Secondary Magazine Issue 59

National Centre for Excellence in the Teaching of Mathematics

Welcome to Issue 59 of the Secondary Magazine.

Contents

Contributors to Issue 59 include: Brian Butterworth, Sue Madgwick, Mary Pardoe, Peter Ransom and Heather Scott.

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Teacher Enquiry

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From the editor

What do we mean by 'teacher enquiry'?

Some of you may be undertaking, or be planning to undertake, an action research project on a particular theme, such as the ways in which you draw on the mathematical understanding that learners bring to your classroom, or how you encourage learners to take risks. You may have been inspired by summaries, in the Research Bulletins, of what other teachers have done, observed and concluded.

But every teacher can engage in enquiry about what they and their learners are doing from day to day in their lessons. In fact, it might be argued that an essential aspect of effective teaching is on-going enquiry into how we are teaching and how learning is resulting from what we are doing.

As a new editor of this magazine for a while, I hope that contributions to this magazine will reflect, and sometimes specifically focus on, our enquiries. I am hoping that some of you will write about your classroom experiences, such as how a learner surprised you, how a rich discussion developed, how you learned something from an incident, how you noticed a learner becoming more independent, how a lesson went an unexpected way, or about a problem and how you overcame it.

It would be good to focus on particular achievements; to read, for example, about one way in which you exposed a misconception, or created a connection between topics, or promoted mathematical thinking by asking a question.

What unusual or interesting examples have your learners created? Have your students come up with some conjectures or questions that they have not yet explored?

I remember a time when I believed that everything always went 'swimmingly' in every mathematics teachers' classroom except mine. But when I led my own department and we were frequently in and out of each others' lessons that idea soon evaporated! Although we can't all visit each others' lessons, we can 'tell' each other about them – we can describe, and learn from, things of all kinds that happen in our lessons.

It would be interesting to read about the different ways in which different teachers have used a particular resource, or introduced a particular topic. Or perhaps you might invite your students to write, or talk to you, about something that they discovered themselves, or something that they don't understand, or why they enjoyed, or didn't enjoy, a particular lesson.

Teacher enquiry and sharing experiences are at the heart of the NCETM. So please talk and write to us about what you and your students are doing.

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It's in the News!
Election

The fortnightly *It's in the News!* resources are explorations of mathematical ideas in topical contexts. Each set of materials is intended to provide a framework that you can adapt for your classroom and your learners, rather than a set of instructions to follow exactly.

A General Election was held on 6 May 2010. Elections generate an enormous amount of data that students can investigate. They are excellent opportunities for students to interpret data in an important and relevant context.

This activity looks at data from the 2010 election. In order to make sense of percentages derived from the data, students need to decide what 100% represents. They are prompted to calculate some alternative percentages, and to represent their data on graphs of their choosing.

Because this resource is not year group specific, you will need to read and possibly adapt it before you use it. For example you may choose to use the resource in a way that enables your learners to address key processes from the Key Stage 3 Programme of Study.

You will find more data from the 2010 election, together with questions and prompts, on the Secondary Forum..

- Download this *It's in the News!* Maths resource - in PowerPoint format

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The Interview

Name: Brian Butterworth

About you: I was Professor of Cognitive Neuropsychology at University College London from 1992 until last year, when I became Emeritus (unpaid). Most of my work at the moment is on a congenital condition called developmental dyscalculia, which is an inability to learn arithmetic. It's rather like dyslexia, but for numbers.

How I got to be doing this, is rather a long story. I started postgraduate work on formal languages and the foundations of mathematics, but moved into research on natural language, very natural language – hesitations in speech. There was a natural progression into disorders of speech and language following brain damage called 'aphasia'. Once you start going to neuropsychology clinics to study the cognitive effects of brain injury, you meet fascinating patients. Some of them seemed to have problems just in mathematics (acalculia), and I began to become more interested in these patients, and discovered that there is a brain network specialised for processing numbers, and if this is damaged then maths is impaired. I thought this might be a way to take an empirical approach to the foundations of mathematics.

What was a surprise was that some of these patients had very specific disabilities. Some were unable to count beyond four, others could do addition but not multiplication, others multiplication but not addition; some had superb calculation skills even though their language had almost completely disappeared. It turned out that the brain not only had a network for numerical processing, but sub-networks for distinct arithmetical tasks.

A natural next step was to find out why and how the brain becomes specialised in this way, and this meant looking at how children learned arithmetic – and in particular, at what kinds of specialised cognitive and neural capacities the child brings to learning arithmetic. This has led to the striking finding that a child can be born with a selective inability to learn arithmetic – not all maths, just numerical aspects of it – these are the dyscalculics. Recently, we have found inherited brain abnormalities in dyscalculic learners, and we have started to go gene hunting, beginning with genetic abnormalities that seem to target maths more than any other cognitive capacity.

Along the way, I have written a popular book about everything I'd learned about the origins of our numerical abilities, *The Mathematical Brain*; and I launched a journal for people like me, *Mathematical Cognition*, which the publishers, Psychology Press, closed down after five years without consulting me, and just as the whole area began to explode, just as we predicted.

One of my jobs now is to persuade government, teachers, parents, and learners, that being unable to learn arithmetic isn't necessarily a sign of stupidity, but it's rather the numerical equivalent of dyslexia and that specialized help is needed for these learners, just as it is for dyslexia. One of the outcomes of this work has been the creation of an inter-institutional *Centre for Educational Neuroscience*, that brings together neuroscientists, psychologists and educators from UCL, Birkbeck and the Institute of Education, to explore other areas where these disciplines can get together to make the learner's life better and happier.

The most recent use of mathematics in your job was… Endless statistics to analyse experimental results.

Some mathematics that amazed you is… I am amazed that non-human species can do mathematics. Chimps are better at remembering number sequences than we are. Newborn chicks can add and subtract. Even fish have a sense of number.

Why mathematics? Mathematics is one of the things that makes human beings what they are. We have inherited a capacity to do wonderful things with numbers. Stone-age cavemen (they didn't really live in caves except when the weather was awful) counted the waxing and waning of the moon. We have discovered that tribes whose languages contain no counting words, and whose culture contains no counting practices (for example, in Australia), nevertheless have the same numerical capacities as members of numerate societies.

A significant mathematics-related incident in your life was… Going through the proof of Gödel's theorems and finding out that my hero, Bertrand Russell, was wrong about the foundations of arithmetic.

A maths joke that makes you laugh is…
A mathematician is a machine for turning coffee into theorems (attributed to Erdős).

The best book you have ever read is…
I really loved G H Hardy's *A Mathematician's Apology*, and the Foreword by C P Snow.

Who inspired you?
Bertrand Russell, whom I never met, alas. Of those who I have met, Tim Shallice and Elizabeth Warrington got me interested in neuropsychology, and showed me that it was possible to test theories of normal cognition by studying extremely abnormal individuals.

If you weren't doing this job you would… Be an archaeologist. It's not too late!

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Focus on…short image sequences

"The mind does not spontaneously adopt a logical approach to the study of a subject but rather acts intuitively on the material presented to it." Caleb Gattegno

Being able to form, hold, modify, manipulate, change and think about our own personal mental images is profoundly useful in doing mathematics. It helps us explore situations and consider possibilities and implications.

How can we help our students cultivate their powers of imagery, explore the ambiguity of words and refine their abilities to describe their mental images?

We can use various resources and different kinds of strategy.

Inviting students to create mind-pictures

Challenge students to conjure up short sequences of mental images from verbal instructions, talk about their images, and possibly draw them. For example:

Picture two points.
Move them about.
Stop them moving.
Are they near to each other, or far apart?
Is one higher than the other?
Bring in a straight-line segment.
Place it somewhere between your two points.
Move it about until it passes through one of the points.
Can you move it so that it also passes through the other point?
What kind of movement?
If you can't make it pass through the other point, why can't you?

Students' images will be personal and idiosyncratic, and in a supportive atmosphere students usually enjoy sharing them.

This way of working with learners is described, with many examples, in 'Geometric Images', a handbook for teachers written by Dick Tahta, Roger Beeney, and others in 1982 for the ATM.

Instructions could be more specific. For example:

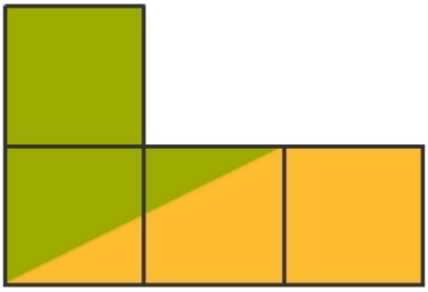
Visualise a square.
Join diagonally opposite corners with straight lines.
Join opposite mid-points of the sides.
Fit a circle inside the square so that it touches the sides of the square.
Join up the points where the circle cuts the diagonals of the square.
What shape have you made?
Join the mid-points of neighbouring sides.
What shape have you made?

How are the two shapes that you have made related to each other?
Join up all neighbouring corners of the two shapes that you have made.
What new shape have you made?

To facilitate more reflection and discussion about this situation, show students Square doodling, pausing it frequently and prompting thought by, for example, asking what they think will appear next. How do they know that the octagon is regular? How do they know that the smaller squares are congruent?

Visualising possibilities

Convince me that the area of this tetromino is halved by the cut:



Picture in your mind other ways of dividing the area in half with one cut.

Challenge students to visualise and describe possibilities before showing this animation, which you can pause and replay as many times as you like. *How would you cut other polyominoes?*

Can you visualise every possible pentomino?
Use this image sequence to confirm students' images and develop discussion.

What about hexominoes?

Does this image sequence generate every possible hexomino? What is its system?

Picture in your mind the 'growing' of a 7-point mystic rose. How many line segments will there be altogether? Invite students to talk about their ideas and their images, which they can confirm here.

Extend your ideas to a 10-point mystic rose. Use this animation to generate questions and confirm students' mind-pictures.

Imagine slicing through a cube.
How do you know that the shape that slicing in this way creates is a rectangle?

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What other two-dimensional shapes can you make by slicing in different ways through the cube?
Use this Animated Geometry film, designed by Gattegno, to prompt, encourage, support and confirm students' ideas.

Picturing paths

Imagine rolling along a line: a line segment, an equilateral triangle, a square, a regular hexagon, a circle. What path is traced by:

one end of the line segment,
one corner of the equilateral triangle,
one corner of the square,
one corner of the regular hexagon,
the point of the circle that is originally touching the line?

Watch this sequence and pause it after just a fraction of a second. How do students think the 'complete' paths will look? Eventually you can ask: Are the paths as you imagined they would be?

Re-creating together what you saw

Encourage students to 'replay' in their minds an image sequence that you all watched, and use their descriptions of their images to stimulate classroom discussion.

You could show this circle doodling sequence, then challenge students to 'replay it' in their minds, describing what they are 'seeing'.
What can you deduce about the centres and radii of the circles?

Invite students to describe what they 'see' when they replay in their minds this parallel lines sequence.

Show this sequence, pausing to let students discuss what they think will happen next. Can they explain how they might create a similar tessellation? What if they started with a triangle, or another shape?
Students might re-create aspects of what they see in Journey to the Center of a Triangle, which is a sequence developed in 1976 by Bruce and Katharine Cornwell. At the end of this stimulating five-minute sequence we see where the four centres go as the triangle changes shape! You can read Margaret Jones' reflections on using this film with a group of 15-year-old pupils in MT206.

Margaret compares working with a moving film to working with software tools.

Using a short image sequence as the starting point for exploration

Image sequences often suggest further explorations and prompt students naturally to look for explanations.

For example, the Triangles in polygons sequence created by the Swiss mathematics teacher Jean Nicolet leads naturally to asking oneself whether a right-angled triangle can be created in a similar way using other regular polygons. And why does it work? Might other polygons lead to other special kinds of triangle?

During the 1940s and 50s Jean Nicolet, Caleb Gattegno and Trevor Fletcher produced short, silent animated films. You can find information about a DVD version of 22 of Nicolet's films here.

A van shooten's theorem animation at the 'Maths Films' website also prompts students to explain what they see.

Using 'artistic' image sequences to encourage creativity, exploration and inquiry

Some image sequences are quite dazzling. For example, think what students might create with the stimulus of Notes sur un triangle by Réné Jodoin. This five-minute film is absolutely fabulous! – packed with arrangements that slide into each other. View it 'full-screen'! A printable sheet of equilateral triangles to use in explorations generated by watching this film is available from NRICH.

Dance Squared is another sequence of the same kind made in 1961 by Réné Jodoin with help from Trevor Fletcher.

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An idea for the classroom – weighty problems

Think of a balance situation!

You can ask a variety of questions and pose problems about weights on balances and so, with encouragement, can your learners. By thinking about what can happen when weights are put on the pans of a balance it is possible to create problems that are weighty also in the sense that they are rich – they may be approached in different ways, and modified or extended.

Here is the starting point for an investigation in which students can set their own rules, and then try to find out as much as they can under the rules that they have set:

Using a set of four weights, for example 1kg, 2kg, 3kg and 4kg, what different amounts of flour can you weigh for customers?

Students are likely to ask: *"Can we put the weights on both sides?"*

You might suggest – *"Why not try to see what happens with a 'weights on one side only' rule, and/or with a 'weights on both sides' rule – you decide what your rules are going to be!"*

Let students decide what weights they want to use.

Encourage them to ask their own questions.

> *What happens if I change my set of four weights?*
> *Can I now weigh more or less amounts?*
> *With what set of four weights can I weigh the greatest number of different amounts of flour?*
> *Does my strategy for finding the four weights that allow me to weigh the greatest number of different amounts still work if I am using five, six, … weights?*

Starting points for other explorations involving weight can be found in problems and activities such as the Swings and roundabouts interactive activity, this Weights problem, and this Number Balance problem, all from NRICH.

Valuing methods more than answers

A fascinating aspect of students solving problems is discovering how they started the problem and then how they found the solution. If students share their ideas with the whole class, and you value methods more than answers, they will get the idea that doing mathematics is partly about finding different ways of getting unstuck.

To build confidence while encouraging creativity, you might present a variety of problems, of different types and degrees of difficulty, and let your students choose just one to try to solve. For example, they might choose one of the following six problems, which are ideal for students to work on in pairs, discussing possibilities and bouncing ideas off each other.

We have nine weights that all look identical.
Eight weights weigh exactly the same.
One weight is slightly less than the other eight weights.
Using a balance scale how can you find out which is the lighter weight?

One brick is one kilogram and half a brick heavy. What is the weight of one brick?

A grocer had just received a 20kg bag of rice.
She had 10 customers that morning and each customer wanted to buy 2kg of rice.
Unfortunately she only had a 5kg and a 9kg weight to go with her balance scale.
How did she weigh out 2kg of rice?

A particular fish's tail weighs 9lbs. The head of the fish weighs as much as the tail + 1/3 of the weight of the body. The body of the fish is the same as the weight of the tail and the head. How much does the fish weigh?

Imagine that you find 10 sacks of gold coins arranged in a row. Only one of the sacks contains true gold coins; the coins in the other nine sacks look like pure gold coins, but actually they are gold plated, and almost worthless. You must identify the one sack of pure gold coins.
A pure gold coin weighs 2 ounces, while a gold-plated copper coin weighs only 1 ounce. You have a weighing scale, but you are allowed only one weighing. You might take a coin from bag number 7 and weigh it. If the scales show 2 ounces you've found the gold, but if it shows just 1 ounce, the gold coins are in one of the other nine sacks. How can you find the sack of gold coins with just one weighing?

A pumpkin grower has five pumpkins. When he weighs them two at a time, he gets the following results 110, 112, 113, 114, 115, 116, 117, 118, 120, and 121 pounds. What is the weight of each pumpkin?

You might find or devise a problem, present it at the end of a lesson, and then let the students think about it before talking about it during the next lesson.

This is Lewis Carroll's 'Monkey Puzzle', which is in *Mathematical Puzzles of Sam Loyd, Volume 2*:

"If to a rope, passed over a frictionless pulley, is suspended a 10-pound weight that exactly balances a monkey at the other end, what happens to the weight if the monkey attempts to climb the rope?"

Inviting students to contribute to a collection of objects can help them develop their understanding of weight.

For example, invite them to bring in, for a classroom display:

- items that weigh exactly 1g
- photographs of items that weigh 1 ton.

Or challenge them to find out about the history of weights, and weights in different cultures – posters showing their findings would also make a good display for the mathematics department.

5 things to do this fortnight

- Interesting new entries have been added to Mathemapedia. One of them is about using logo, an environment that provides brilliant opportunities for students to combine rigour with imagination. If your learners have recently been working with logo you might like to add your own Case Study. Also, have you experienced a 'Great Mistakes' incident that you could add to this new entry?

- If you live in or near London, a free London Mathematical Society Popular Lecture at the Institute of Education in the evening of Wednesday 30 June could show you how understanding knots has helped us to understand DNA better. You might get in the mood by watching this video about the mathematics of knots.

- Have you investigated the Further Mathematics Support Programme Online Revision Sessions? They are free to attend and in order to take part students need only their brains and a computer with an internet connection.

- If you are teaching mathematics, but don't have a qualification in mathematics, you might consider applying for the Mathematics Development Programme for Teachers (MDPT) course that starts at the Institute of Education in July.

- The birth date of the mathematician and meteorologist Edward Norton Lorenz is 23 May. The term 'butterfly effect' is attributed to him, and he was a pioneer of chaos theory. If you would like to introduce your students to this mathematician, you can read about his life and work, and learn something about (and listen to!) the Lorenz Attractor, which is named after him.

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Diary of a subject leader

Issues in the life of an anonymous Subject Leader

Wow! What a week it was at the end of last term. Many years ago, when I started teaching, I never imagined how stimulating my job would be. I enjoy the thrill of not knowing exactly what each day will bring.

The last week of term was only three days – 1 April was a disaggregated INSET day and as a faculty we had decided to do two 'twilights' instead of being in school with no students. Therefore, on Monday after school we started our faculty meeting as usual with 'Sharing good practice' – this is when we show and talk about some things we have come across that have inspired work in the classroom.

I decided to do something explosive that I had experienced at a Texas Instruments conference in Atlanta last month. (Yes, teach mathematics and travel the world – schools will let you out if you ask and can justify it. I can always find something in our Faculty or School Improvement Plan that will justify attending conferences.) The work links science and mathematics and looks at the surface area and volume of smaller and smaller pieces of Alka-Seltzer and the speed of their reaction with water. By adding the water into a small container with half a tablet in it (broken into 2, 4, 8 or smaller pieces) and then putting the lid on you can time how long it takes for the top to blow. If you build a large cube using small unit cubes, and note the volume and surface area; then break it into smaller cubes, noting the total volume and surface area of the cubes that you now have; then break up each of the smaller cubes in the same way; then do that again and again, you can graph the results using hand-held technology – which is far more portable than a suite of PCs. What the graph shows explains some phenomena in nanotechnology (a nanometre is a billionth of a metre, and nanotechnology is the study of matter on an atomic and molecular scale). Working through that took an hour and then we spent another 90 minutes discussing people's ideas for September relating to the new style GCSE – we have just been given another hour per fortnight with Year 11 and will be changing our KS4 scheme of work for September.

Tuesday started well – a request for someone to come and observe before starting their PGCE in September. No problem there – just needed to sort out a suitable date and give them some idea of our dress code. That makes two visitors for the first week of next term. It all helps us get used to having observers in the classroom. Then, the Assistant Head emails me regarding a meeting to go through the applications for jobs we have advertised – including an AST and two mathematics teachers, since the faculty is expanding and some staff are moving on. We agree to meet tomorrow afternoon. There's an incident with an awkward student to deal with who is disrupting a class, so I go to see what the matter is and remove the student from their audience to find out what it is about – gives everyone a bit of time to cool off. The day finishes and it's off to see our grandson for an hour, home for tea, then out to play bridge.

There's a reference to do when I get back, but it is just one that requests numbers for certain qualities, so that does not take long. However, that doesn't reflect the true worth of the teacher, so I add a few sentences to clarify.

Wednesday starts with our faculty briefing: 10 minutes to remind the team about deadlines for marking the Y10 exam, tell them about the visitors, and let them know about the faculty 'do' at my house the following week. We've had some babies born into faculty families this year, including one set of twins, so they will be coming also. Nobody can accuse the faculty of not being able to multiply! I sign letters to some parents whose children need to improve their statistics coursework over Easter, letting them know when I will be in school to help.

I meet with the Assistant Head to look at the application forms for the jobs – a good field with enough applicants worthy of having references taken up. Interestingly, only one person has mentioned that they are registered with the NCETM! Our entire faculty registered two years ago, so I wonder how many of them will read this diary.

Well, the day finishes and its sunny and the students are streaming home so I go out and help clear the site, then it's back to my room to thank each teacher for their sterling work this week and this term before scrolling through the emails that have built up in the last hour. I take some books out to the car knowing that they will probably not mark themselves this vacation, but one can always hope!