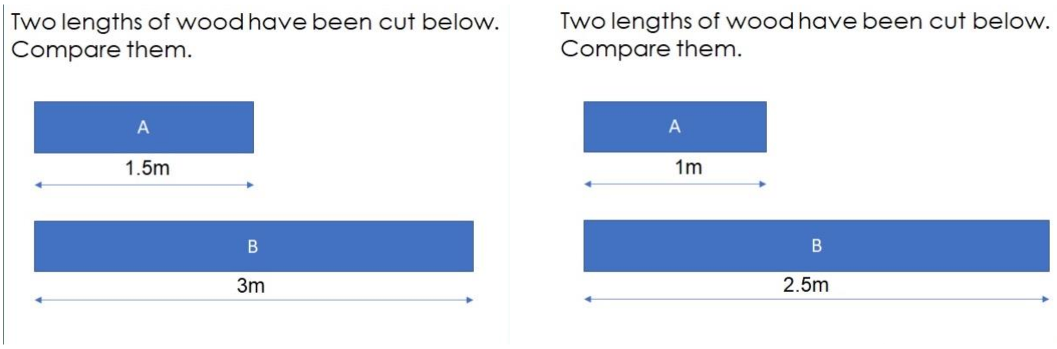
- past research has indicated that many/most students reason multiplicatively 'with much less ease', and more reluctantly, than they do additively … for example, when they are trying to find an unknown length in a similarity/enlargement situation many students 'are **not sure if you have to add something or times it by something**' … 'how do students spot when to scale and when to add';

- teachers briefly discussed how focussing on the difference (with respect to meaning or lack-of-meaning) between expressions such as ‘2cm + 2’ and ‘2cm × 2’ can help to move students towards **knowing whether it will be appropriate to reason multiplicatively or additively in any particular given situation** … ‘it comes down to units, 3m + 3m = 6m, 3m × 2 = 6m’;

stumbling blocks in multiplicative reasoning, and why they arise:

- a teacher remarked that ‘when topics are taught in isolation students don’t see the links between them’ … some teachers have **a learning unit called ‘multiplicative reasoning’** in which multiplicative reasoning is applied to situations that may be explored by drawing on the mathematics of different maths-curriculum ‘topics’;

the host invited contributors to say what they would learn about students’ ways of thinking by observing the students’ responses to the following two tasks:



- teachers thought that ‘they would spot the left-hand one being twice as long’, but that **they would struggle to express the relationship between the lengths in the right-hand one multiplicatively**;
- they also thought that **in both cases students would be likely to try to express the longer length in terms of the shorter length**, rather than the other way round;

although most of the discussion about encouraging students to express ‘scale factors’ as ‘single fractional multipliers’ rather than as ‘multiple steps of multiplication and division’, is shown in the sequences of screenshots above, there was one other response to that question:

- expressing an operation that is initially seen as ‘divide by n’ as ‘multiply by n’ is harder when n is given or seen as a decimal or percentage, rather than as a whole number or a fraction;

how teachers encourage students to move away from additive-in-all-cases thinking to multiplicative-sometimes thinking … the host tweeted these example tasks from Don Steward:

use *two* steps to get from the given number to 12

for both of the steps you can only use digits
you can × or ÷ (and you can repeat an operation)

8 → → 12	180 → → 12
18 → → 12	192 → → 12
27 → → 12	216 → → 12
28 → → 12	480 → → 12
30 → → 12	672 → → 12
300 → → 12	972 → → 12

use *two* steps to get from the given number to 100

for both of the steps you can only use digits
you can × or ÷ (and you can repeat an operation)

150 → → 100	66⅔ → → 100
350 → → 100	5 → → 100
125 → → 100	37½ → → 100
4200 → → 100	6¼ → → 100
60 → → 100	62½ → → 100
75 → → 100	2½ → → 100

- a contributor commented that ‘some students will “see” what they can add or subtract rather than what they can multiply or divide’ … another **contributor asked whether ‘they are using rated addition [as in 4/6 = (4 + 2)/(6 + 3)]’… if so they are making good sense … or are they ‘blindly’ thinking additively [as in 4/6 = (4 + 2)/(6 + 2)]** … the first contributor thought that they would be thinking additively, to which the second contributor responded by raising the further question as to **whether these students are ‘not “looking” to multiply, or not making much sense of fractions’** … a further comment was that the students may ‘have limited understanding of fractions because they’ve just been drilled to high heaven w/out understanding’;

the host asked what mathematical (or not-specifically-mathematical) understanding/ability is facilitated by being able ‘to do proportional reasoning’ competently (by being competent with thinking multiplicatively):

- in response to this question contributors mentioned **the following ‘topics’** … percentages, scale factors, direct and inverse proportion, equivalent fractions, standard form, currency conversions, unit conversions, trigonometry, compound measures, rationalising the denominator, similar figures, gradients, pie charts, pictograms … magnification in A level Biology, heamocytometer calculations, genetic cross outcomes, population estimates using Lincoln index, rate of enzyme reactions;
- a contributor asked ‘how does one “**do** proportional reasoning”?’
- that ‘you can do away with **formula triangles**!’;

introducing inversely proportional relationships:

- **in Y10, ‘once pupils are already secure in thinking multiplicatively’** … ‘always as “the product is constant”’ … ‘xy = k is my default here, not y = k/x’ … “Which value stays constant?”;

- some teachers introduce inverse proportion in the **context of speed/time/distance** situations and tasks (for example, ‘What happens to the time if my speed increases?’) … other teachers’ ‘go-to’ context is number-of-builders and time-to-complete-a-building-job;
- teachers commented that it is important to **avoid generating confusion between the idea of inverse proportion and the idea of negative gradient**.