MathsHUBS Resources

National Centre for Excellence in the Teaching of Mathematics

Mastery Professional Development Materials

6 Geometry

Theme overview

Guidance document | Key Stage 3

Making connections

'Teaching for mastery' describes the elements of classroom practice and school organisation that combine to give students the best chance of developing a deep, connected, embedded and sustainable understanding of mathematics.

At any one point in a student's journey through school, achieving mastery means acquiring a secure understanding of the mathematics that has been taught to enable them to move on to more advanced material.

To achieve this, students need to understand the interconnected nature of mathematics and how one idea builds on and develops from other ideas. To this end, the NCETM has identified a set of six 'mathematical themes' within Key Stage 3 mathematics that bring together a group of connected ideas or 'core concepts'.

The theme *Geometry* covers the following interconnected core concepts:

6.1 Geometrical properties

6.2 Perimeter, area and volume

6.3 Transforming shapes

6.4 Constructions

Please note that these materials are principally for professional development purposes. Unlike a textbook scheme they are not designed to be directly lifted and used as teaching materials. The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

Why is this mathematical theme important?

Geometry is a central part of mathematics. Geometrical thinking is a fundamental and often intuitive and accessible way to engage students in mathematical activity. The study of geometry helps to develop students' spatial awareness, intuition and visualisation. Encouraging students to draw diagrams and to see problems in a visual and geometrical form can help them gain insights and hence solve many types of problems. There are many applications of geometry relevant to employment and everyday life. Other subjects in the curriculum, such as science and technology, also make use of geometrical ideas and techniques. Geometry is an important area of the curriculum where students can engage in making conjectures, using logical chains of reasoning and mathematical proof.

Key underpinning knowledge

Several important considerations are key to students gaining a secure and deep understanding of the mathematics within this theme, namely:

- that knowing about and understanding the properties of various geometrical shapes and configurations is an important idea – simply knowing the names of shapes is not enough
- that various facts (for example, angle sum of a triangle) and formulae (for example, for areas and volumes of shapes, Pythagoras' theorem, etc.) are derived from properties and as such should not be learned blindly by rote but in relation to these properties in order to support deep and sustainable understanding
- that transformations allow students to explore the key mathematical ideas of variance and invariance, considering what's the same and what's different when looking at objects and their images
- that standard ruler and compass constructions are not a collection of actions to be learnt mechanically; they work because they are based on fundamental geometrical properties and should be taught and learnt in a way that relates to an understanding of these properties.

Statements of knowledge, skills and understanding

Each of the four core concepts within the theme *Geometry* has been broken down further into a set of statements of knowledge, skills and understanding, as listed below.

6.1 Geometrical properties

6.1.1 Understand and use angle properties

6.1.2 Understand and use similarity and congruence

6.1.3 Understand and use Pythagoras' theorem

6.2 Perimeter, area and volume

6.2.1 Understand the concept of perimeter and use it in a range of problem-solving situations

6.2.2 Understand the concept of area and use it in a range of problem-solving situations

6.2.3 Understand the concept of volume and use it in a range of problem-solving situations

6.3 Transforming shapes

6.3.1 Understand and use translations

6.3.2 Understand and use rotations

6.3.3 Understand and use reflections

6.3.4 Understand and use enlargements

6.4 Constructions

6.4.1 Use the properties of a circle in constructions

6.4.2 Use the properties of a rhombus in constructions

We have produced guidance documents that offer an overview of each core concept, as well as an overview of the content of each statement of knowledge, skills and understanding. We have also broken down each of the latter into a series of key ideas to support planning, with some of the key ideas exemplified as to what teaching for mastery may look like.

We make no suggestion that each key idea represents a lesson. Rather, the fine-grained distinctions we offer in these key ideas are intended to help you think about the learning journey irrespective of the number of lessons taught.

Not all key ideas are of equal weight and the amount of classroom time required for them to be mastered will vary, but each step is a noteworthy contribution to the statement of knowledge, skills and understanding with which it is associated.

These materials are designed for teachers to use collaboratively when planning how they will teach for a secure and deep understanding of mathematics throughout Key Stage 3. They are underpinned by a clear set of pedagogical principles and practices.

The *Geometry* core concept guidance documents[1] can be downloaded from the NCETM website.

Links to the national curriculum

A mapping[2] of all statements of knowledge skills and understanding to the national curriculum Key Stage 3 programme of study is available on the NCETM website.

Previous learning

From Upper Key Stage 2, students will bring experience of:

- drawing 2D shapes using given dimensions and angles
- recognising, describing and building simple 3D shapes, including making nets
- comparing and classifying geometric shapes based on their properties and sizes, and finding unknown angles in any triangles, quadrilaterals and regular polygons
- illustrating and naming parts of circles, including radius, diameter and circumference, and knowing that the diameter is twice the radius
- recognising angles where they meet at a point, are on a straight line or are vertically opposite, and finding missing angles
- drawing and translating simple shapes on the coordinate plane and reflecting them in the axes.

