MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development

Number, Addition and Subtraction

1.11 Addition and subtraction: bridging 10

Teacher guide | Year 2

Teaching point 1:

Addition of three addends can be described by an aggregation story with three parts.

Teaching point 2:

Addition of three addends can be described by an augmentation story with a '***first…, then…, then…, now…'*** structure.

Teaching point 3:

The order in which addends (parts) are added or grouped does not change the sum (associative and commutative laws).

Teaching point 4:

When we are adding three numbers, we choose the most efficient order in which to add them, including identifying two addends that make ten (combining).

Teaching point 5:

We can add two numbers which bridge the tens boundary by using a 'make ten' strategy.

Teaching point 6:

We can subtract across the tens boundary by subtracting *through* ten or subtracting *from* ten.

Overview of learning

In this segment children will:

- learn about the addition of three or more single-digit numbers in the context of both aggregation and augmentation (see segments *1.5 Additive structures: introduction to aggregation and partitioning* and *1.6 Additive structures: introduction to augmentation and reduction*)
- understand the importance of the laws of commutativity and associativity in the context of adding three or more numbers
- practise applying written and mental strategies for the addition of three or more addends, using partitioning, commutativity and associativity.

Children should already be familiar with aggregation and augmentation structures for the addition of two numbers. The teaching sequence begins by extending this to the addition of three or more single-digit numbers. Throughout, children should be encouraged to make connections between different representations, including real-world stories, tens frames, part–part–part–whole diagrams, number lines and symbolic notation (e.g. $3 + 1 + 4$). The learning sequence begins with three addends which sum to ten or less, before exploring bridging ten.

Once children have developed an understanding of the meaning of addition with more than two addends (parts), they can begin to explore the effect of changing the order of the addends in the sum. This depends on the laws of commutativity and associativity, but children do not need to use these terms at this point. Note that it is much more obvious that we can change the order of the addends in the context of aggregation, than in the context of augmentation.

Children then explore how to make efficient choices when deciding in what order to add the three addends, including identifying when two addends sum to ten. This is then combined with partitioning to give a useful strategy for adding two numbers which bridge the tens boundary; in order for children to use this strategy independently, they will need to be fluent at partitioning numbers up to ten in different ways.

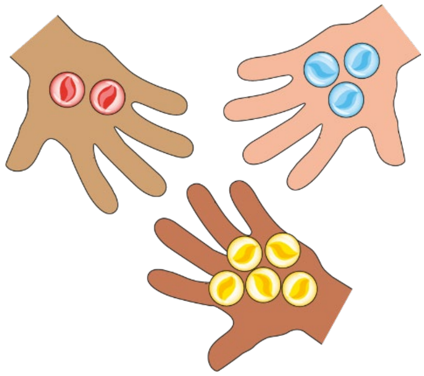
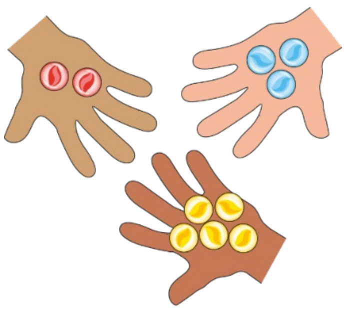
The final teaching point explores two different strategies for subtracting across ten: subtracting *through* ten and subtracting *from* ten. These strategies exemplify the idea that whatever part of a set we subtract from, the resulting difference will be the same. Children will discuss which subtraction method is the most efficient according to the particular numbers involved. This important principle can be applied to other calculations children will meet through the rest of the spine.

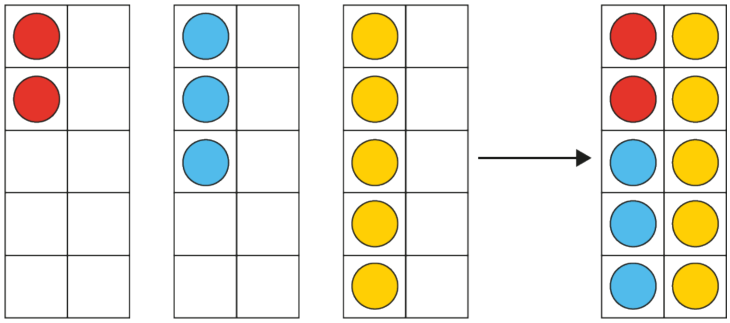
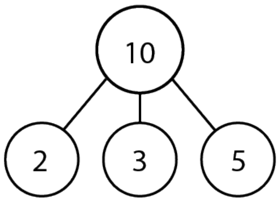
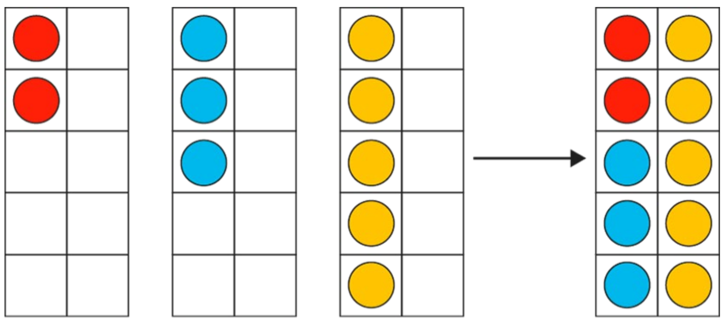
An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast*. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks*

Teaching point 1:

Addition of three addends can be described by an aggregation story with three parts.

Steps in learning

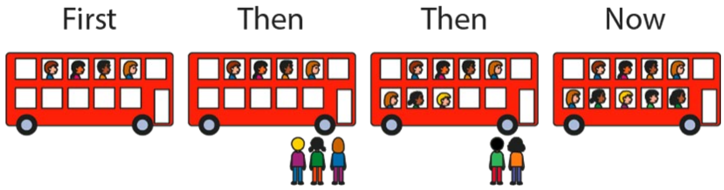
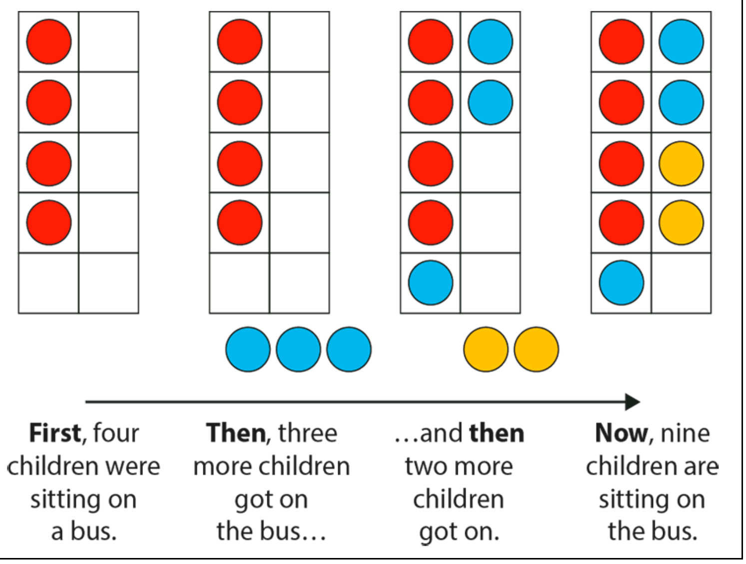
	Guidance	Representations
1:1	Build on previous work on aggregation (see segment *1.5 Additive structures: introduction to aggregation and partitioning*) by extending to include three addends. Initially consider addends that sum to ten or less. Begin using a practical context with an aggregation structure in three parts, for example, three groups of marbles, or rolling three dice. At this stage, for the dice representation, use pictures rather than actually rolling dice to avoid values that sum to greater than ten. Make sure you include examples where one or more of the addends is zero, for example: *'I have six yellow marbles, no blue marbles and three red marbles. How many marbles do I have altogether?'* Use a stem sentence with the structure: ***'There are ___, ___ and ___. Altogether there are ___.'***	*'Madison has two red marbles, Charlie has three blue marbles and Asif has five yellow marbles. They have ten marbles altogether.'*  Children can use real marbles as manipulatives. *'Tom rolls three dice. He rolls a 2, a 3 and a 4. What is Tom's total roll?'* 
1:2	Once children are confident with the use of practical and pictorial representations of three-part aggregation, begin to introduce generalised representations, including: • three tens frames and ten counters • the part–part–part–whole model (note that the cherry representation is used throughout this teacher guide, but you could also use the bar model). To help children make a connection between the concrete/practical and	*'Madison has two red marbles, Charlie has three blue marbles and Asif has five yellow marbles. They have ten marbles altogether.'* Concrete/pictorial: 

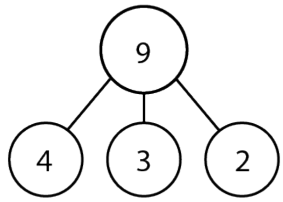
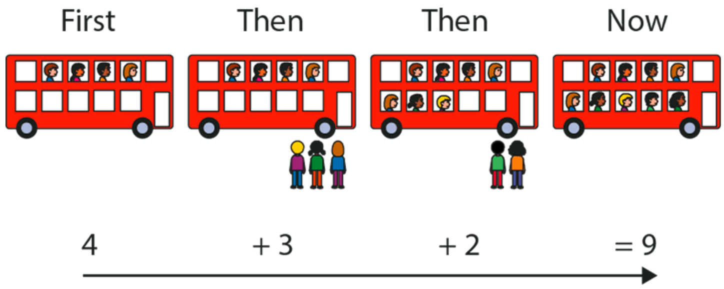
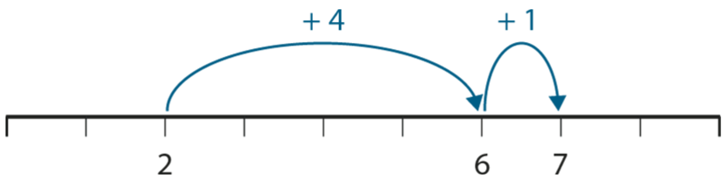
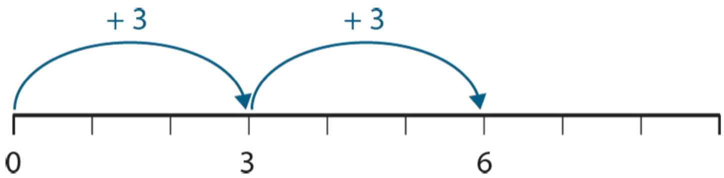
	the generalised representation of the tens frame, introduce the matching generalised representation alongside the practical or pictorial representation. When using the part–part–part–whole representation, make sure you explicitly connect each number with what it represents; ask the children to describe in full sentences, for example: • *'The 2 represents the two red marbles.'* • *'The 3 represents the three blue marbles.'* • *'The 5 represents the five yellow marbles.'* • *'The 10 represents the total number of marbles.'* Apply variation to strengthen the connections: • Present children with a part–part–part–whole representation and ask them to write an aggregation story to go with it. • Present children with a number sentence (e.g. *10 = 6 + 3 + 1*) and ask them to draw the part–part–part–whole model.	Tens frames:  Part–part–part–whole representation: 
1:3	Introduce abstract representations alongside the practical and pictorial to support children in making a connection between the representations. Ensure that children are confident expressing equations with the equals symbol in different positions, for example: • 10 = 2 + 3 + 5 • 2 + 3 + 5 = 10	 2 + 3 + 5 = 10 2 + 3 + 5 = 10

Teaching point 2:

Addition of three addends can be described by an augmentation story with a ***first…, then…, then…, now…'*** structure.

Steps in learning

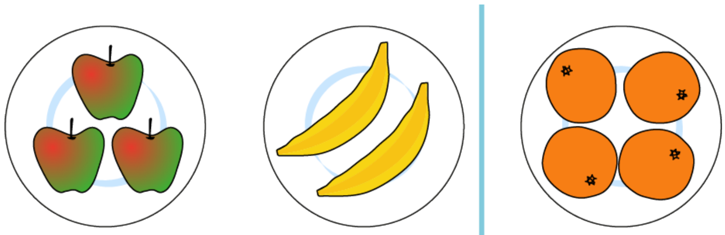
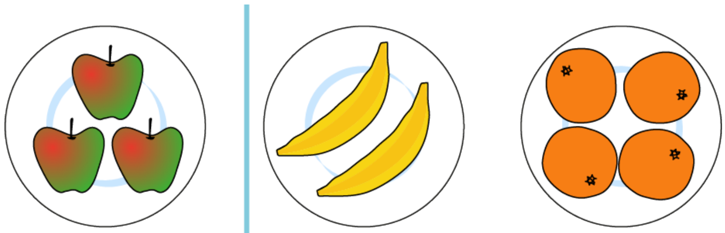
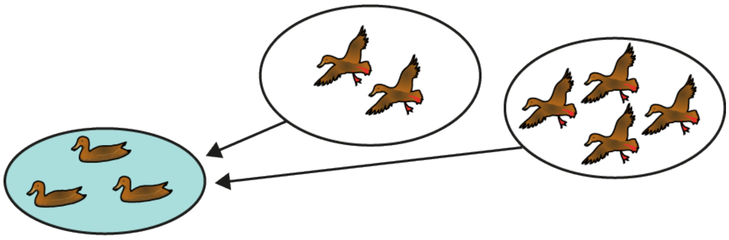
	Guidance	Representations
2:1	Build on previous work on augmentation (see segment *1.6 Additive structures: introduction to augmentation and reduction*), again by extending to include three addends that sum to ten or less. Again, you can begin with practical work, with children acting out a range of relevant augmentation stories. Use a stem sentence with the structure: ***'First…, then…, then…, now…'*** You can use a similar approach to segment *1.6* to move from a concrete to a pictorial representation.	Practical: ***First****, four children were sitting on the bus.* ***Then****, three more children got on the bus, and* ***then*** *two more children got on.* ***Now****, nine children are sitting on the bus.'* Chairs could be arranged to support acting out this story. Pictorial: 
2:2	Encourage children to connect practical representations with a generalised structure by recording using tens frames: as the story is told add the counters. Then introduce the further abstraction of the part–part–part–whole model (cherry or bar representation): as before, when using this model, explicitly ask children what each number represents.	Tens frames: 

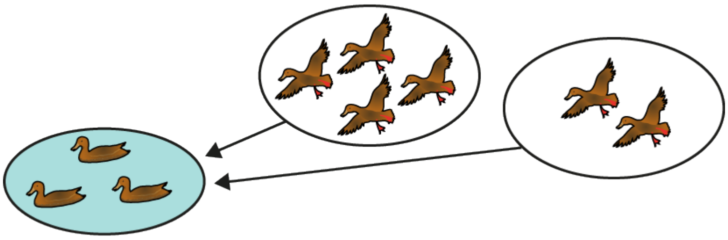
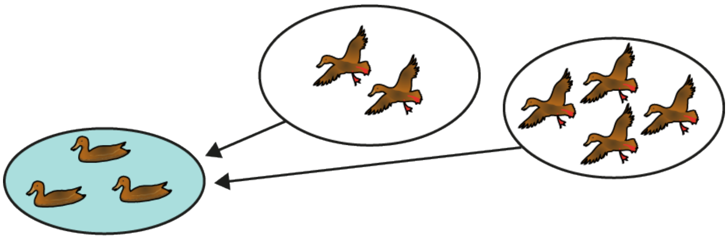
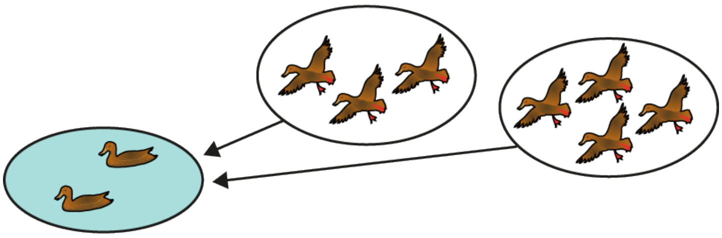
		Part–part–part–whole representation: 
2:3	Now ask the children how augmentation involving three numbers can be written as an equation. Ensure that children are able to link the pictorial to the abstract and say what each number represents. Note that, in an augmentation context, it does not make sense to place the equals symbol at the start of the equation since the resulting expression would no longer represent the ***'first…, then…, then…, now…'*** structure.	 $4 + 3 + 2 = 9$
2:4	It is also important to represent augmentation on a number line – clearly demonstrate the two increases in quantity. This representation will prepare children for bridging through ten, covered later in the segment (see *Teaching point 5* below).	*'A snail is two centimetres up a flower stalk. Then it crawls up four centimetres, and then one centimetre more. How high up the stalk is it now?'* 
2:5	To promote depth of understanding, present children with a number line depicting two increases, along with a multiple-choice selection of equations; ask children to select the correct equation and justify their choice. To increase the challenge ask children to write the equation from scratch, or provide an equation and ask children to draw the corresponding number line. Throughout this teaching point, apply variation and look at the same context using different representations.	 *'Which equation matches this number line?'* $3 + 3 = 6$ $0 + 3 + 3 = 6$ $3 + 3 + 6 = 12$

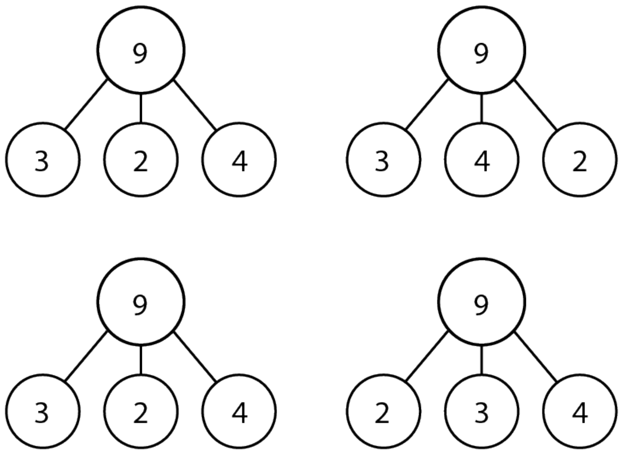
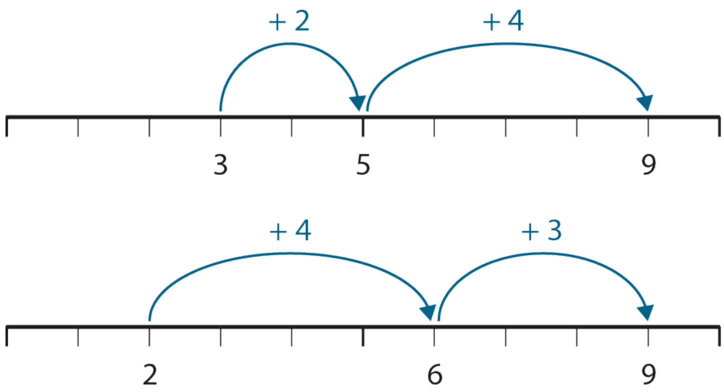
Teaching point 3:

The order in which addends (parts) are added or grouped does not change the sum (associative and commutative laws).

Steps in learning

	Guidance	**Representations**
3:1	Since this is the first time children are formally adding three addends this is the first time they will be exploring the associative law. Discuss equivalent number sentences using concrete materials to represent the addends. Children can explore the conservation of number (i.e. the total remains the same irrespective of which pair of addends are added first). Introduce this generalised statement to capture what children have learnt: ***'When we add three numbers, the total will be the same whichever pair we add first.'***	*'I have three apples, two bananas and four oranges. How many pieces of fruit do I have?'*  3 + 2 + 4 5 + 4 $\mathbf{3+2}+4=\mathbf{5}+4=9$  3 + 2 + 4 3 + 6 $3+\mathbf{2+4}=3+\mathbf{6}=9$
3:2	Children should already be familiar with commutativity in the context of addition with two addends (see segment *1.7 Addition and subtraction: strategies within 10*). It is much more obvious that we can change the order of the addends in the context of aggregation, than in the context of augmentation. For example, children can see much more easily that: $3+1+4=1+3+4$	*'At first there were three ducks in the pond, then two ducks arrived, and then four more ducks arrived. Now there are nine ducks in the pond.'* First Then Then  3 + 2 + 4

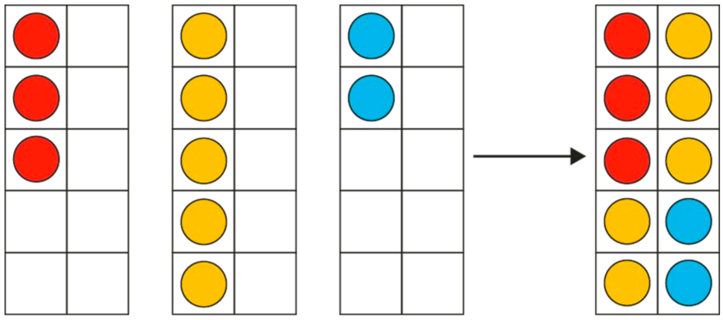
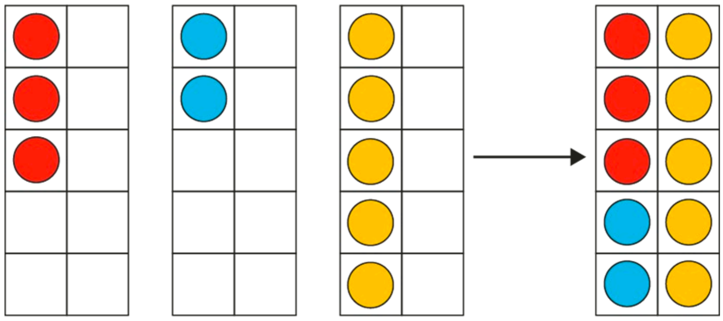
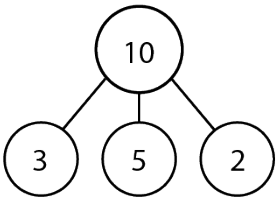
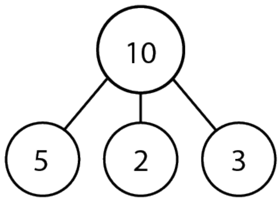
	in the context of an aggregation story: *'I have three red cars, one yellow car and four blue cars. How many cars do I have altogether?'* than in the context of an augmentation story: *'I had three red cars. On my birthday, my sister gave me one yellow car. The next day my friends gave me four blue cars. How many cars do I have now?'* After exploring commutativity in the context of aggregation, move the focus onto augmentation, using ***'first…, then…, then…, now…'*** stories to illustrate commutativity with three addends. Ensure that you consider both reordering the 'thens' as well as switching one of the 'thens' with the 'first'. In the representations given here, we first consider a fixed starting number of ducks, with two more groups arriving in a different order. We then change the number of ducks at the start, but retain the same three addends so that the sum also remains the same.	vs. *'At first there were three ducks in the pond, then four ducks arrived, and then two more ducks arrived. Now there are nine ducks in the pond.'* First Then Then  3 + 4 + 2 3 + 2 + 4 = 3 + 4 + 2 *'At first there were three ducks in the pond, then two ducks arrived, and then four more ducks arrived. Now there are nine ducks in the pond.'* First Then Then  3 + 2 + 4 vs. *'At first there were two ducks in the pond, then three ducks arrived, and then four more ducks arrived. Now there are nine ducks in the pond.'* First Then Then  2 + 3 + 4 3 + 2 + 4 = 2 + 3 + 4

3:3	Transfer each pair of duck examples to the part–part–part–whole structure. Compare each pair, asking children: • *'What's the same?'* • *'What's different?'* Draw attention to the fact that they are the same even though the parts are in a different order. The second duck example (swapping the 'first' with a 'then') is less intuitive; using the part–part–part–whole diagram transforms the problem into an aggregation structure to help children understand what is happening.	
3:4	You can also use a number line to explore commutativity in a situation where it looks less obvious. By the end of these steps children should know the generalised statement: ***'If you change the order of the addends, the sum stays the same.'***	 $3 + 2 + 4 = 2 + 4 + 3$

Teaching point 4:

When we are adding three numbers, we should choose the most efficient order in which to add them, including identifying two addends that make ten (combining).

Steps in learning

	Guidance	Representations
4:1	Now explore the different ways of calculating the sum of three addends, using the knowledge that if you change the order of the addends, the sum stays the same. At this stage it is important that you give children the answers to the calculations to ensure that their focus is on the methods and not the solution. Make sure that the connection between the context, pictorial and abstract representations is made explicit. Ask children: • *'What does the 3 represent?'* • *'What does the 5 represent?'* • *'What does the 2 represent?'* • *'What does the 10 represent?'* Children should respond in full sentences, for example *'the 3 represents the three red cars'.* Compare the two different methods illustrated here and ask children which they think is easier. Draw attention to the second method and discuss how creating two fives may make the calculation easier. Children are learning number bonds and should be fluent with *5 + 5 = 10.*	*'In my toy-box, I have three red cars, five yellow cars and two blue cars. I have ten cars altogether.'*  3 + 5 + 2 = 10  3 + 2 + 5 = 10
4:2	After using tens frames to represent the different orders of addition, move to the part–part–part–whole model. Continue to use the symbolic representation alongside the pictorial representations to help children make connections.	 3 + 5 + 2 = 10  5 + 2 + 3 = 10

<table>
<tr>
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<td>Note that pictorial representations are used both to model the structure and to provide support for more challenging calculations, for which children have not yet been taught efficient mental strategies.</td>
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<tr>
<td>4:3</td>
<td>Use the pictorial representations to build towards a stepped symbolic notation, including the correct use of the equals symbol. Children commonly use the equals sign incorrectly when explaining their thinking, for example:

$2 + 3 = 5 + 5 = 10$ ✗

Remind children that expressions on both sides of the equals sign must be equal, for example:

$2 + 3 + 5 = 5 + 5 = 10$ ✓

Provide children with missing number problems for practice in identifying the most efficient strategies as well as correct symbolic notation.</td>
<td>Different strategies for adding two, four and three:
'Fill in the missing numbers.'

$2 + 4 + 3 = \square + 3 = \square$

$2 + 4 + 3 = \square + 4 = \square$

$2 + 4 + 3 = \square + 2 = \square$

Different strategies for adding two, one and four:
'Fill in the missing numbers.'

$2 + 1 + 4 = \square + \square = \square$

$2 + 1 + 4 = \square + \square = \square$

$2 + 1 + 4 = \square + \square = \square$</td>
</tr>
<tr>
<td>4:4</td>
<td>To promote depth, use a dòng nǎo jīn question such as the one shown here.</td>
<td>Example dòng nǎo jīn question:
'Fill in the missing squares, using the digits 0, 1, 2, 4, 5 and 6, so that each row and column adds up to the same number.'
<table>
<tr><td> </td><td> </td><td> </td></tr>
<tr><td>3</td><td>3</td><td>3</td></tr>
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Solution:
<table>
<tr><td>4</td><td>5</td><td>0</td></tr>
<tr><td>3</td><td>3</td><td>3</td></tr>
<tr><td>2</td><td>1</td><td>6</td></tr>
</table>
</td>
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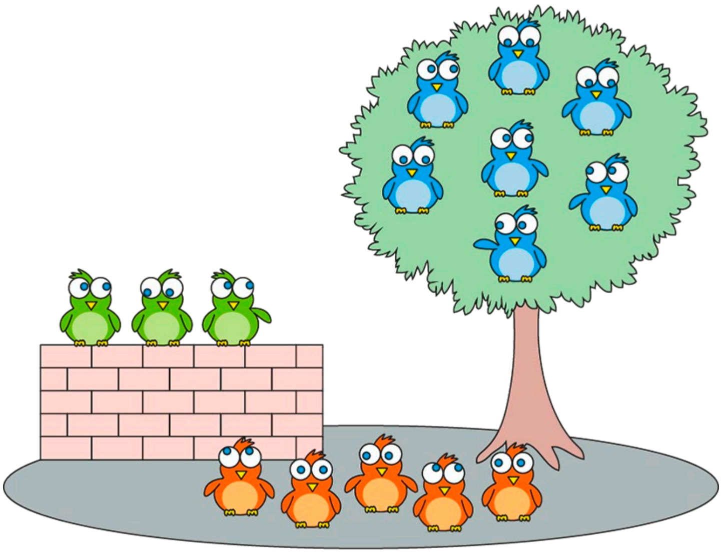
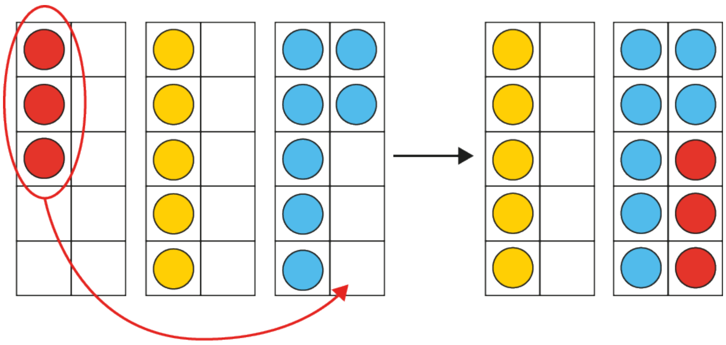
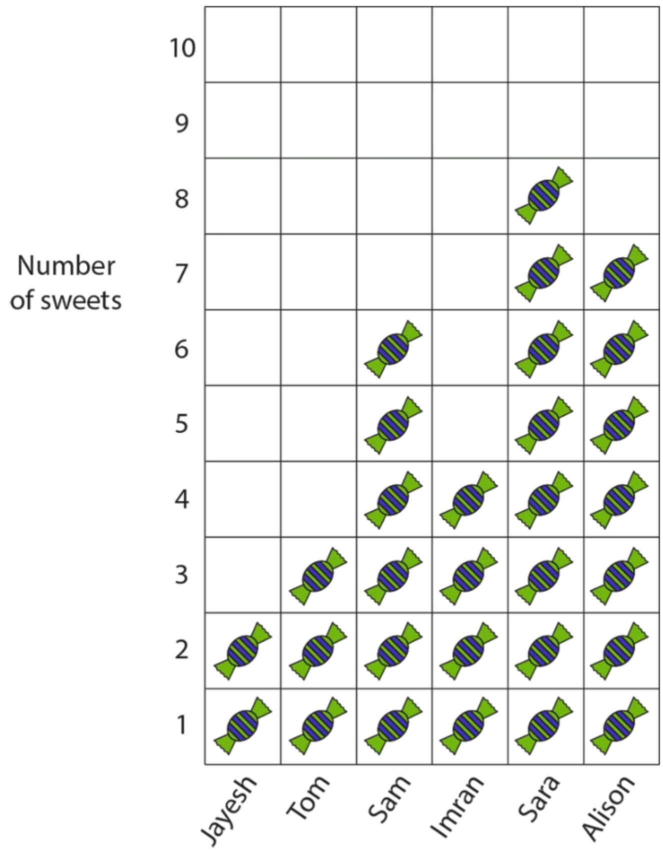
4:5	Now move on to the first context where children will meet a total greater than ten. Begin with calculations in which two of the addends sum to ten. Since children should now have mastered swapping the order of addends in a calculation, they can use this strategy to efficiently add three numbers by making ten first. From previous work, children should be able to recognise pairs of numbers which sum to ten (see segment *1.7 Addition and subtraction: strategies within 10*). They also need to have mastered adding a single-digit number to ten (see segment *1.10 Composition of numbers: 11–19*). Begin by representing a problem pictorially, and make a clear link with the tens frame representation to allow children to see the pairs of addends which sum to ten. For the first problem, consider recalling addition as aggregation to support children in recognising that they can add the addends in any order. The example given is *3 + 5 + 7*; from the previous step, children should be able to suggest three ways of solving this problem, as well as selecting and justifying the most efficient method. This leads to a useful generalised statement when adding three or more addends: ***'We can look for pairs of addends which sum to 10.'***	*'There are three birds on the wall, five birds on the ground and seven birds in the tree. How many birds are there altogether?'* Pictorial:  Using tens frames:  $3 + 5 + 7 = 5 + 10$
4:6	Once children are confident in making tens using the tens frames, move on to the use of symbolic representations alone. Children will now need to recall their number bonds to ten in order to find the most efficient calculation strategies.	*'Fill in the missing numbers.'* $5 + 6 + 5 = \square$ $3 + 3 + 7 = \square$ $7 + 4 + 3 = \square$

<table>
<tr><td></td><td></td><td>$8 + 4 + 2 = \square$

$2 + 8 + 6 = \square$</td></tr>
<tr><td>4:7</td><td>Once children have mastered the ‘making ten’ strategy, present them with a mix of calculations and ask them to sort them into two groups – those for which the ‘making ten’ strategy can be applied, and those for which it can’t.</td><td>Make ten where possible.
$7 + 5 + 5$
$7 + 6 + 2$
$1 + 3 + 9$
$4 + 8 + 5$
<table><tr><th>Can make 10</th><th>Cannot make 10</th></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><td>4:8</td><td>To promote further depth, present children with four-addend problems which include:
two pairs of number bonds to ten
three addends which sum to ten.</td><td>Two pairs of number bonds to ten:
$2 + 4 + 6 + 8 = \square$
Three addends sum to 10:
$8 + 3 + 6 + 1 = \square$</td></tr>
<tr><td>4:9</td><td>You can use a pictogram-based question to provide children with the opportunity to apply the strategy in a different context. This alternative representation reflects variation. Note that each column in the pictogram sums to ten to help children visualise opportunities to make ten.

Ask questions such as ‘How many sweets do Jayesh, Sam and Sara have together?’ (2 + 6 + 8).</td><td>
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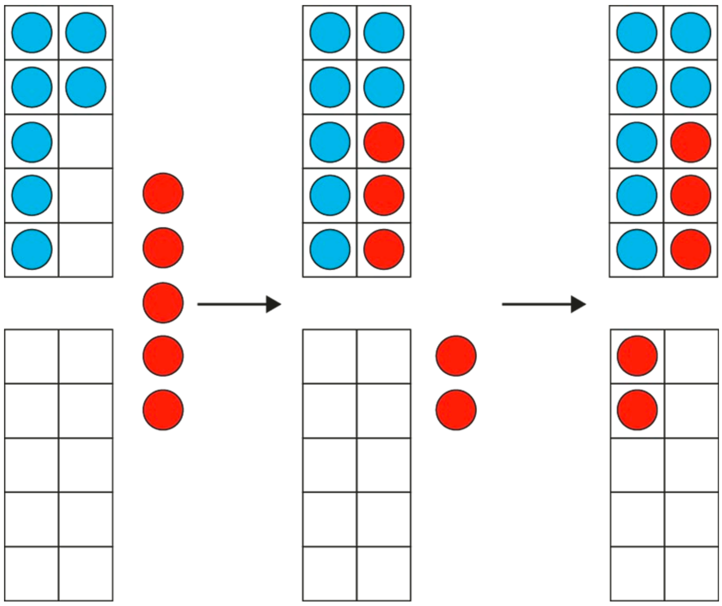
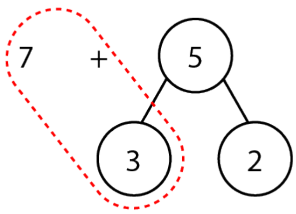
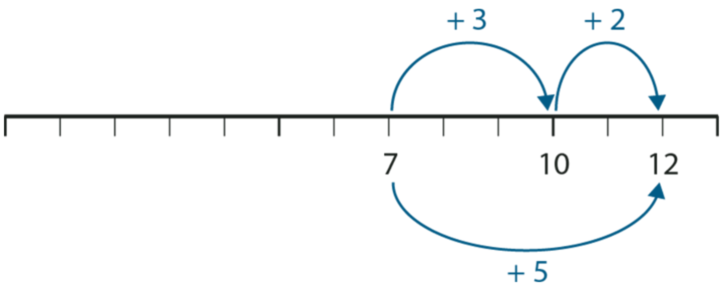
4:10	By the end of this teaching point, when you show children expressions (e.g. *7 + 3 + 4*), they should be able to describe their answers in full, in a consistent way (for example, *'seven plus three is equal to ten, then ten plus four is equal to fourteen'*). This language is important because it prepares children for the next step. Use a stem sentence with the structure: ***'___ plus ___ is equal to ten, then ten plus ____ is equal to ___.'***	

Teaching point 5:

We can add two numbers which bridge the tens boundary by using a 'make ten' strategy.

Steps in learning

	Guidance	Representations
5:1	This teaching point builds on elements of segments *1.2 Introducing 'whole' and 'parts': part–part–whole*, *1.3 Composition of numbers: 0–5* and *1.4 Composition of numbers: 6–10*. Introduce the idea that we can transform a sum of two addends (e.g. $7 + 5$) into a sum of three addends (e.g. $7 + 3 + 2$) by partitioning one of the addends. This will enable us to use the strategies from *Teaching point 3*. Begin by using a story with a practical representation. Then transfer the context to tens frames to help children see the opportunity to make ten.	Story and practical representation: *'A ride at the funfair has ten seats in each carriage. You have to fill up the whole carriage before children can get in a new one. There are seven children in the first carriage. Five more get on. How many children are there altogether?* *'Three children can get in the first carriage and two children get in the next carriage.'* Chairs could be arranged to support acting out this story. Tens frames: 
5:2	You can use the part–part–whole cherry representation to help children move on to an equation representing the partitioning. Encourage children to describe the steps in full, for example: • *'First I partition the five: three plus two is equal to five.'* • *'Then seven plus three is equal to ten…'*	 $7 + 3 = 10$ $10 + 2 = 12$

	• *'…and ten plus two is equal to twelve.'* Stay with the same example until the children are confident. The generalised stem sentence has the structure: • ***'First I partition the ___: ___ plus _____ is equal to ____'*** • ***'Then ___ plus ___ is equal to ten…'*** • ***'…and ten plus ___ is equal to ___.'***	
5:3	Provide children with several examples to explore in pairs using the tens frame. Encourage them to repeat the stem sentence as they move the counters around. Ensure that children use the tens frames to expose the mathematical structure and that they do not, for example, use them to 'count all' or 'count on' without making ten.	
5:4	You can also represent this strategy using a number line.	
5:5	Children should build towards using just the symbolic notation. Begin with equations with terms on both sides to practise and embed, before moving to a less scaffolded approach.	*'Fill in the missing numbers.'* $7 + 5 = 7 + 3 + \square$ $8 + 5 = 8 + 2 + \square$ $6 + 5 = 6 + \square + \square$ $9 + 5 = 9 + \square + \square$ $6 + 6 = 6 + \square + \square$

		8 + 8 = $\boxed{}$ + $\boxed{}$ + $\boxed{}$ 8 + 4 = $\boxed{}$ + $\boxed{}$ + $\boxed{}$
5:6	Children will need regular practice at using this strategy in order to develop fluency. It is acceptable for some children to need to use concrete resources for longer than others – in these cases, think carefully about how to move their thinking on, for example by encouraging them to first visualise how to partition the addend and 'make ten'. Children who grasp this strategy quickly will benefit from the opportunity to find the answer in as many ways as possible. They may also enjoy partitioning both addends into *5 + 'something'*, for example: 8 + 6 = (5 + 3) + (5 + 1) = 14 8 + 6 = (5 + 5) + (3 + 1) = 14	*'Can you think of more than one strategy to find the answer to each calculation?'* 7 + 6 9 + 4 9 + 8 7 + 7 7 + 4 8 + 8 6 + 8 9 + 3 7 + 9 8 + 6 8 + 3 9 + 7 8 + 5 8 + 4 9 + 6 7 + 5 4 + 8 7 + 8 6 + 6 5 + 7 5 + 8
5:7	To promote depth, use this dòng nǎo jīn question: provide a set of digits with a one-to-one correspondence with a series of expressions with missing numbers, as shown in the example.	*'Use the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 to complete the following expressions. Use each digit exactly once.'* 8 + $\boxed{}$ > 10 + 5 $\boxed{}$ + 7 = 15 $\boxed{}$ + $\boxed{}$ > 11 10 = $\boxed{}$ + $\boxed{}$ + $\boxed{}$ + $\boxed{}$ 6 + 9 = 10 + $\boxed{}$ Solution: 8 + $\boxed{9}$ > 10 + 5 $\boxed{8}$ + 7 = 15

		$\boxed{6} + \boxed{7} > 11$ $10 = \boxed{1} + \boxed{2} + \boxed{3} + \boxed{4}$ $6 + 9 = 10 + \boxed{5}$

Teaching point 6:

We can subtract across the tens boundary by subtracting *through* ten or subtracting *from* ten.

Steps in learning

6:1

Guidance

Revisit the fairground ride scenario from step *5:1*, this time beginning with more than ten children, and using a subtraction *through* ten strategy. To subtract *through* ten, we subtract down to ten, and then subtract the rest of the subtrahend from ten, for example:

$12 - 4 = 12 - 2 - 2 = 8$

Either show the reduction context pictorially, as opposite, or act it out with chairs arranged as the carriages. Practise telling the story as a class until children are confident describing it: '*First there were twelve children on the ride. Then four got off. Now there are eight children on the ride.*'

Now give children an opportunity to work through the story themselves with counters and tens frames. Tell them:

- *'We are going to partition the four into two and two.'*
- *'We first subtract two from twelve to get to ten.'*
- *'Then we subtract the remaining two from the ten – we already know that ten minus two is equal to eight.'*

Have children describe the process in sentences: *'First we took away two counters to make ten. Then we took away another two counters to make eight.'*

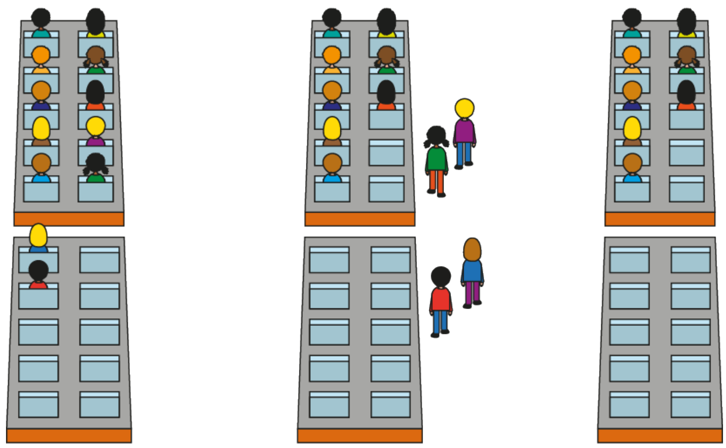
Model the process on a bead bar and number line, continuing to describe in full sentences. The bead bar helps to

