Fractions

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

Question 8

How confident are you that you understand and can support children to add and subtract fractions?

1 ☑ 2 ☐ 3 ☐ 4 ☐

How would you respond …?

a. Look at the examples. Are they true or false?

$$\frac{2}{9}+\frac{6}{9}=\frac{8}{18}$$

$$\frac{1}{11}+\frac{3}{11}+\frac{5}{11}=\frac{9}{11}$$

$$\frac{3}{8}+\frac{5}{8}=1$$

b. What is the largest possible numerator that can be used to complete this comparison statement?

$$\frac{11}{20}+\frac{\square}{20}<\frac{17}{20}$$

c. Can you solve this word problem?

A beetle was $\frac{8}{10}$m away from its hole.
It walked $\frac{2}{10}$m towards the hole.

How much further is it to the beetle's hole?

Responses

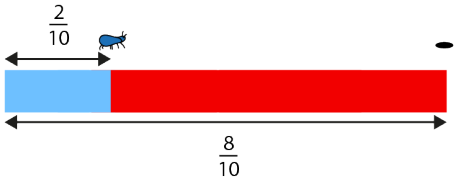
Note your responses to the questions here before you engage with the rest of this section:

Did you notice that…?

a. The first example is false. In this case, the fractional amounts have been operated on as though they were numbers and added together. Here we have two one-ninths and six one-ninths so the total should have been eight one-ninths.
The second example is correct. It demonstrates that more than two fractional amounts can be added.
The third example is correct. Three one-eighths and five one-eighths equal eight one-eighths, which is the same as one whole.

b. Here, we need to work out how many one-twentieths need to be added to eleven one-twentieths to make a total that is less than seventeen one-twentieths.
To make an amount less that seventeen one-twentieths, I could add $\frac{1}{20}$ to make $\frac{12}{20}$, $\frac{2}{20}$ to make $\frac{13}{20}$, $\frac{3}{20}$ to make $\frac{14}{20}$, $\frac{4}{20}$ to make $\frac{15}{20}$, $\frac{5}{20}$ to make $\frac{16}{20}$. The largest of these is $\frac{5}{20}$ so a five should be recorded as the numerator.

c. The beetle is now $\frac{6}{10}$ m away from its hole. This image shows one way of representing it.



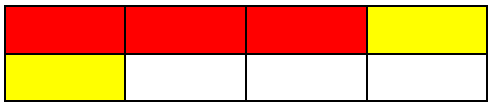
Adding and subtracting fractions

In order for children to understand how to add and subtract fractions, they will need to have a good understanding of non-unit fractions as the repeated addition of unit fractions. A deep conceptual understanding that, for example, five one-eighths make five-eighths, and that this is also the sum of three one-eighths and two one-eighths, will help to minimise the chance of children mistakenly adding or subtracting the denominator.

A common error that children make when adding and subtracting fractions is to look at the parts of the fractions as separate numbers and calculate with those numbers. By emphasising the composition of non-unit fractions, this risk should be reduced. If children are not secure with the composition of fractions or are only given the calculation without the representations, they may revert to applying their understanding of number, rather than using their understanding of fractions.

For example, by stating that $\frac{3}{8} + \frac{2}{8}$ equals $\frac{5}{16}$.

A representation can be used alongside the calculation to help quickly address these misconceptions.



Here we can see that the addition of these two amounts cannot equal $\frac{5}{16}$.

Representations can be coupled with the careful use of language to draw attention to the unit of the fraction. In this case, three one eighths, add two one-eighths.

Counting in a given unit is an important experience for children and previous experiences where they have unitised the count will support them with making connections. For example:

- 3 cats + 2 cats = 5 cats
- £3 + £2 = £5
- 3 kg + 2 kg = 5 kg
- 3 million + 2 million = 5 million
- 3 one-eighths + 2 one-eighths = 5 one-eighths.

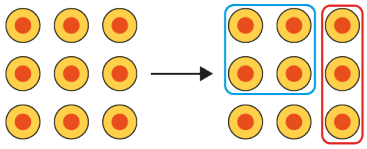
To support the ability to unitise (i.e. to see a non-unit fraction as composed of several unit fractions), teachers should use dual-naming of fractions. For example, *'two one-eighths add three one-eighths equals five one-eighths'* as well as *'two-eighths add three-eighths equals five-eighths.'* Children must be able to interchange fluently between these when adding and subtracting fractions. They should already be used to adding and

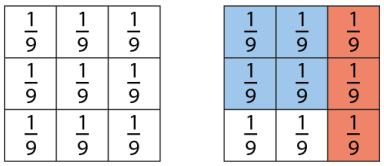
subtracting in all sorts of other units and this approach links addition and subtraction of fractions to their previous learning.