Future learning

In Key Stage 4, students will build on the core concepts in this mathematical theme to:

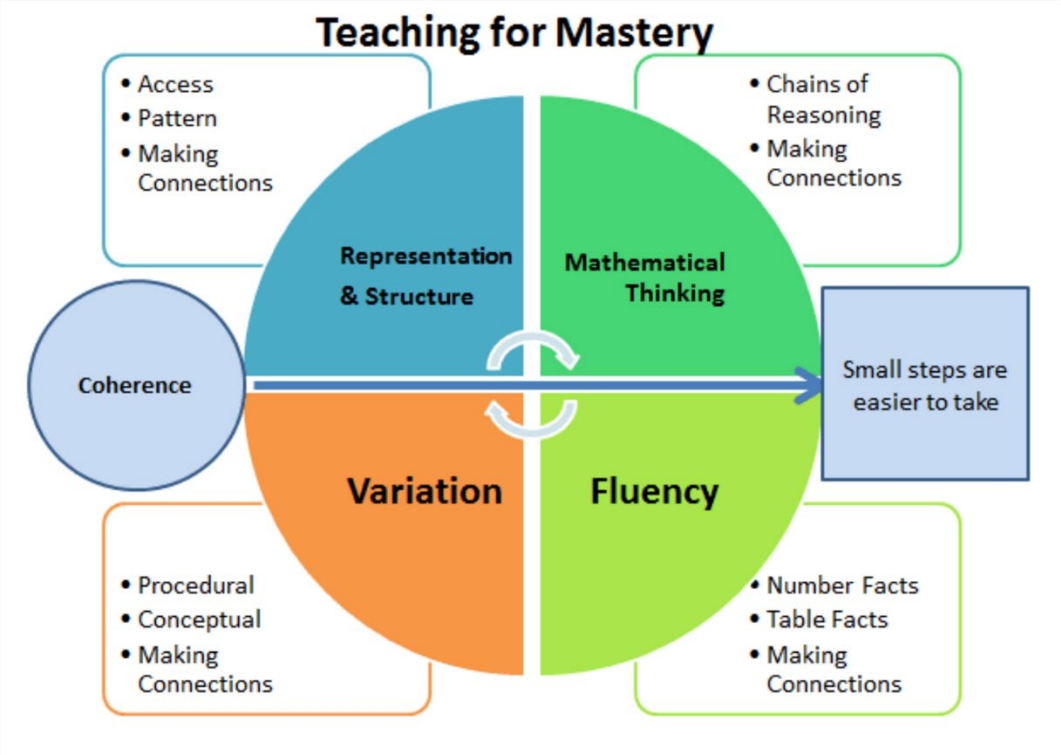
- interpret and use fractional {and negative} scale factors for enlargements
- {describe the changes and invariance achieved by combinations of rotations, reflections and translations}
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- {apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results}
- construct and interpret plans and elevations of 3D shapes
- interpret and use bearings
- calculate arc lengths, angles and areas of sectors of circles
- calculate surface areas and volumes of spheres, pyramids, cones and composite solids
- apply the concepts of congruence and similarity, including the relationships between lengths, {areas and volumes} in similar figures
- apply Pythagoras' theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two {and three} dimensional figures
- know the exact values of sin θ and cos θ for $\theta = 30°, 45°, 60°$ and 90°; know the exact value of tan θ for $\theta = 30°, 45°, 60°$
- {know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$, to find unknown lengths and angles}
- {know and apply Area = $\frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle}
- describe translations as 2D vectors
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}.

Please note: Braces { } indicate additional mathematical content to be taught to more highly attaining students.

Teaching for mastery

A central component in the NCETM/Maths Hubs programmes to support the development of teaching for mastery has been discussion of Five Big Ideas[3] underpinning teaching for mastery. These are:

- Coherence
- Representation and structure
- Variation
- Fluency
- Mathematical thinking

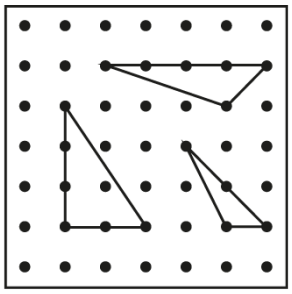
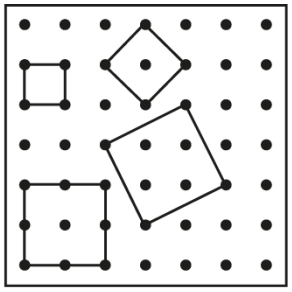
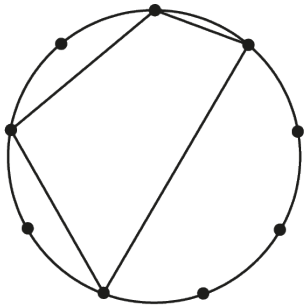


The sections below offer guidance about how these ideas relate to *Geometry*.

Coherence

It is important to find a balance between focusing in on important elements of this theme where it is useful to plan a coherent set of small steps (for example, when introducing areas of shapes, deciding to tackle parallelograms before triangles and trapezia) and an appreciation of how each idea is connected to others in the theme. For example, when teaching the standard ruler and compass constructions for constructing a perpendicular bisector of a line segment and for bisecting an angle, linking them both to the properties of a rhombus.

Representation and structure

Representations	Structural understanding
Geoboards	The use of both square and circular grid pin boards can support students' exploration of the properties of various shapes made by stretching elastic bands around the pins or by drawing lines on a virtual board.  Exploring how many different triangles can be made on a square grid can support students' understanding of different types of triangles (isosceles, equilateral, scalene, right-angled), their properties and the concepts of congruence and similarity.  Thinking about how many different squares can be made on a pin board, and calculating their areas, can provide an engaging introduction to Pythagoras' theorem.  Circular pin boards allow exploration of angles and angle properties within shapes.
Dynamic geometry software	The guided use of dynamic geometry software can provide an environment in which students are able to make and test conjectures. By dragging and changing features, they are able to identify variant and invariant properties, leading to a deeper understanding of generalisations.
Mental imagery	All concrete and pictorial representations have their limitations in the way they are able to represent the abstract world of mathematics. The ultimate purpose of all representations is to support students' mental construct of an idea or concept. Students do not use the representation to do the mathematics; rather, the representation supports the student in making a mental representation of the idea. It is therefore important (particularly in the area of geometry – although not exclusively) to engage students in activities which require them to generate and manipulate their own mental images.

	For example, imagining a triangle, fixing two of the vertices and exploring what happens when the other vertex moves, can help students appreciate that the angle sum is a fixed amount. You could ask, for example: • *'When the angle at this vertex is very small what happens to the other angles?'* • *'When the angle is very large (how large could it be?) what happens to the other angles?'*

Further guidance on using representations[4] in Key Stage 3 is available on the NCETM website.

Variation

Three aspects of variation that can be usefully employed:

1. Careful **choice of exercises** to 'home in' on the important concept. For example, when offering some exercises on constructing rotations, design them to draw attention to the importance of the centre of rotation by systematically varying this while keeping other aspects the same.
2. Careful **choice of examples** to include *'what it is'* (using non-standard as well as standard examples) and *'what it is not'*. For example, when offering examples of particular shapes, include non-standard examples of where a base is not always horizontal, a right-angle is not always in a bottom right or left corner, the hypotenuse is sometimes vertically or horizontally oriented, or where angles are not always acute but also include concave or re-entrant angles.
3. Rather than focusing on the answer and asking only that students solve a problem, inviting students to see **in how many different ways they can solve a problem** can prompt important discussions about methods and processes, and support students' development of increasingly efficient, creative and elegant approaches. For example, asking students to find the area of a composite shape in a number of different ways (such as by partitioning it in a number of different ways or by thinking about it as a larger shape with some elements removed).

Fluency

A key aspect of fluency in this context is the ability to recall and use efficiently the techniques for certain ruler and compass constructions. When students know and understand the geometrical principles on which these techniques depend, their retention and efficient use of these constructions will be much stronger.

Mathematical thinking

Throughout all the work that falls within *Geometry*, the emphasis is on understanding the properties of shapes and the over-arching principles on which these are based. It is vital throughout all of the work in this theme that students are prompted to reason, explain, conjecture and prove through carefully planned teacher–student and student–student discussion, and not merely to listen to and follow carefully constructed teacher demonstrations and explanations. For example, asking students how many different triangles they can find on a square geoboard can help them to think deeply about such connected ideas as similarity, congruence, reflection, rotation and area.

Further reading

NCETM secondary assessment materials[5]
Exemplar questions, tasks and activities, which may be used to support teaching and assessment. The assessment materials are mapped against the key mathematical skills and concepts within the national curriculum Key Stage 3 programme of study. Of particular relevance to *Geometry* are the sections

focusing on: area and perimeter (pages 35–37), geometrical properties (pages 38–40), angles and proof (pages 41–43) and transformations (pages 44–46).

Weblinks

1 Theme 6: *Geometry* – core concept guidance documents
https://www.ncetm.org.uk/resources/53535

2 NCETM Key Stage 3 mastery curriculum structure, including national curriculum mapping
https://www.ncetm.org.uk/secondarymasterypd#curriculum_structure

3 Five Big Ideas in teaching for mastery
https://www.ncetm.org.uk/resources/50042

4 Representations in Key Stage 3 – guidance documents
https://www.ncetm.org.uk/resources/53609

5 NCETM secondary assessment materials
https://www.ncetm.org.uk/resources/51246