Representations

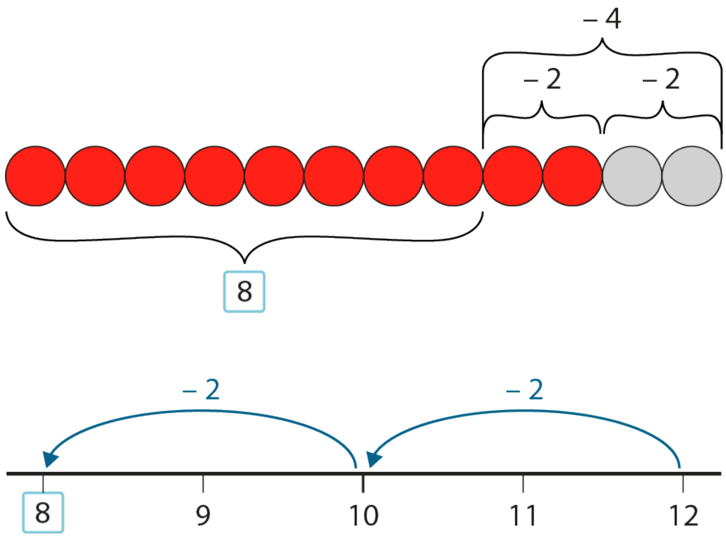
Subtraction *through* ten – pictorial:

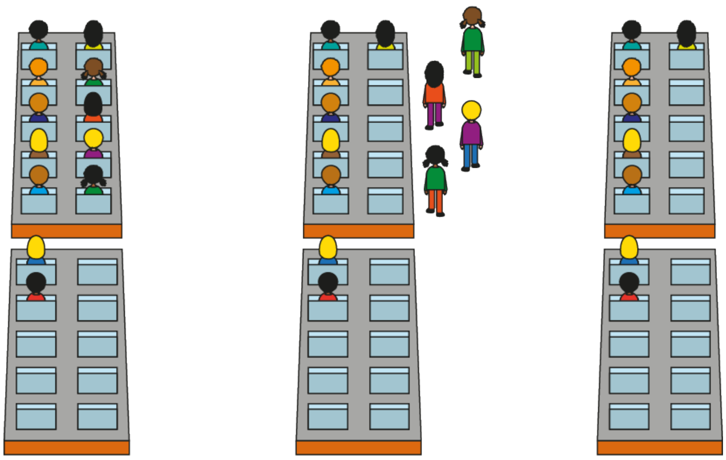
'First there were twelve children on the ride. Then four got off. Now there are eight children on the ride.'

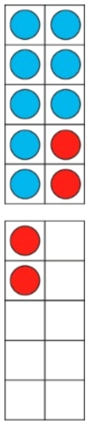
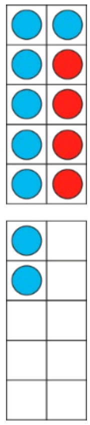
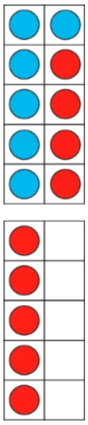
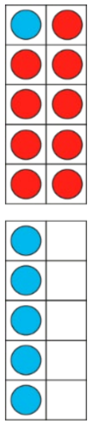
First Then Now

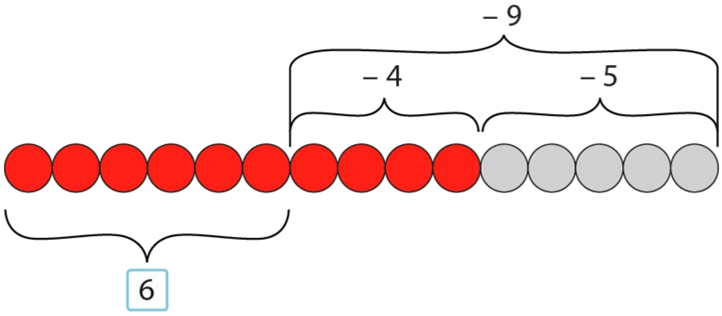
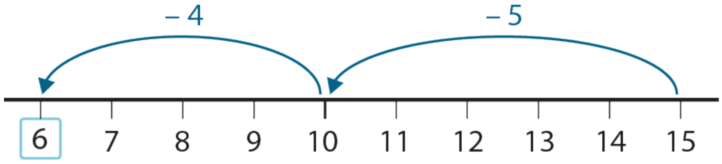
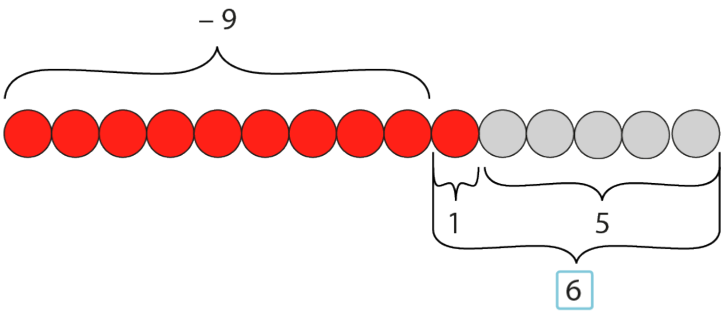
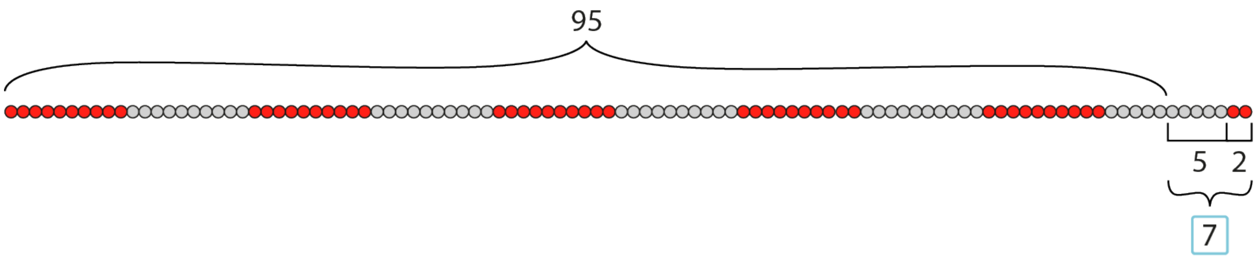


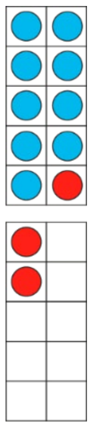
Subtraction *through* ten – bead bar and number line:

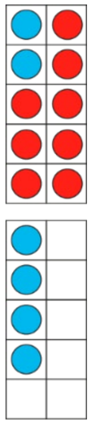


	make the link between the tens frame representation and the number line. Finally write equations to express the two-stage process, showing the children how to jot down partitioning of the subtrahend and express the two steps. Some children may write: $12 - 2 = 10 - 2 = 8$ × Make sure that you pick up this error, and show that *10 – 2 = 8* must be written as a separate equation to avoid differing values on either side of the equals symbol.	Subtraction *through* ten – abstract: 12 – 4 2 2 $12 - 2 = 10$ $10 - 2 = 8$ so $12 - 4 = 8$
6:2	Now tell the story with the same numbers, but explaining that a *different* four children get off of the ride – i.e. now using a subtraction *from* ten strategy. To subtract *from* ten, we subtract the subtrahend from ten, and then add on the difference between the minuend and ten, for example: $12 - 4 = (10 + 2) - 4$ $= (10 - 4) + 2 = 8$ Emphasise that we still have four children getting off, as before, but it is a different four. Follow a similar teaching pattern to that used in step *6:1*: • Tell the story while acting out or showing the pictures. • Use tens frames and counters to model the story – *'First we take away four counters from the ten which means we have six left in the first tens frame. Then we add on the other two; six plus two is equal to eight.'* • Show the partitioning of the minuend, and equations, to represent the calculation. Note the omission of the bead bar and number line, since these	Subtraction *from* ten – pictorial: *'First there were twelve children on the ride. Then four got off. Now there are eight children on the ride.'* First Then Now  Subtraction *from* ten – abstract: 12 – 4 10 2 $10 - 4 = 6$ $6 + 2 = 8$ so $12 - 4 = 8$

<table>
<tr><td></td><td>representations are not as helpful for supporting this strategy.</td><td></td></tr>
<tr><td>6:3</td><td> Show the two methods alongside each other, asking children, 'What's the same? What's different?' Depending on the numbers involved, sometimes one of the approaches will seem more efficient than the other; the aim is for children to be able to fluently subtract across ten, whichever of these approaches is used. </td><td>
<table>
<tr><th>Subtraction through ten:</th><th>Subtraction from ten:</th></tr>
<tr><td></td><td></td></tr>
<tr><td>12 – 4
2 2</td><td>12 – 4
10 2</td></tr>
<tr><td>12 – 2 = 10
10 – 2 = 8
so
12 – 4 = 8</td><td>10 – 4 = 6
6 + 2 = 8
so
12 – 4 = 8</td></tr>
</table>
</td></tr>
<tr><td>6:4</td><td> Revisit both methods for a new equation which bridges ten. The example opposite (15 – 9) has been chosen as a case for which subtracting from ten may be 'easier' than subtracting through ten, since the arithmetic involved is simpler in the from ten strategy. Remember to highlight that if we are subtracting 9 items from 15 items, whichever 9 items we subtract, there will always be 6 remaining. At first glance, you may feel the subtraction from ten strategy seems unusual or unintuitive; however the 'from ten' strategy is quite similar to the 'counting on' method (for example, using a number line where the subtrahend is taken away from the </td><td>
<table>
<tr><th>Subtraction through ten:</th><th>Subtraction from ten:</th></tr>
<tr><td></td><td></td></tr>
<tr><td>15 – 9
5 4</td><td>15 – 9
10 5</td></tr>
<tr><td>15 – 5 = 10
10 – 4 = 6</td><td>10 – 9 = 1
1 + 5 = 6</td></tr>
</table>
</td></tr>
</table>

	beginning of the number line and the remaining quantity is counted up), although the presentation of the strategies differ. The subtraction *from* ten bead bar representation below serves as a useful link between the idea of 'counting on' on a number line and the '*from* ten' strategy; although not necessarily appropriate for children at this stage, this representation can be useful for teachers new to the '*from* ten' strategy. The idea of subtracting *from* ten prepares children for subtracting from other 'units' (e.g. 100) in future segments. For example, when calculating *102 – 95*, subtracting 95 *from* 100 to give 5, and adding this to the 2 to give 7, is probably more efficient than subtracting *through* 100, i.e. subtracting 2 and then 93 more to give 7.	so 15 – 9 = 6 | so 15 – 9 = 6 Subtraction *through* ten – bead bar and number line:  
6:5	Subtraction *from* ten concept – bead bar representations: 15 – 9 = 6  102 – 95 = 7 	

	Continue until the children are confident describing and using both methods, when working with counters, and choosing between the methods. Subtracting *through* ten may be easier when the subtrahend is smaller (e.g. *12 – 3*) and subtracting *from* ten possibly lends itself to larger subtrahends (e.g. *17 – 9*). However, there isn't always a 'best' choice, and decisions about approach should always be made according to the particular minuend and subtrahend, and the relationship between them. For both strategies, fluency in number facts for the numbers to 10 (for example, being able to partition four into two and two, and knowing that *10 – 2 = 8*) is important for success.	
6:6	Because subtractions that bridge ten require several steps (with either strategy), at this stage in Year 2 children are likely to need lots of practice in order to become fluent. You will need to carefully support children in progressing to the completion of these calculations without using counters and without counting back in ones to get an answer. Initially, move away from using physical counters by presenting children with images of tens frames to work with alongside expressions which already show either the subtrahend or minuend partitioned. Then move slowly to just giving the children the equation to solve – first remove the scaffold of the pictorial tens frames, then remove the scaffold of the 'pre-partitioned' subtrahend/minuend.	Scaffolded subtraction *through* ten:  12 – 3 2 1 12 – 2 = 10 10 – 1 = 9 so 12 – 3 = 9

<table>
<tr>
<td></td>
<td></td>
<td>Scaffolded subtraction from ten:

$$\begin{array}{ccc} 14 & - & 8 \\ \swarrow\ \searrow & & \\ 10 \quad 4 & & \end{array}$$
10 – 8 = 2
2 + 4 = 6
so
14 – 8 = 6</td>
</tr>
<tr>
<td>6:7</td>
<td>Once children are confident with the two methods, provide practice including:
equations with missing differences
contextual questions, for example:
'There are eleven apples in a bowl. We eat seven. How many are left?' (reduction)
'I have fourteen paintbrushes to wash. I have washed eight already. How many more do I have to wash?' (partitioning)
To promote and assess depth of understanding, present dòng nǎo jīn questions such as those shown opposite.</td>
<td>Missing difference equations:
'Fill in the missing numbers.'
$11 - 3 = \square$
$14 - 6 = \square$
$12 - 9 = \square$
$16 - 7 = \square$

Dòng nǎo jīn problems:
'What is the smallest number for which this expression is true?'
$13 - \square < 7$
'Fill in the missing numbers.'
$13 - 3 - 4 = 10 - \square$
$12 - 2 - \square = 12 - 7$
$16 = 3 + \square + 6$</td>
</tr>
</table>

6:8	Complete the segment by presenting intelligent practice for both addition and subtraction bridging ten, highlighting and using the inverse relationship between addition and subtraction.	*'Fill in the missing numbers.'* $5 + 8 = \square$ $9 + \square = 16$ $\square + 6 = 11$ $\square - 8 = 5$ $16 - \square = 9$ $11 - 6 = \square$