

Mastery Professional Development

Number, Addition and Subtraction



1.6 Additive structures: introduction to augmentation and reduction

Teacher guide | Year 1

Teaching point 1:

An addition context described by a '**first..., then..., now...**' story is an example of augmentation. We can link the story to a numerical representation – each number represents something in the story.

Teaching point 2:

A subtraction context described by a '**first..., then..., now...**' story is an example of reduction. We can link the story to a numerical representation – each number represents something in the story.

Teaching point 3:

Given any two parts of the story we can work out the third part; given any two numbers in the equation we can find the third one.

Teaching point 4:

Addition and subtraction are inverse operations.

Overview of learning

In this segment children will:

- develop an early understanding of addition as augmentation, using stories
- develop an early understanding of subtraction as reduction, using stories
- relate addition (augmentation) and subtraction (reduction) as inverse operations
- use symbolic notation to represent augmentation and reduction stories.

This segment develops children's early understanding of addition and subtraction by introducing the structures of augmentation for addition, and reduction for subtraction. The structure of augmentation is where a quantity increases; it is 'augmented'. The structure of reduction is the opposite – a quantity decreases; it is 'reduced'.

Teaching will approach:

- addition as augmentation (the inverse of reduction) using situations where there is one element – the initial quantity of the element (the augend) is increased by an amount (the addend) and an addition is required to find the augmented value (the sum)
- subtraction as reduction (the inverse of augmentation) using situations where there is one element – the initial quantity of the element (the minuend) is decreased by an amount (the subtrahend) and a subtraction is required to find the reduced value (the difference).

A '**first...**, **then...**, **now...**' story representation will be used to expose the structures. Teachers are encouraged to plan a range of story contexts relevant to the structures. Careful consideration must be given to the language used within the stories to ensure augmentation and reduction are exposed over other structures (for example, aggregation or partitioning).

The *structures* rather than the *solutions* should remain the focus throughout. To support this, teachers should plan using stories with quantities within ten, such that children are not focused on calculating. This will further negate any need to tell children to '*put the bigger number in your head and count on/back*', which would draw attention away from the structures. That said, it is the augmentation and reduction structures that give purpose to counting on and back from fixed starting points by a set value.




1.6 Augmentation and reduction







An explanation of the structure of these materials, with guidance on how teachers can use them, is contained in this NCETM podcast: www.ncetm.org.uk/primarympdpodcast. The main message in the podcast is that the materials are principally for professional development purposes. They demonstrate how understanding of concepts can be built through small coherent steps and the application of mathematical representations. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks

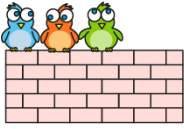
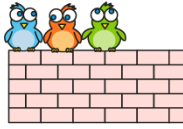
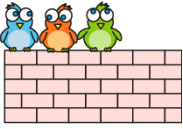


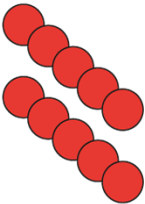
Teaching point 1:

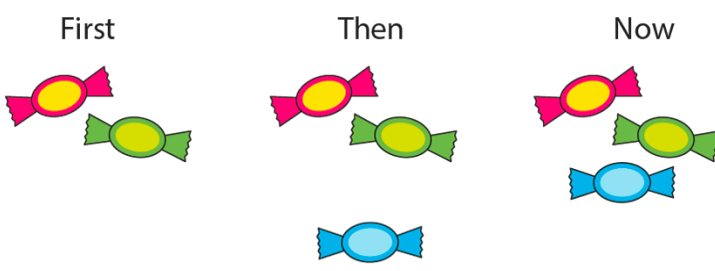
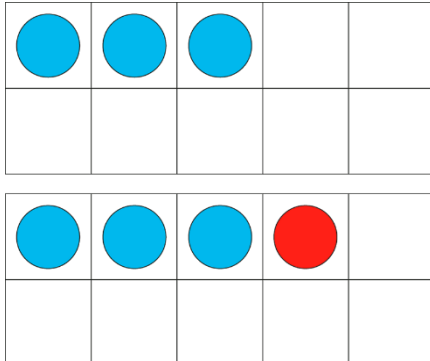
An addition context described by a **'first..., then..., now...'** story is an example of augmentation. We can link the story to a numerical representation – each number represents something in the story.

Steps in learning

	Guidance	Representations
1:1	In a practical way, and by planning a range of augmentation contexts (such as number of people on the bus, number of cars in the car park, etc.) introduce the 'first..., then..., now...' story representation.	'First, three children were sitting on the carpet. Then, two more children sat on the carpet. Now, five children are sitting on the carpet.' Children can act out this story.
1:2	Once children are confident with the use of concrete/practical representations, begin to introduce pictorial representations. To help children make a connection between the concrete/practical and pictorial representation, introduce the matching pictorial representation alongside the practical representation. When working pictorially, continue to use a range of augmentation contexts, but ensure that these are 'paired up' with the corresponding practical representation to facilitate progression. Within the story context, focus on the value by which the element is increased (the addend). Carefully consider how to represent (when acting or drawing) the 'then' part of the story to demonstrate the increase in value of the story's element (for example, children waiting to board the bus; a child standing next to the carpet, ready to sit).	Concrete/practical: 'First, four children were sitting on the bus. Then, three more children got on the bus. Now, seven children are sitting on the bus.' Chairs could be arranged to support acting out this story. Pictorial: <div style="display: flex; justify-content: space-around; text-align: center;"> <div> <p>First</p>  </div> <div> <p>Then</p>  </div> <div> <p>Now</p>  </div> </div>

<p>1:3 Introduce abstract representations <i>alongside</i> the pictorial to support children in making a connection between the pictorial and abstract representations. Record the numbers and symbols alongside the pictures as the story is told.</p> <p>Explicitly encourage children to connect each number/symbol to each part of the story, and the language within it, by asking questions:</p> <ul style="list-style-type: none"> • 'What does the 4 represent?' • 'What does the + 3 represent?' • 'What does the 7 represent?' <p>Children should answer in full sentences, for example: 'The 4 represents the number of children on the bus at the start.'</p> <p>Explicitly connect the addition symbol with the language of the increase.</p>	<div style="display: flex; justify-content: space-around; text-align: center;"> <div> <p>First</p>  </div> <div> <p>Then</p>  </div> <div> <p>Now</p>  </div> </div> <div style="text-align: center; margin-top: 10px;"> <p>4 + 3 7</p> <hr style="width: 100%; border: 0.5px solid black;"/> <p style="margin-left: 150px;">$4 + 3 = 7$</p> </div>
<p>1:4 When planning augmentation story contexts, make sure you include examples that involve adding zero. Consider a range of stories in which either the augend or the addend is zero.</p>	<p>Augend is zero:</p> <p>'First, there were no people in the car. Then, two people got into the car. Now, there are two people in the car.'</p> <div style="display: flex; justify-content: space-around; text-align: center;"> <div> <p>First</p>  </div> <div> <p>Then</p>  </div> <div> <p>Now</p>  </div> </div> <div style="text-align: center; margin-top: 10px;"> <p>0 + 2 2</p> <hr style="width: 100%; border: 0.5px solid black;"/> <p style="margin-left: 150px;">$0 + 2 = 2$</p> </div> <p>Addend is zero:</p> <p>'First, there were three birds sitting on the wall. Then, no more birds sat on the wall. Now, there are still three birds sitting on the wall.'</p>

		<div style="display: flex; justify-content: space-around; text-align: center;"> <div> <p>First</p>  <p>3</p> </div> <div> <p>Then</p>  <p>+ 0</p> </div> <div> <p>Now</p>  <p>3</p> </div> </div> <div style="text-align: center; margin-top: 10px;">  <p>$3 + 0 = 3$</p> </div>
<p>1:5</p> <p>To deepen children’s understanding and strengthen the links they are making between the abstract representations and each part of the story, provide them with a ‘first..., then..., now...’ card and ten counters.</p> <p>Children can use the resource to tell and record their own stories, for example:</p> <ul style="list-style-type: none"> • ‘First, three children were sitting on the carpet.’ Place three counters in first box. • ‘Then, two more children sit on the carpet.’ Move the three counters across and add two more. • ‘Now, there are five children on the carpet.’ Move all five counters to the final box. <p>Question children on each part of their story, for example (referring to the ‘first’ box), ‘How many sweets did the boy have at the beginning?’ or (referring to the ‘then’ box) ‘How did the number of sweets change?’</p> <p>In a similar way, question children on the meaning of each number at different points in their stories.</p>		<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 10px; margin-right: 20px;"> <div style="display: flex; justify-content: space-around; text-align: center;"> <div> <p>First</p> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 20px; height: 20px; margin: 5px auto;"></div> </div> <div> <p>Then</p> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 0 auto;"></div> <div style="display: flex; align-items: center; margin: 5px auto;"> + <div style="border: 1px solid black; width: 20px; height: 20px; margin: 0 5px;"></div> </div> </div> <div> <p>Now</p> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 20px; height: 20px; margin: 5px auto;"></div> </div> </div> <div style="text-align: center; margin-top: 10px;">  <p>$\square + \square = \square$</p> </div> </div> <div style="margin-left: 20px;">  </div> </div>
<p>1:6</p> <p>Increase the challenge, encouraging children to continue using their cards and counters to help them with the following scenarios:</p> <ul style="list-style-type: none"> • Present children with a story. Can they find the corresponding 		

	<p>expression or equation from a selection? Can they justify their selection? Can they disprove others from the selection?</p> <ul style="list-style-type: none"> In a similar way, present children with an equation. Can they find the corresponding story from a selection? Can they justify their selection? Can they disprove others from the selection? <p>Note that there will only be one way of recording the augend and addend to match the 'first..., then..., now...' story context. Give children the opportunity to select the correctly ordered expression and justify their choice. At this point we are <i>not</i> drawing attention to commutativity; this will be addressed in the segment <i>1.7 Addition and subtraction: strategies within 10</i>.</p> <p>Once children have mastered choosing from a selection, further increase the challenge:</p> <ul style="list-style-type: none"> Present children with a story. Can they write the corresponding equation? In a similar way, present children with an equation. Can they draw a corresponding story? 	<p>'First, Tom had two sweets. Then, Tom got one more sweet. Now, Tom has three sweets.'</p> <p>First Then Now</p>  <p>$2 + 1$ or $1 + 2$?</p>
<p>1:7</p>	<p>You can vary the representation of the augmentation structure by applying the stem sentence ('first..., then..., now...') to the tens frame, bar model, bead string and the number line.</p> <ul style="list-style-type: none"> Bar model: you can use squared paper, and unitise the squares, to support the use of this representation in exposing the augmentation structure without requiring calculation. Number line: it is the augmentation (and reduction) structure that lies behind the concept of counting on (and back) along a number line from 	<p>Tens frame:</p> <p>'First, James wrote three sentences. Then, he wrote one more sentence. Now, he has four sentences written down.'</p>  <p>$3 + 1 = 4$</p>

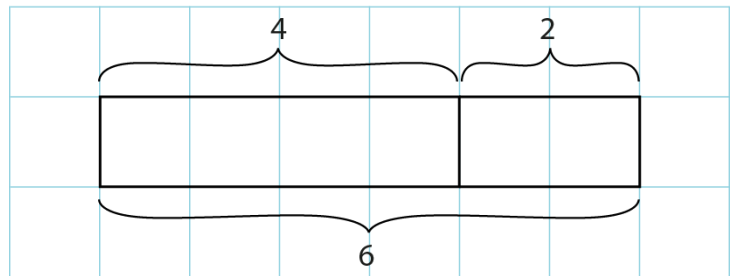
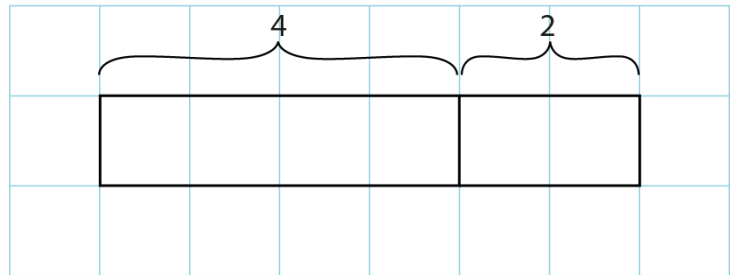
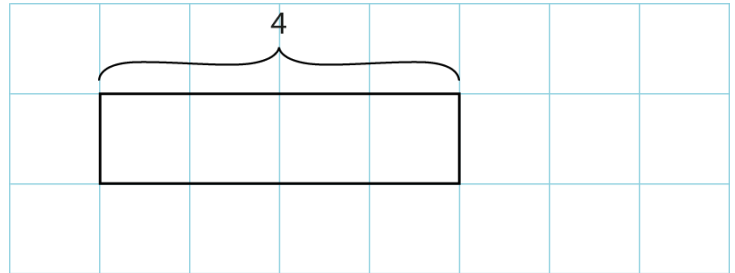
a fixed starting point. At this stage, the context of the stories could broaden to include measurement (for example, money, length, age or mass).

Bar model with squared paper:

'First, Sarah had four pounds.

Then, she was given two pounds more.

Now, she has six pounds.'



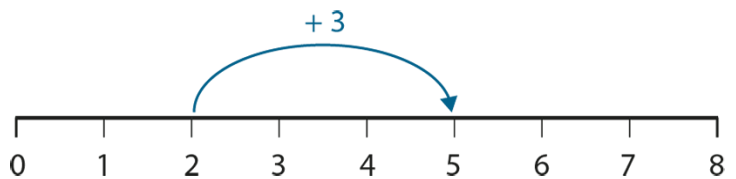
$$4 + 2 = 6$$

Number line:

'At first, the tower was two bricks tall.

Then, three more bricks were added.

Now, the tower is five bricks tall.'

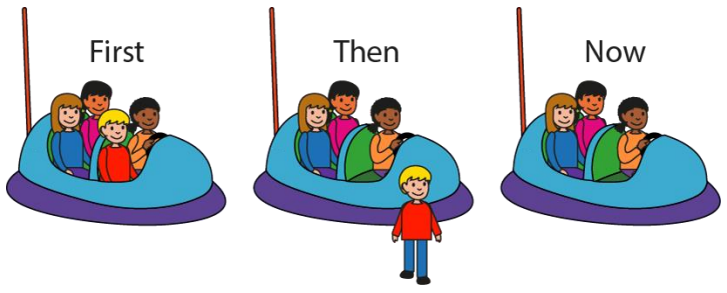


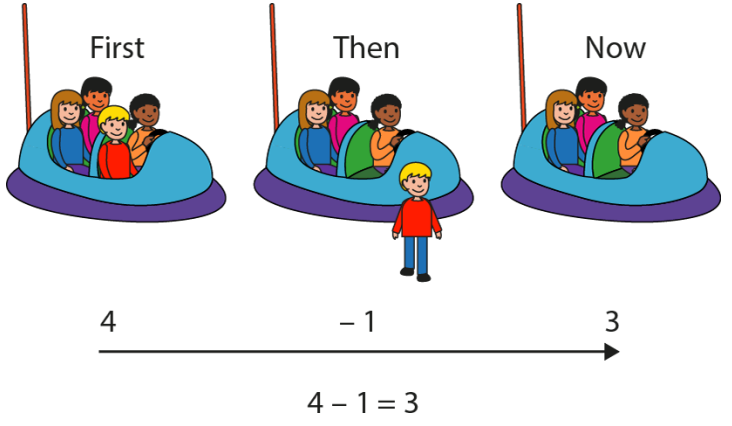
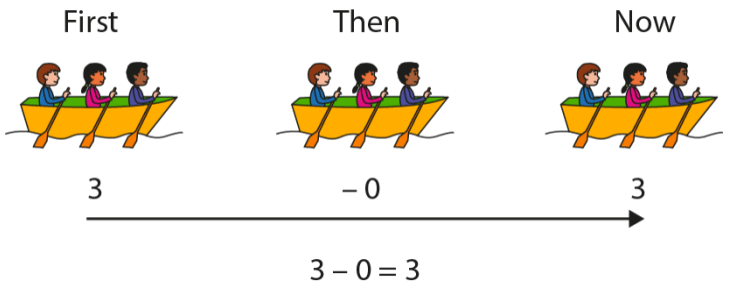
$$2 + 3 = 5$$

Teaching point 2:

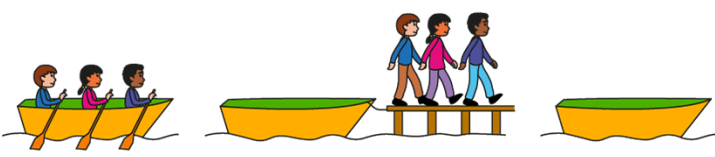
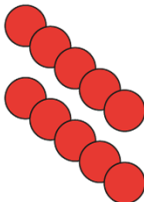
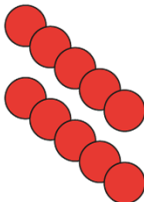
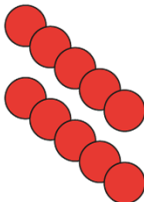
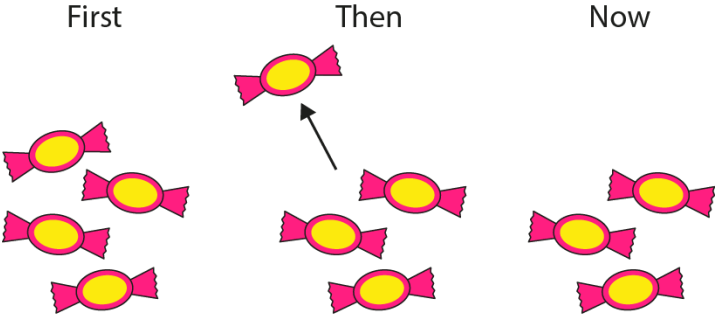
A subtraction context described by a **'first..., then..., now...'** story is an example of reduction. We can link the story to a numerical representation – each number represents something in the story.

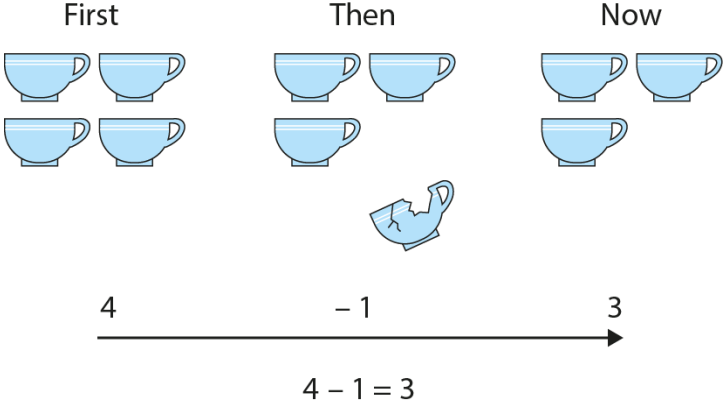
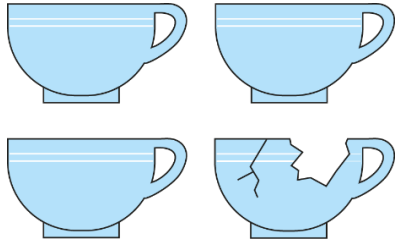
Steps in learning

	Guidance	Representations
2:1	The introduction to reduction will mirror the introduction to augmentation seen above in steps 1:1–1:3. Introduce the reduction structure in a practical way. Plan a range of reduction contexts to explore using the 'first..., then..., now...' story representation.	'First, there were five children in the book corner. Then, two children left the book corner. Now there are three children in the book corner.' Children could act out this context, along with others.
2:2	Now begin to introduce pictorial representations for the reduction structure. To help children make a connection between the concrete/practical and pictorial representation, introduce the matching pictorial representation alongside the practical representation. When working pictorially, continue to use a range of reduction contexts, but ensure that these are 'paired up' with the corresponding practical representation to facilitate progression. Within the story context, focus on the value by which the element is decreased (the subtrahend). Carefully consider how to represent (when acting or drawing) the 'then' part of the story to demonstrate the decrease in value of the story's element (for example, a child climbing out of the car).	Concrete/practical: 'First, there were four children in the car. Then, one child got out. Now, there are three children in the car.' Chairs could be arranged to support acting out this story. Pictorial: 

<p>2:3</p> <p>As before (with augmentation), begin to introduce abstract representations <i>alongside</i> the pictorial to support children in making a connection between the pictorial and abstract representations. Record the numbers and symbols alongside the pictures as the story is told.</p> <p>Explicitly encourage children to connect each number/symbol to each part of the story, and the language within it, by asking questions:</p> <ul style="list-style-type: none"> • 'What does the 4 represent?' • 'What does the - 1 represent?' • 'What does the 3 represent?' <p>Children should answer in full sentences, for example: 'The minus one represents the number of children that got out of the car.'</p> <p>Explicitly connect the subtraction symbol with the language of the decrease.</p>	 <p>First Then Now</p> <p>4 - 1 3</p> <p>4 - 1 = 3</p>
<p>2:4</p> <p>When planning reduction story contexts, explore zero explicitly:</p> <ul style="list-style-type: none"> • Consider cases where the subtrahend is zero (i.e. the difference is equal to the minuend). You will need to consider carefully how to represent '- 0' in the 'then' box. This box should not be empty; the '- 0' signifies that 'nothing changes'. • Also consider cases where the subtrahend is equal to the minuend, giving a difference of zero. 	<p>Subtrahend is zero:</p> <p>'First, there were three children in the boat. Then, no children got out of the boat. Now, there are three children in the boat.'</p>  <p>First Then Now</p> <p>3 - 0 3</p> <p>3 - 0 = 3</p>

1.6 Augmentation and reduction

		<p>Subtrahend is equal to the minuend: <i>First there were three children in the boat. Then three children got out of the boat. Now there are no children in the boat.'</i></p> <p>First Then Now</p>  <p style="text-align: center;">3 - 3 0</p> <hr style="width: 100%; border: 0; border-top: 1px solid black; margin: 5px 0;"/> <p style="text-align: center;">$3 - 3 = 0$</p>										
<p>2:5</p>	<p>Use the <i>first..., then..., now...</i> card and counters, as described for the augmentation structure (see steps 1:5 and 1:6 above) to deepen children's understanding and strengthen the links they are making between the abstract representations and each part of the story.</p>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center; padding: 5px;">First</td> <td style="text-align: center; padding: 5px;">Then</td> <td style="text-align: center; padding: 5px;">Now</td> <td rowspan="3" style="text-align: center; vertical-align: middle;">  </td> </tr> <tr> <td style="text-align: center; height: 60px;"><div style="border: 1px solid black; width: 100%; height: 100%;"></div></td> <td style="text-align: center; height: 60px;"><div style="border: 1px solid black; width: 100%; height: 100%;"></div></td> <td style="text-align: center; height: 60px;"><div style="border: 1px solid black; width: 100%; height: 100%;"></div></td> </tr> <tr> <td style="text-align: center;"><div style="border: 1px solid black; width: 30px; height: 20px; margin: auto;"></div></td> <td style="text-align: center;"><div style="display: flex; align-items: center; justify-content: center;">- <div style="border: 1px solid black; width: 30px; height: 20px; margin: auto;"></div></div></td> <td style="text-align: center;"><div style="border: 1px solid black; width: 30px; height: 20px; margin: auto;"></div></td> </tr> </table> <hr style="width: 100%; border: 0; border-top: 1px solid black; margin: 5px 0;"/> <p style="text-align: center;"><div style="border: 1px solid black; width: 30px; height: 20px; margin: auto;"></div> - <div style="border: 1px solid black; width: 30px; height: 20px; margin: auto;"></div> = <div style="border: 1px solid black; width: 30px; height: 20px; margin: auto;"></div></p>	First	Then	Now		<div style="border: 1px solid black; width: 100%; height: 100%;"></div>	<div style="border: 1px solid black; width: 100%; height: 100%;"></div>	<div style="border: 1px solid black; width: 100%; height: 100%;"></div>	<div style="border: 1px solid black; width: 30px; height: 20px; margin: auto;"></div>	<div style="display: flex; align-items: center; justify-content: center;">- <div style="border: 1px solid black; width: 30px; height: 20px; margin: auto;"></div></div>	<div style="border: 1px solid black; width: 30px; height: 20px; margin: auto;"></div>
First	Then	Now										
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<p>2:6</p>	<p>Explore the importance of the order of the minuend and the subtrahend by giving children the opportunity to select the correctly ordered expression and justify their choice. There is no need to teach children that <i>'you must start with the biggest number'</i> or that subtraction is not commutative. At this stage, the order of the <i>first..., then..., now...</i> story will provide children with the language required to justify their selections.</p>	<p><i>First, Tom had four sweets. Then, Tom gave one sweet away. Now, Tom has three sweets.'</i></p> <p>First Then Now</p>  <p style="text-align: center;">$1 - 4$ or $4 - 1$?</p>										

<p>2:7</p>	<p>As with augmentation (see step 1:7 above), further representations can be used to expose the reduction structure; the stem sentence can be applied to the tens frame, bar model, bead string and number line.</p>	
<p>2:8</p>	<p>Now is a good time to revisit the subtraction symbol in the context of partitioning – i.e to express the evaluation of a missing part. The subtraction symbol was first introduced in segment 1.5 <i>Additive structures: introduction to aggregation and partitioning</i>, but at that stage some children may not have achieved mastery. Now that the subtraction symbol is fully understood in the context of reduction, consider some contexts that bridge the reduction and partitioning structures to reinforce the use of the subtraction symbol to represent the partitioning of a set. Examples include:</p> <ul style="list-style-type: none"> • bananas and eaten bananas (banana skins) • cups and broken cups <p>Using the example shown on the right you can think about the context from a reduction point of view, before moving to a partitioning point of view:</p> <ul style="list-style-type: none"> • Initially think about a 'first..., then..., now...' reduction story: <i>'First I had four cups, then I broke one and threw it away. How many cups do I have now?'</i> This can be represented by subtraction: $4 - 1 = 3$. • Then ask <i>'What happens if I don't throw the broken cup away?'</i> • Now think about the context in terms of partitioning: <i>'I have four cups. One of the cups is broken. How many cups are not broken?'</i> This too can be represented by subtraction: $4 - 1 = 3$. 	<p><i>'First I had four cups, then I broke one and threw it away. How many cups do I have now?'</i></p> <p>First Then Now</p>  <p style="text-align: center;">$4 - 1 = 3$</p> <p><i>'I have four cups. One of the cups is broken. How many cups are not broken?'</i></p> 

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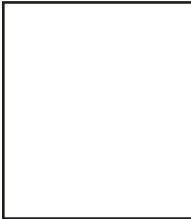
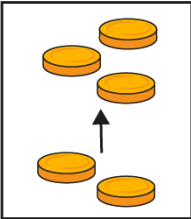
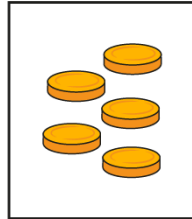
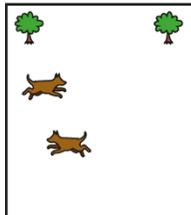

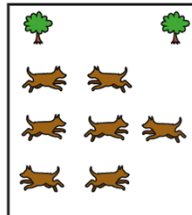
Once children are confident with both augmentation and reduction structures, provide them with further opportunities to deepen their understanding and strengthen the links they are making between the abstract representations and each part of the story. Use similar methods to those used in step 1:6 above, but now challenge children to select from both augmentation and reduction expressions and stories:

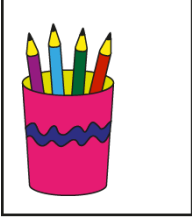
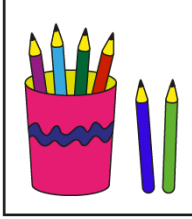

- Present children with a story. Can they find the corresponding expression or equation from a selection including both addition and subtraction? Can they justify their selection? Can they disprove others from the selection?
- In a similar way, present children with an equation. Can they find the corresponding story from a selection including representations of both augmentation and reduction? Can they justify their selection? Can they disprove others from the selection?


Teaching point 3:


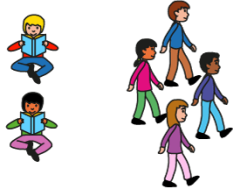





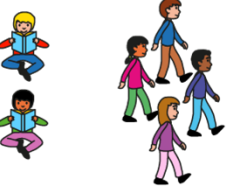

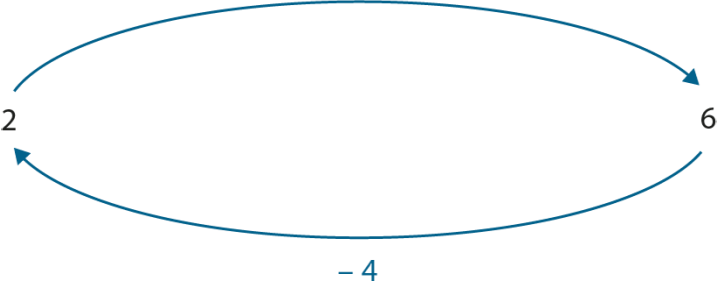
Given any two parts of the story we can work out the third part; given any two numbers in the equation we can find the third one.

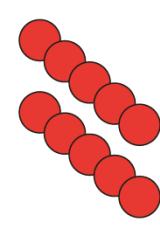
Steps in learning

	Guidance	Representations
<p>3:1</p>	<p>Once children understand the two structures, you can introduce 'missing number' problems to facilitate depth of understanding. Beginning with augmentation, present children with 'first..., then..., now...' story images that show only two parts, with one missing part. Represent the equation in the same way, with one missing term.</p> <ul style="list-style-type: none"> • Ask children to both draw the missing image and record the missing term in the equation. • Make sure that you vary which term is missing across a series of examples. • When the first term (augend) is missing, make sure that the 'now' drawing explicitly demonstrates the increase. 	<p>Missing augend:</p> <p><i>'We don't know how many coins Jamal had at first. Then, he got two more coins. Now, he has five coins. How many coins did Jamal have at first?'</i></p> <div style="display: flex; justify-content: space-around; text-align: center;"> <div data-bbox="758 712 949 996"> <p>First</p>  </div> <div data-bbox="1029 712 1220 996"> <p>Then</p>  </div> <div data-bbox="1284 712 1476 996"> <p>Now</p>  </div> </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;"> <div style="border: 1px solid black; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center; margin-right: 10px;">□</div> <div style="margin-right: 10px;">+ 2</div> <div style="margin-right: 10px;">→</div> <div style="margin-right: 10px;">5</div> </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;"> <div style="border: 1px solid black; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center; margin-right: 10px;">□</div> <div style="margin-right: 10px;">+ 2 = 5</div> </div> <p>Missing addend:</p> <p><i>'First, there were two dogs in the park. We don't know what happened then. Now, there are seven dogs in the park. How many more dogs came into the park?'</i></p> <div style="display: flex; justify-content: space-around; text-align: center;"> <div data-bbox="758 1433 949 1713"> <p>First</p>  </div> <div data-bbox="1029 1433 1220 1713"> <p>Then</p>  </div> <div data-bbox="1284 1433 1476 1713"> <p>Now</p>  </div> </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;"> <div style="margin-right: 10px;">2</div> <div style="margin-right: 10px;">+</div> <div style="border: 1px solid black; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center; margin-right: 10px;">□</div> <div style="margin-right: 10px;">→</div> <div style="margin-right: 10px;">7</div> </div> <div style="display: flex; justify-content: center; align-items: center; margin-top: 10px;"> <div style="margin-right: 10px;">2 +</div> <div style="border: 1px solid black; width: 30px; height: 30px; display: flex; align-items: center; justify-content: center; margin-right: 10px;">□</div> <div style="margin-right: 10px;">= 7</div> </div>

		<p>Missing sum: <i>'First, there were four pencils in the pot. Then, two more pencils were put in the pot. How many pencils are there now?'</i></p> <p style="text-align: center;">First Then Now</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>4</p> </div> <div style="text-align: center;">  <p>+ 2</p> </div> <div style="text-align: center;">  <p><input type="text"/></p> </div> </div> <div style="text-align: center; margin-top: 10px;"> $4 + 2 = \square$ </div>
<p>3:2</p>	<p>Work through the steps described for augmentation (see step 3:1 above), making the relevant changes to expose the reduction structure of subtraction. Example contexts include:</p> <ul style="list-style-type: none"> • <i>'We don't know how many cakes were on the plate at first. Then, two cakes were eaten. Now, there are five cakes on the plate. How many cakes were on the plate at first?'</i> • <i>'First, there were five skittles standing. Then, some skittles were knocked down. Now, there are two skittles standing. How many skittles were knocked down?'</i> • <i>'First there were three children in the pool. Then, two children got out of the pool. How many children are in the pool now?'</i> <p>The stem sentence will vary depending on which story part is missing; children must explain the processes taken to solve the problem.</p>	
<p>3:3</p>	<p>For both augmentation and reduction stories, consider how children will identify the missing images/values and how children should explain their</p>	

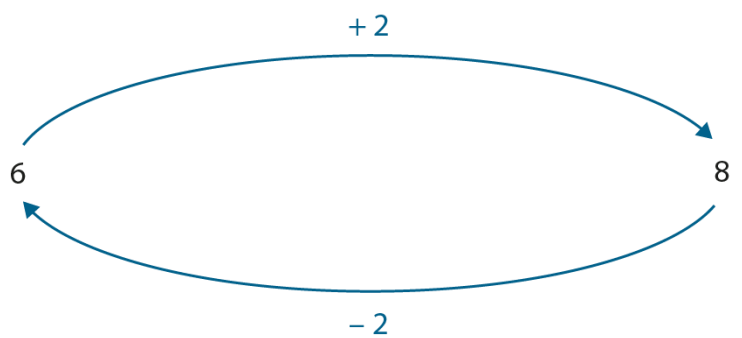
	<p>processes. For example, for a story corresponding to</p> $\square + 4 = 6$ <p>the inverse relationship means that the calculation $6 - 4 = 2$ will solve the problem. Likewise, for a story corresponding to</p> $\square - 2 = 5$ <p>the calculation $5 + 2 = 7$ will solve the problem.</p> <p>However, careful use of the pictures and the language related to the augmentation/reduction should remain the focus, and you should encourage children to use these to find the missing part. The inverse equation should not be introduced at this stage; the inverse relationship is exposed further in <i>Teaching point 4</i> below.</p>	
<p>3:4</p>	<p>To provide further challenge and promote depth of understanding, you can use a dòng não jīn problem: given only one part of the story, can children find a way to satisfy the missing parts? Can they find another way?</p>	<p>'Can you write a story which ends with six children on the bus?'</p> <p>First Then Now</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <div style="border: 1px solid black; width: 100px; height: 100px; margin: 0 auto;"></div> <div style="margin-top: 10px;"> \square </div> </div> <div style="text-align: center;"> <div style="border: 1px solid black; width: 100px; height: 100px; margin: 0 auto;"></div> <div style="margin-top: 10px;"> $+$ \square </div> </div> <div style="text-align: center;">  <div style="margin-top: 10px;"> \square </div> </div> </div> <div style="text-align: center; margin-top: 10px;"> $\square + \square = \square$ </div>

		<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>First</p>  </div> <div style="text-align: center;"> <p>Then</p>  </div> <div style="text-align: center;"> <p>Now</p>  </div> </div>
<p>4:3 Introduce abstract representations <i>alongside</i> the pictorial to support children in making a connection between the pictorial and abstract representations:</p> <ul style="list-style-type: none"> Record the numbers and symbols alongside the pictures as the story is told. Explicitly support children in connecting each number/symbol to each part of, and the language within, the story. Draw children's attention to the addend and subtrahend in the story context. <p>Again, the order of the expressions must match the story sequence; although $2 + 4 = 6$ could be recorded as $6 - 4 = 2$ or $6 - 2 = 4$, only one expression will match the story.</p>		<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>First</p>  <p>2</p> </div> <div style="text-align: center;"> <p>Then</p>  <p>+ 4</p> </div> <div style="text-align: center;"> <p>Now</p>  <p>6</p> </div> </div> <p style="text-align: center;">$2 + 4 = 6$</p> <hr style="width: 100%; border: 0.5px solid black; margin: 5px 0;"/> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>First</p>  <p>6</p> </div> <div style="text-align: center;"> <p>Then</p>  <p>- 4</p> </div> <div style="text-align: center;"> <p>Now</p>  <p>2</p> </div> </div> <p style="text-align: center;">$6 - 4 = 2$</p> <div style="text-align: center; margin-top: 20px;"> <p>+ 4</p>  </div>

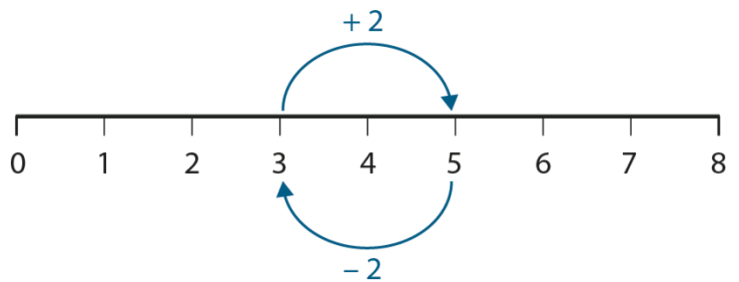
<p>4:4</p>	<p>As before, use the 'first..., then..., now...' cards and counters (see steps 1:5, 1:6 and 2:5 above) to deepen children's understanding.</p> <p>Now provide children with a <i>pair</i> of 'first..., then..., now...' cards and ten counters.</p> <ul style="list-style-type: none"> • Children can use the resource to tell and record their own augmentation–reduction and reduction–augmentation stories. • Present children with an augmentation–reduction and reduction–augmentation story. Can they find the corresponding expression or equation from a selection? Can they justify their selection? Can they disprove others from the selection? • In a similar way, present children with the equations. Can they find the corresponding story from a selection? Can they justify their selection? Can they disprove others from the selection? • Present children with a story: can they write the corresponding equations? • In a similar way, present children with the equations: can they draw the corresponding story? • Encourage children to continue using the stem sentence, in pairs: 'First..., then..., now...' 'First..., then..., now...' along with the number stories: <i>'Six plus two equals eight.'</i> <i>'Eight minus two equals six.'</i> 	<div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 10px; margin-right: 20px;"> <p style="text-align: center;">First Then Now</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 5px;"></div> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 5px;"></div> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 5px;"></div> </div> <div style="display: flex; justify-content: space-around; margin-top: 5px;"> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="font-size: 24px;">+</div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> </div> <hr style="border: 0.5px solid black; margin: 5px 0;"/> <div style="display: flex; justify-content: center; align-items: center; margin-top: 5px;"> <div style="border: 1px solid black; width: 20px; height: 20px; margin-right: 5px;"></div> <div style="font-size: 24px; margin-right: 5px;">+</div> <div style="border: 1px solid black; width: 20px; height: 20px; margin-right: 5px;"></div> <div style="font-size: 24px; margin-right: 5px;">=</div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> </div> </div> </div> <div style="border: 1px solid black; padding: 10px; margin-right: 20px;"> <p style="text-align: center;">First Then Now</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 5px;"></div> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 5px;"></div> <div style="border: 1px solid black; width: 60px; height: 60px; margin: 5px;"></div> </div> <div style="display: flex; justify-content: space-around; margin-top: 5px;"> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="font-size: 24px;">-</div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> </div> <hr style="border: 0.5px solid black; margin: 5px 0;"/> <div style="display: flex; justify-content: center; align-items: center; margin-top: 5px;"> <div style="border: 1px solid black; width: 20px; height: 20px; margin-right: 5px;"></div> <div style="font-size: 24px; margin-right: 5px;">-</div> <div style="border: 1px solid black; width: 20px; height: 20px; margin-right: 5px;"></div> <div style="font-size: 24px; margin-right: 5px;">=</div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> </div> </div> <div style="margin-left: 20px;">  </div>
<p>4:5</p>	<p>You can further highlight the inverse relationship using multilink or the number line.</p> <p>Repeatedly join and partition the multilink, accompanied by the stem sentence and number story, for</p>	<p>'First, there were six apples in the bowl. Then, two apples were added to the bowl. Now, there are eight apples in the bowl. Then, two apples were taken out of the bowl. Now, there are six apples in the bowl.'</p>

example, 'Six plus two equals eight and eight minus two equals six.'

Try using condensed story language: **'First..., then..., now..., then..., now...'**



'First, there were three marbles in the jar.
Then, two marbles were put into the jar.
Now, there are five marbles in the jar.
Then, two marbles were taken out of the jar.
Now, there are three marbles in the jar.'



4:6

Set the children some simple written work to consolidate their understanding of the inverse relationship between augmentation and reduction (addition and subtraction). Ensure you vary how the maths is represented to develop children's flexibility in thinking about the concepts.