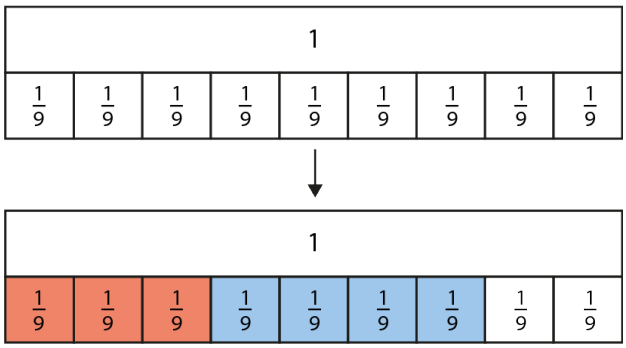
Children will also need to be exposed to different representations to support their understanding.

Dad baked a tray of biscuits. Olivia took $\frac{3}{9}$ of the biscuits and Dinesh took $\frac{4}{9}$ of the biscuits. What fraction of the biscuits did they take altogether?

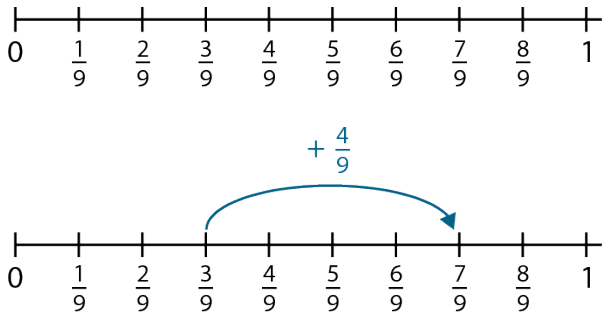
Method 1 – using a diagram







Method 2 – using a number line



Method 3 – verbal reasoning

$\frac{3}{9}$ is three lots of $\frac{1}{9}$

$\frac{4}{9}$ is four lots of $\frac{1}{9}$

I know that 3 + 4 = 7….

so, I know that $\frac{3}{9} + \frac{4}{9} = \frac{7}{9}$.

Equation: $\frac{3}{9} + \frac{4}{9} = \frac{7}{9}$

Initially, children will be given fractions with the same denominator to add or subtract. Using representations alongside this, children will be encouraged to calculate and draw out a generalisation, such as: '*When adding (or subtracting) fractions with the same denominators, just add (or subtract) the numerators.*'

Generalisations are extremely useful and adults often draw on them heavily. For instance, 'just knowing' to add only the numerators when adding fractions with the same denominator, saves us from having to stop and think from first principles every time.

Recognising mathematical relationships and forming generalisations about how those relationships hold true to relevant contexts is the essence of learning mathematics. However, it is really important that children don't just rote learn the generalisation, but also have an underlying understanding of 'why' that generalisation holds true. As children meet different types of fraction calculations (addition and subtraction of fractions with different denominators, and multiplication and division of fractions), they will learn further generalisations.

The examples illustrated are based on addition calculations; the same steps in learning would need to be considered when addressing subtraction calculations.

Children in Year 3 will be expected to have experiences of adding and subtracting fractions with the same denominator. In Year 4, they will develop their understanding of equivalent fractions and then in Year 5, they will begin to add and subtract fractions with denominators that are multiples of the same number. When working on calculations with different denominators, children will be taught to find the equivalent fractions first and convert so the denominators are in the same unit and then calculate accordingly.

Common errors in this area may include:

- children not applying their understanding of unitising when calculating with fractions, instead they see the fraction as numbers and apply this understanding, e.g.
- stating that $\frac{3}{8} + \frac{2}{8}$ equals $\frac{5}{16}$ because 3 + 2 = 5 so that goes on the top and 8+8=16 so that goes on the bottom.

What to look for

Can a child:

- use the language of three one-eighths add two one-eighths to emphasise the composition of non-unit fractions and use this to calculate with?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 3: Fractions:

- Topic 3.4: Adding and Subtracting Within One Whole

Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice: