NCETM
NATIONAL CENTRE FOR EXCELLENCE
IN THE TEACHING OF MATHEMATICS

Additive Reasoning

This document is part of a set that forms the subject knowledge content audit for Key Stage 1 and Key Stage 2 maths. Each document contains: audit questions with tick boxes that you can select to show how confident you are (1 = not at all confident, 2 = not very confident, 3 = fairly confident, 4 = very confident), exemplifications; explanations; and further support links. At the end of each document, there is space to type notes to capture your learning and implications for practice. The document can then be saved for your records.

Question 6

How confident are you that you understand and can support children to use manipulatives when developing a written algorithm for addition?

1 ☐   2 ☐   3 ☐   4 ☐

How would you respond …?

a. What does the 1 represent in this calculation and where has it come from?

$$
\begin{array}{r}
2\ 5 \\
+\ 4\ 7 \\
\hline
7\ 2 \\
\end{array}
$$
₁

b. Can you sort these calculations into those that need regrouping once, twice or not at all?

$$
\begin{array}{r}
1\ 2\ 4 \\
+\ 2\ 3\ 3 \\
\hline
\end{array}
\qquad
\begin{array}{r}
6\ 4\ 4 \\
+\ 1\ 7\ 2 \\
\hline
\end{array}
\qquad
\begin{array}{r}
3\ 6\ 6 \\
+\ 2\ 7\ 7 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
5\ 7\ 9 \\
+\ 2\ 2\ 1 \\
\hline
\end{array}
\qquad
\begin{array}{r}
7\ 9\ 1 \\
+\ 1\ 6\ 3 \\
\hline
\end{array}
\qquad
\begin{array}{r}
5\ 6\ 7 \\
+\ 2\ 3\ 3 \\
\hline
\end{array}
$$

c. Sherrin says that the missing number is 1. Raya disagrees and says no number will work.
What errors have been made and how can manipulatives be used to support their understanding?

$$
\begin{array}{r}
3\ 2\ 2 \\
+\ 1\ 6\ \boxed{} \\
\hline
4\ 9\ 1 \\
\end{array}
$$

Responses

Note your responses to the questions here before you engage with the rest of this section:

Did you notice that…?

a. In part **a**, five ones and seven ones have been added to make 12 ones. This cannot be recorded so 12 ones need to be regrouped into one ten and two ones. The one ten is carried over to be considered in the next part of the calculation so needs to be recorded within the tens column. Some people may record the additional ten in different places but it should always be within the tens column.

b.
- The first calculation (124 + 233) does not require any regrouping.
- 644 + 172 requires regrouping the tens into one hundred.
- 366 + 277 requires regrouping twice; the ones will need regrouping and the tens will need regrouping.
- 579 + 221 will require regrouping twice as the additional group of ten made from regrouping the ones will mean the tens also need regrouping.
- 791 + 163 will require regrouping once as the tens will need regrouping.
- 567 + 233 requires regrouping twice as the additional group of ten made from regrouping the ones will mean the tens also need regrouping.

c. Children may initially look at the numbers and not take notice of the calculation. Sherrin has used a known number fact, 2 − 1 = 1 to fill in the missing number, not taking note of the operation.

Raya may only be thinking about number bonds within 10; she needs to calculate all single digits that could be bonded with 2, to find a sum with one 1. In this case, the missing number would be nine, as two ones and nine ones are 11 ones. For 1 to be in the ones place, the sum of the ones column has to be 11 ones, which can be regrouped into one 10 and one 1. This will ensure the sum of the tens is also correct, as the additional group of ten, added to the two tens and six tens, will make nine tens.

Using manipulatives to support understanding

There are a number of different manipulatives that expose the structure of written calculations and some of these will be discussed below, as well as why these deepen children's understanding.

Before beginning work on column addition, it is important to ensure that children have mastered writing calculations in the horizontal format and are confident in using a range of mental strategies, otherwise they can become reliant on written calculations, even when they are not the most efficient strategy to use.

Initially, when developing children's understanding of the alignment of the columns for calculation, arrow cards can be used. By showing the tens and ones separately, before sliding the ones over the tens card, children will be able to understand and describe what each digit represents.

- *'The three is in the ones column – it represents three ones; the five is in the ones column – it represents five ones.'*
- *'The four is in the tens column – it represents four tens; the two is in the tens column – it represents two tens.'*

Children should be confident partitioning numbers into tens and ones, as well as unitising language, which will help prepare them for the column layout. Laying out the addends correctly and describing what they represent, will ensure children have a firm foundation in their understanding of each column, rather than focusing on the digits within it. Opportunities to work with numbers containing a different number of digits, or where the larger number is written first, should be provided, to ensure misconceptions do not develop.

In column addition, the calculation starts to be worked out from the least significant digit, or the right-hand side. It is important that this is developed in preparation for regrouping.

When introducing regrouping in addition, for any column that sums to ten or greater, a manipulative with all the ones visible, such as straws, can be used. This is because the 10 straws become one bundle, rather than having to swap for one ten, such as when using a manipulative like Dienes. Children are then able to physically group the straws together into bundles, supporting their understanding of unitising.

For example, there are seven ones and four ones, which equal 11 ones, or one 10 and one 1. Children will then be able to see that this new group needs to be in line with, or in the same column as, the other bundles of ten and record this appropriately.

Other representations, such as place value counters, where the unit is not representative of the size of the manipulative, can also be used.

By writing the expanded version of the calculation for the image shown, children will have a clear understanding of what each digit represents in each column when the calculation is compacted.

$200 + 10 + 5 \rightarrow$ 215

$+ 300 + 40 + 1 \rightarrow + 341$
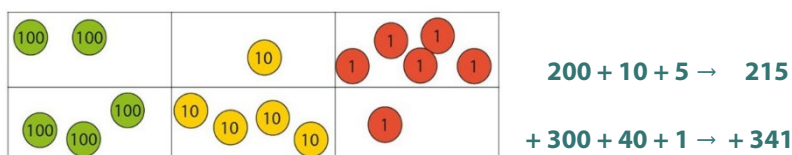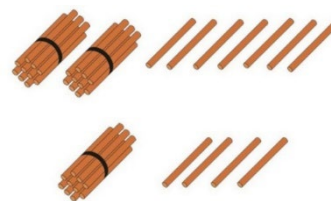
As with Dienes, children will follow the same steps when regrouping. For example, when the sum is greater than 10, they will group the ones counters and exchange for a tens counter.

Common errors in this area may include:

- children having underdeveloped fine motor skills leading to errors in the presentation of calculations
- having more than ten units in a column
- children not understanding where to write the digits when regrouping.

What to look for

Can a child:
- confidently lay out a column addition?
- use unitising language to describe the calculation?
- regroup when required?

Links to supporting materials:

NCETM Primary Professional Development materials, Spine 1: Number, Addition and Subtraction
- Topic 1.20: Algorithms: column addition

Notes:

Key learning from support material and self-study:

What I will focus on developing in my classroom practice: