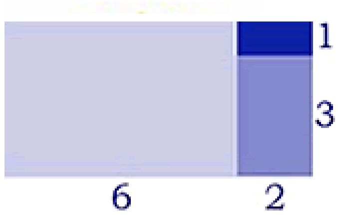
Secondary Magazine 130




National Centre for Excellence in the Teaching of Mathematics

## Welcome to Issue 130 of the Secondary and FE Magazine

Half term already: on the one hand, marvellous – it's a week's break; on the other – GCSE, AS and A levels are six weeks nearer! No doubt we're all starting to think about revision programmes and strategies to help our pupils and students maximise their exam performance so that they achieve the results that give them a broad range of choices next year: what do you do? What do you recommend? How do you ensure that the learners with whom you work continue to deepen their conceptual understanding alongside their determination to achieve procedural fluency? Let us know, by email to info@ncetm.org.uk or on Twitter @NCETM, and we'll share ideas in the near future.

## Contents



## Heads Up

If the new GCSE still feels, well, 'new' to you, you might want to take advantage of some of the abundant sources of free help and guidance that is out there on the Internet. Among many others that specifically target the new parts of GCSE Maths are Resourceaholic, Just Maths and Mr Barton Maths.

And if in a GCSE lesson you face that frequent question from a pupil, 'What's the point of doing quadratics?' you might like to use some of the arguments in this recent blog post from Anne Watson of Oxford University.

This might be a first for the Secondary Magazine, but we're going to suggest you tune your radio to Radio Three, where mathematician Marcus du Sautoy is just finishing a series The Secret Mathematician, in which he uncovers the maths within literature, art and music. All five episodes are still available via the BBC iPlayer.

There's a good deal of maths in an essay about risk, linked to the 30th anniversary of the Challenger space shuttle disaster, on the BBC website.

If you're fed up with fellow teachers banging on about what they've 'seen on Twitter,' perhaps it's time to adopt the tactic, 'if you can't beat 'em, join 'em.' This Beginner's guide to Twitter for teachers from Mark Anderson (aka @ICTevangelist) can ease you in gently.

**STOP PRESS:** Congratulations from all of us at the NCETM to Colin Hegarty, maths teacher at a London comprehensive, for being named in the top ten finalists for a global teaching prize, largely due to his phenomenally popular maths teaching website and Twitter account, @hegartymaths.

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# Building Bridges

*A qualitative study exploring learners' experiences with mathematics in a Further Education College by R.C.D. (Chinty) Pettitt*

I recently completed a qualitative study in a Further Education (FE) College exploring vocational learners' experiences with mathematics for my MA Ed dissertation. This study was supervised by Dr Nick Pratt of Plymouth University, Associate Professor (Senior) Lecturer in Education (IMP), Plymouth Institute of Education (Faculty of Arts and Humanities).

The aims of the study were:

1. To understand elements of vocational learners' experiences that increased or decreased their motivation to learn mathematics as they studied Functional Skills Mathematics.
2. To reflect on my practice and critically analyse the most effective ways to generate positive learning experiences in context.

Through experience, I have noted that many learners have difficulty with fractions, decimals and percentages and I also wanted to explore their experiences as they studied these topics. Participants in the study were given additional mathematics support either as 1:1 or in groups during their tutorial classes. They were asked to complete questionnaires, charts, and short interviews to get an insight into their experiences with mathematics in FE and school.

The findings indicate a general trend of increasingly negative experiences with mathematics from the early years of school to Year 3 and highly negative by Years 10 and 11. Comments ranged from *'Great'; 'Very good'; to 'Frustrated, Boring'; 'Not that good';* and *'Didn't like maths'*. Experiences with mathematics topics at school were generally positive with addition and subtraction, and to a lesser extent with multiplication and multiplication tables. However, confidence was less with fractions, decimals, percentages, area and perimeter, and word problems. The most negative comments were reserved for algebra and geometry, for example *'I am feeling stupid'; Struggle'* and *'Not getting the letters/irritating'*.

In FE, despite learners appearing extrinsically motivated to study maths alongside their vocational courses to progress, many still felt negative towards mathematics in September. They expressed concerns about attending compulsory mathematics sessions with comments such as *'Nervous but happy to improve knowledge'* and *'Don't understand difficult information which was complicated and adds pressure'*. Nevertheless, there were positive comments about attending compulsory college mathematics sessions, and over time some learners appeared to accept and adapt to their situations, and by October were attending additional Learning Support sessions or changed classes, although others continued to feel frustrated and unsupported.

During a group session in November, many learners had difficulty determining which denominator to use when adding fractions. Reflecting in and on action (Schön 1984), I realised that learners were also having difficulties with subtracting, dividing and multiplying, many not knowing their multiplication tables. In November, I was introduced to Singapore Maths (Hoople 2011) during a conference with the Cornwall and West Devon Maths Hub. I used Singapore Maths resources and mathematical tables to help learners develop their confidence when calculating with fractions. I also developed a 'personalised maths dictionary', with learners using their choice of coloured paper to record calculations and mathematical language; this was then used as a revision aid, linking colour to topic. This holistic approach to supporting learners at their pace and current level of competence in mathematics successfully increased confidence and motivation in most learners; for example one learner noted *'Doing homework and keeping notes really benefitted me'*, whilst another stated *'I feel more confident with fractions and percentages'* and later said, *'Feeling better, beginning to understand maths better than previously. I get the work and can do the questions'*.

If FE colleagues would like to read further about this study, my thesis will be at Plymouth University Library and I aim to publish further details in due course.

*In the meantime if there are specific questions, please email Chinty.*

**References**

Hoople, K. (2011) *Singapore Math Mental Math, Grade 4: Strategies and Process Skills to Develop Mental Calculation* Frank Schaffer Publications Ltd: Greensboro

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You can find previous *Building Bridges* features here.

**Image credit**



## Sixth Sense

Back in Issue 123, we advocated an approach to teaching mechanics that begins by introducing the key principles in one dimension; in this article we discuss how you can extend the ideas into two dimensions with your M1 students. The very significant benefit of this approach is that your students have already understood all of the mechanical concepts: components are fiddly to work out correctly, and if you try to teach them components from the start they can become bogged down in the trigonometrical arguments and totally miss the mechanics that you're trying to introduce. By separating these processes, students can "learn mechanics" first, and then later can apply the skills that they've been honing with their core teacher to solve more complicated mechanical problems.

So let's assume that your students have developed a reasonable amount of procedural fluency in using Newton's Laws to find unknown forces, and with the constant acceleration formulae to calculate further information given a set of initial conditions. It should be a small step for them to tackle the question:

> *An object moves with constant horizontal velocity 4ms$^{-1}$and constant vertical velocity 3ms$^{-1}$: how far has it travelled after 1 second? Where is it at this moment?*

You may prefer to use "*x* direction" and "*y* direction" or simply "to the right" and "upwards" depending on the language you've used to date, but whatever you say students tend to be relatively comfortable with the idea that "every second the object moves 4m to the right and 3m up, so that's 5m from its start point on a diagonal path (whose angle we can find using arctan 0.75)". In our experience, this comes so naturally to students who are, by now, very familiar with Pythagoras and trigonometry, that it barely feels like we've introduced a new idea here. In fact, we've set up the entire process that allows all of our previous work to extend into two dimensions.

Next, let's ask the reverse question:

> *An object travels with constant speed 8ms$^{-1}$ at an angle of 30$^{o}$ above the x axis. Find its "rightwards velocity" and its "upwards velocity".*

A quick sketch enables students to use more trigonometry to find the correct answers (and if you don't let them use a calculator, you can check that they remember their "trig special values" too!).

So, we're claiming that it seems entirely natural and untroubling to split velocities into components, and this is the key to the next few lessons on projectile motion. By the time students have mastered all of the techniques involved they will be quite happy to use the same ideas to split forces into components too.

Time for another question, and perhaps worth starting with something like:

> *In playing a shot, a cricketer hits the ball horizontally and to the right so that, at the moment of impact, it has horizontal velocity 7ms$^{-1}$. If this happens 44.1cm above the ground, how far away is it when it first bounces?*

It is sensible at this point to encourage students to get into the habit of always drawing a sketch of the motion; and, certainly, to summarise all of the information in the question:

| | Horizontal (to the right) | Vertical (downwards) |
|---|---|---|
| *s* | *x* | 44.1 |

| | | |
|---|---|---|
| *u* | 7 | 0 |
| *v* | | |
| *a* | 0 | 9.8 |
| *t* | *t* | *t* |

The conceptually difficult line of the table is the "*a*" line: why do we model there being no acceleration horizontally, but some acceleration (*g*, specifically) downwards? Your students will need to be confident with the Newtonian model that forces cause accelerations, and so modelling there being no force horizontally (modelling air resistance as negligible) means no acceleration in that direction. Their riposte may well be "but there is a force on the ball – it's been hit by the bat!", and so there will need to be discussion of the distinction between the *instantaneous* impulse that, at the moment of contact between bat and ball, changes the ball's velocity (momentum, to be precise) – both its direction and magnitude (speed) – and thereafter the absence of any additional or further force acting on the ball, other than gravity. A comparison with a rocket may be helpful.

Then we can discuss -

*What do we know? What do we want to know? Why is it not enough just to say "horizontal" and "vertical"? What's the link between the two columns?*

- and we're now in a position to apply the constant acceleration formulae to complete the question:

*Vertically:* $s = ut + \frac{1}{2}at^2 \Rightarrow 44.1 = 0 + 4.9t^2 \Rightarrow t = 3$

*Horizontally:* *same formula* $\Rightarrow x = 7t = 21m$

A few practice questions like this to ensure that the tabular structure is in place (but keep asking them about the values in the "*a*" row) should prepare students for the next step:

*A ball is kicked off the ground. It is given an initial velocity of 9ms$^{-1}$ at an angle of 20°. For how long is it airborne? How far from the kicker does it land?*

We can head straight for another table, but it makes sense here to think "upwards" rather than "downwards" [though either will work, if handled correctly!]:

| | Horizontal (to the right) | Vertical (upwards) |
|---|---|---|
| *s* | *x* | 0 |
| *u* | 9cos20° | 9sin20° |
| *v* | | |
| *a* | 0 | -9.8 |
| *t* | *t* | *t* |

There's often a conversation here about the "*v*" row: some students are quick to suggest that when the ball lands, either or both of these should be 0. Have your response prepared: we're modelling the motion in flight with only gravity acting on the ball and our model doesn't know about the ground. In the instant before hitting the ground, the ball must have velocity as per our model. As soon as the ball hits the ground our model is no longer valid as we have to model a new force acting on the ball (the instantaneous impulse of the ball-ground contact force) and the ball will start to behave differently. Using the same formula as before (it gets used a lot in projectile motion – make sure your students are factorisation / quadratic formula experts!):

*Vertically:* $0 = 3.07818t - 4.9t^2$ *so* $t = 0$ *or* $t = 0.628$ *(3sf)*

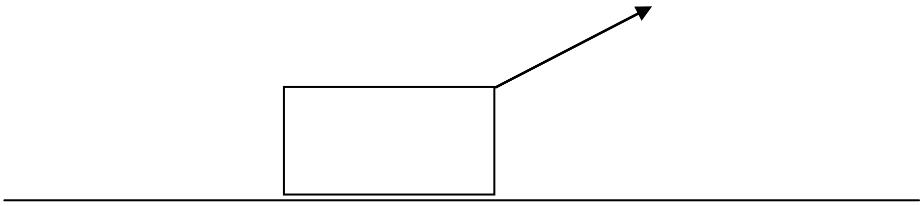
*Horizontally:* $t = 0$ *is the start of the motion, so we want to use the other value to find* $x = 9\cos 20 \times$ *answer still on calculator* $= 8.46m$

The other "classic" question is about the maximum height achieved and it is worth thinking about what this means: again, students often find it difficult at first to comprehend that their "vertical $v$" must be 0 for an instant – but once they have thought about $v$ as a positive quantity on the way up, and a negative quantity on the way down, they can be convinced that $v$ must be 0 *somewhere*, and that that place must be where the maximum height is reached.

There are several lessons to spend here developing fluency, and procedural variation presents itself in some of the more challenging set-ups:

- *For how long is the object more than 1m above ground?*
- *Does the object clear a wall that is 4m tall and 6m away?*
- *An object is projected from 5m above ground, where does it land?*
- *Two objects are projected towards each other, do they collide?*

Having done all of this work on projectiles, the next time you discuss forces, the students shouldn't (!) bat an eyelid when you draw a piece of rope pulling an object at an angle:



It will seem only natural, given all of our work with angles in projectiles, to suggest that the force can be split into components and from this that unknown forces and accelerations can be calculated using Newton's 2nd Law both horizontally and vertically.

There's very little teaching to be done now: if you've done all the work in one dimension, students just need lots and lots of interesting problems to solve – blocks on slopes, trains pulling trucks up hills – to ensure that they are entirely confident with the component process; further mathematicians can also revisit impulse, work done and impact at this point. Once they have had practice revisiting all the one-dimensional contexts but now with two-dimensional information, then they can look at more abstract questions such as "here is a set of forces, what's the magnitude and direction of the resultant force?" – which is leading them to think about vectors.

You can find previous *Sixth Sense* features here.

## From the Library


**The Approximate History of Prime Numbers**

DURATION: 03:09

In January it was announced that the new largest-so-far prime number had been discovered: a number with some 22 million digits. Like many of the previous discoveries of large primes, this was a so-called Mersenne prime, of the form $2^n - 1$. But why are people looking for big primes, and who is this Mersenne that these special primes are named after? Which ones have been discovered so far and how many more primes there are to search for? Has no one yet found a formula to generate them all?

As Maths teachers, do we have the knowledge to answer these questions confidently? How important do we feel it is that we can discuss these issues in our classes? In particular, what are the arguments for the use of the history of Mathematics in classroom teaching, and what does the research say about its benefits?

John Fauvel (1991) collated a number of reasons that have been advanced for using the history of mathematics, including:

- Helps to increase motivation for learning
- Gives mathematics a human face
- Historical development helps to order the presentation of topics in the curriculum
- Changes pupils' perceptions of mathematics
- Helps to develop a multicultural approach
- Provides opportunities for investigations
- Past obstacles to development help to explain what today's pupils find hard
- Pupils derive comfort from realizing that they are not the only ones with problems
- Helps to explain the role of mathematics in society
- Provides opportunity for cross-curricular work with other teachers or subjects

Kaye (2008) identified that while "a number of benefits to the integration of a historical dimension into mathematics education" had often been suggested, "there is little research into the effectiveness of such an approach". She investigated a project where students were studying Babylonian mathematics. The conclusions of the research suggested that the majority of students "were able to appreciate that mathematics has developed over millennia and that there are culturally different but equally valid ways of doing mathematics" and "they were able to appreciate a human element to mathematics." However there was "little evidence…that students came to see mathematics as a creative discipline or appreciated that they could make their own mathematics."

Georgiou (2010) researched her own teaching of a much wider range of topics, covering both mathematics history and culture. She taught pupils from Years 7 to 10 with a range of prior attainment. The material included Egyptian numerals, the Babylonian Plimpton tablet, texts on Eratosthenes and Al-Khwarizmi, as well as the examination of the economics of fair-trade coffee. She found that students reacted intensely to the variety of written texts, with negative facial expressions to some texts and comments such as "This is English!", but with enthusiasm for a SMILE activity resembling a comic-book page. The work on number systems drew out mathematical misconceptions as well as demonstrating quite fixed views when comparing other notations, insisting that "ours are numbers, theirs are shapes". While the researcher emphasised the provisional nature of the findings, there were indications that positive and negative reactions were spread across the groups and that "what many students seemed to appreciate in their comments was the variety in the approach." However, there were also significant challenges for the teacher, such as incidents of feeling exposed when discussion led into unfamiliar territory despite extensive planning: "the challenges a teacher may face with such an approach are either related to a tiring and sometimes daunting preparation or some unexpected themes emerging during the lesson."

Given the challenge of involving historical elements into the lesson, are some teachers more predisposed to this approach than others? Goodwin *et al* (2014) analysed questionnaire responses from 4663 teachers in the USA. They concluded that "[t]eachers with high history scores were more likely to believe that investigating is more important than knowing facts and that mathematics is ongoing and shows cultural differences. On the other hand, teachers with low history scores were more likely to believe that mathematics is a disjointed collection of facts, rules and skills."

Understanding how mathematical ideas developed through human history is recognised as influential in the work of Piaget. "Piagetian psychology called for historical analysis of mathematical … concepts" assuming "that the individual's development replays that of the species" (Lerman 2000). Rogers takes issue with this and argues that "what we learn from the history of mathematics is that the richness, complexity and variety of human endeavour" is such that "differences in cultural contexts give rise to what are essentially different cognitive structures when it comes to handling number, magnitude and space" (Rogers 1997).

A report on the history of mathematics in the Higher Education curriculum argues that "[s]etting historical context can motivate and enthuse learning, but it also enriches the curriculum, shows connections between different branches of the subject, and helps to produce students with a greater sense of the breadth and, what might be termed, the creative life of mathematics as a discipline" (McCartney, 2012). What is your view on using the history of mathematics in the classroom? How do you incorporate history and how have the students reacted? Let us know your opinions and experiences: email info@ncetm.org.uk, or tweet us @NCETM.

**Resources**

There is a wealth of resources. As well as for their quality, the following are selected for their availability and their practicality.

A new competition invites young people aged 11 to 19 to explore the history of maths for a chance to win cash prizes. Launched jointly by Plus Magazine and the British Society for the History of Mathematics. The deadline for entries is Thursday 24 March 2016.

Babylonian Mathematics – a programme of study including worksheets and videos, featuring Eleanor Robson, a leading Near East archaeologist and co-author of books on the history of mathematics.

The Approximate History of Maths – a collection of animation clips from the BBC programme.

The Story of Maths with Marcus du Sautoy – a collection of clips from the BBC series of 2008.

In a video from Teachers TV, Matthew Tosh presents a half hour introduction to the history of Maths.

Maths is good for you! "History of mathematics for young mathematicians."

Searching the NRICH website with "History of mathematics" brings up a wide range of pages, from problems to historical articles.

In the 1990s Nuffield provided a module on the History of Mathematics as part of their A Level Mathematics course. The textbook, challenging and fascinating, is available online, hosted by the National STEM centre.

'Bite-Sized' History of Mathematics Resources for use in the teaching of mathematics

The extensive MacTutor History of Mathematics archive contains a multitude of biographies as well as articles on historical topics.

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You can find previous *From the Library* features here.

Secondary Magazine 130

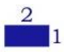
National Centre for Excellence in the Teaching of Mathematics

# It Stands to Reason

In this article we explore a set of activities through which pupils can deepen their conceptual understanding of, and increase the fluency of their reasoning about, **similarity**. Similarity is one of the five key geometrical ideas (the others are symmetry, invariance, transformation and congruence) that should be used to support pupils to build sound spatial and geometrical reasoning skills (ref. *Key Ideas in Teaching Mathematics*, Anne Watson, Keith Jones, Dave Pratt). As Tony Gardiner explains in *Teaching Mathematics at Secondary Level*, deductive reasoning in geometry is based on three organizing principles: the congruence criterion, the parallel criterion and the ***similarity criterion***, the latter of which allows pupils to deal with ratios, scaling and enlargement. It is best taught after the basic consequences of congruence and parallelism have been explored, and once pupils are confident when working with ratios and reasoning multiplicatively.

**Example 1**

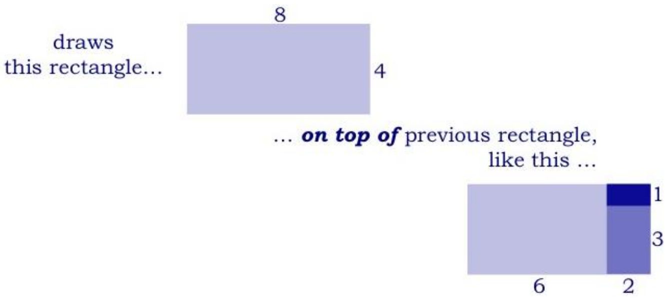
A pupil draws a 2-by-1 rectangle …



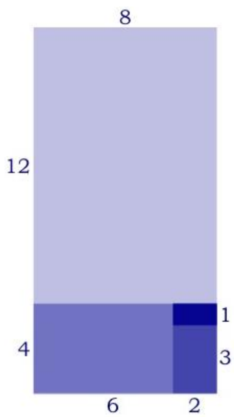

… and 'grows' it, stage by stage, in the following way …

*(Stage 1)*

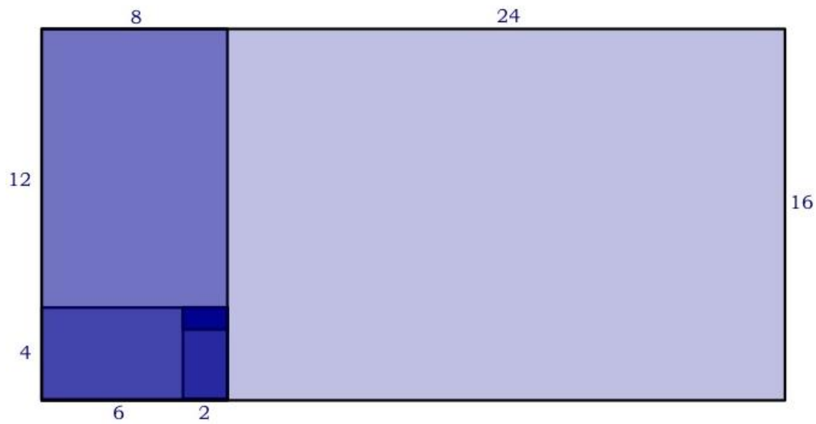

*(Stage 2)*




*(Stage 3)*

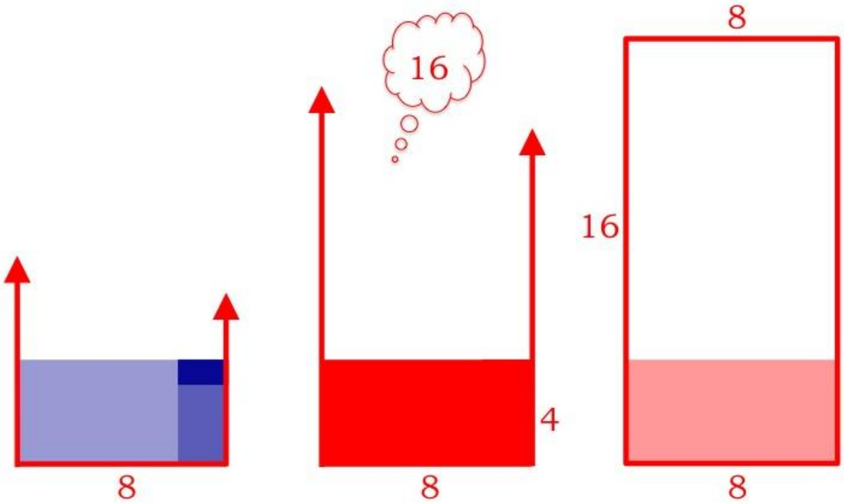


*(Stage 4)*



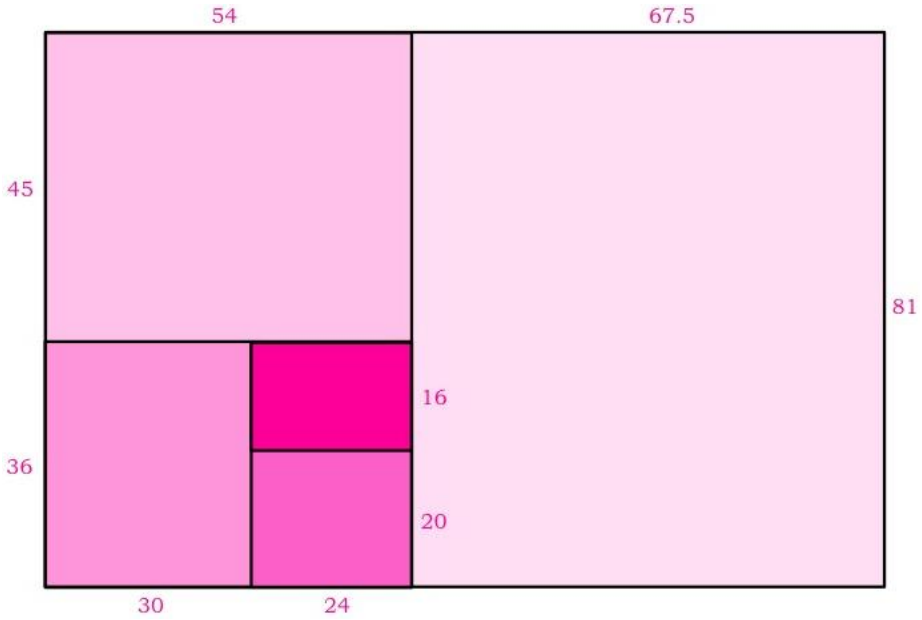

…and so on …

To proceed from one stage to the next, a new rectangle is created from the current stage, drawn such that the shorter side of the "new rectangle" is the same length as the longer side of the current whole rectangle, and the side-lengths of the new rectangle are in the ratio 1 : 2. For example, at Stage 2 above the longer side-length of the current whole rectangle is 8. To create the Stage 3 diagram the pupil now visualizes one of those two sides of length 8 as the shorter side of the next rectangle that she is about to create.  Because she wants the side-lengths to be in the ratio 1 : 2, while she is drawing the longer sides she has in mind that she has to keep going until they are of length '16' (2 × 8). She draws the longer sides perpendicular to the side of length 8 in the direction that will make the new rectangle 'cover', or include, the whole existing rectangle …

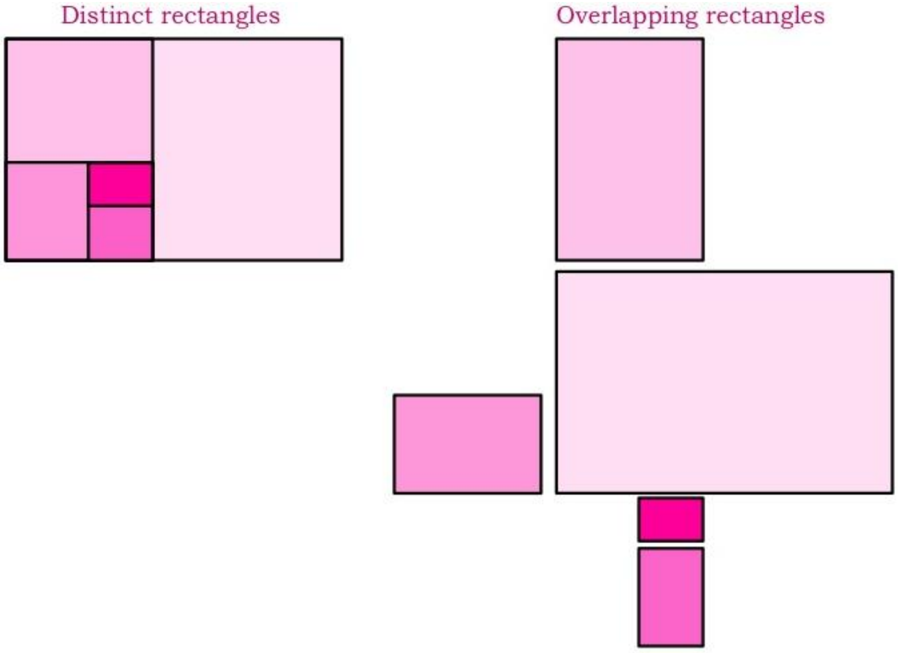



The selection, out of the two possibilities, of the longer side of the previous whole rectangle to become the shorter side of the new rectangle (and the direction of the 'extension') at each stage proceeds clockwise. Pupils will probably realize that at each stage they can create the whole new rectangle by merely appending ('sticking-on') a new rectangle to the other longer side of the current whole rectangle. It is hoped that pupils will also notice for themselves that the ratio of the side-lengths of every appended rectangle is always the same, although not this time 1 : 2. It is important that pupils both see, and endeavour to explain, the following: how does a rectangle with sides in the ratio 1 : 2 plus a rectangle with sides in the ratio 2 : 3 combine to make another rectangle with sides in the ratio 1 : 2?

This recursive process is fairly straightforward for pupils to carry out if the ratio 1 : 2 is changed to any other ratio 1 : n, where n is a whole number, but becomes more tricky with a ratio such as 2 : 3, e.g.




Whatever ratio is chosen, and whichever stage you stop at, the figure is always an interesting combination of overlapping rectangles that are all similar to the outside rectangle, and distinct rectangles that are again similar to each other, and your pupils can reason about the relationship between the two ratios.

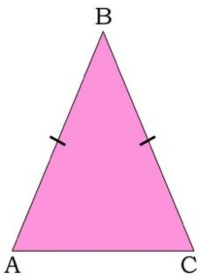



In devising, and then carrying out, this process, the pupil is experimenting with a ***recursive rule*** and the idea of ***'same shape'***. The pupil is using the spatial aspects of geometry (spatial thinking and visualization) which, although distinct from, are very closely entwined with the deductive aspects of geometry, in particular deductive reasoning from geometrical axioms. Pupils need to be clear about when they are experimenting and conjecturing, and when they are working deductively. The recursive rules explored in this article lead to pupils creating mathematically ***similar*** shapes, which then provide contexts and opportunities for both spatial thinking and deductive reasoning. Whether the recursive rules are given to them or made-up by them, your pupils will encounter and use, again and again, the ***similarity criterion***, which they should become confident to express conventionally as two polygons, ABCD… and $A'B'C'D'$…. are **similar** if

- corresponding ***angles*** are ***equal***, $\angle A = \angle A'$, $\angle B = \angle B'$ etc,
- and corresponding ***sides*** are ***proportional***, $AB : A'B' = BC : B'C'$ etc.

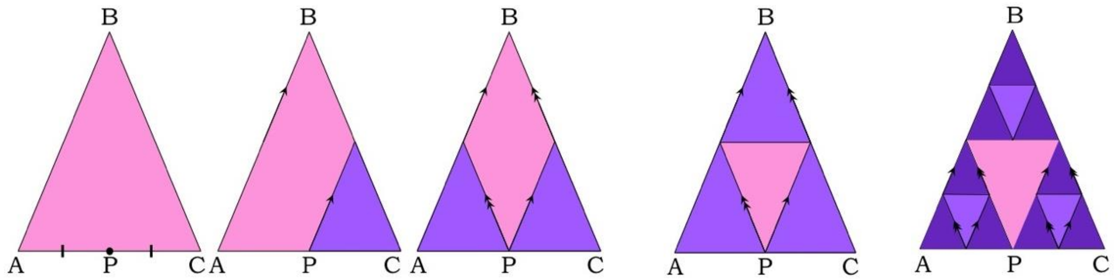
The following examples show how the spatial thinking involved in creating the images goes 'hand-in-hand' with deductive reasoning: engaging successfully in one supports successful engagement in the other.

**Example 2**
*Starting point:* an isosceles triangle.

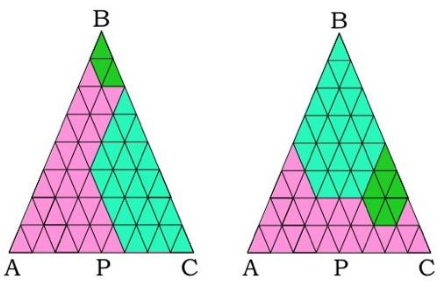



*Recursive procedure:* from the centre-point of the side between the equal angles of any existing isosceles triangle draw line segments parallel to the other two sides to meet the other two sides at points that you then join with another line segment.

The next five images (from left to right) show the first stage, and part of the second stage, of this procedure.

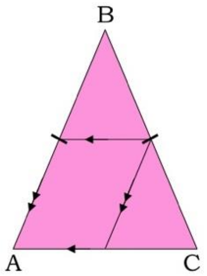



Pupils can decide themselves how many times they will apply the procedure. Invite them to describe relationships between shapes in their image – for example they may pick out 'same shapes' that are not the same size (similar, non-congruent, shapes). They should realize for themselves that labelling points makes it easier to identify particular shapes - although they could also use colours, for example

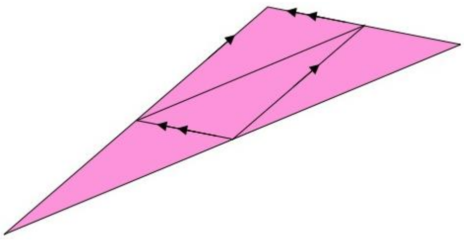



It is important to challenge pupils to use mathematical facts that they know to justify their assertions that particular shapes are the 'same shape'. As usual, encourage discussion and constructive challenge: does pupil B agree with pupil A's assertion? Can pupil C refine and improve pupil A's justification. Does pupil D think that A, B or C has given the clearest reasoning? And so on.

What happens if the lines parallel to sides are drawn from the midpoint of one of the equal sides? Why?

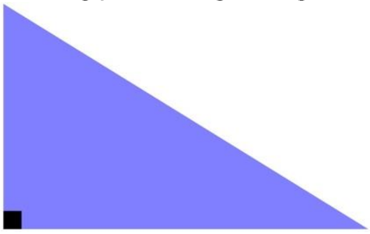



Does the same procedure when applied to ANY (scalene) triangle produce similar shapes?

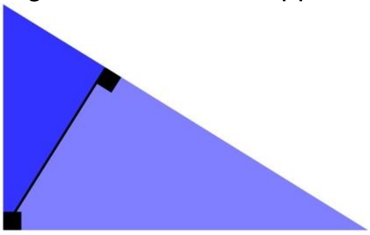


This question provides many opportunities for pupils to reason deductively. Pupils who struggle to create sufficiently accurate diagrams could work on triangle dotty paper.

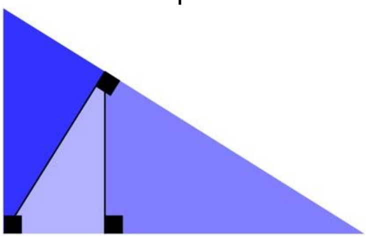
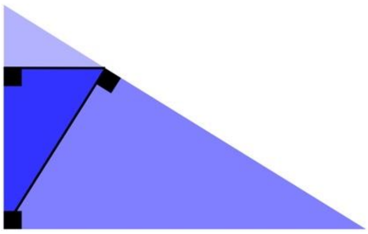
**Example 3**
*Starting point:* a right-angled triangle



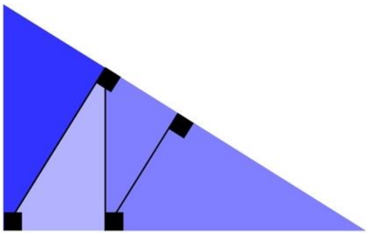
*Recursive procedure:* from the right angle of a right-angled triangle in the image draw a perpendicular line segment to meet the opposite side of that triangle

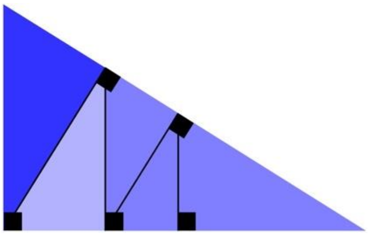


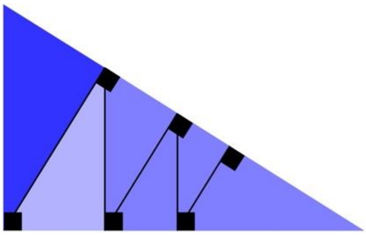
There are two possible second stages …

If the left-hand option is taken, a pupil could continue to create an interesting image in this way …







… but there are many other equally effective systematic ways to continue!

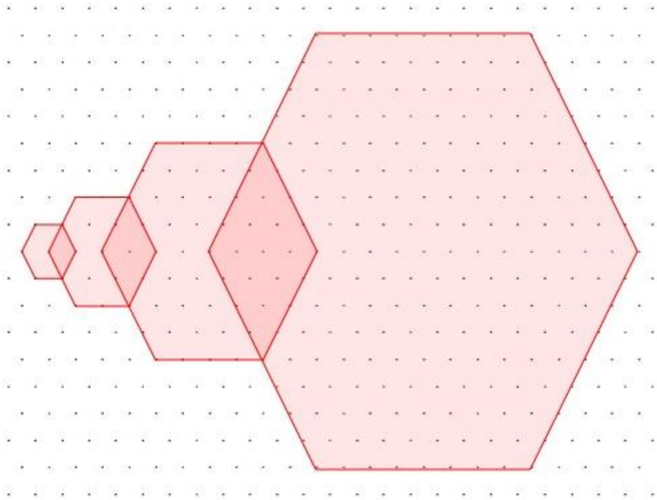
There are lots of opportunities, in any of the pleasing images that can be created, for pupils to use reasoning, in particular to prove that ALL the triangles are similar!

In examples 2 and 3 the application of the recursive rule repeatedly cuts-up shapes to create smaller and smaller similar shapes: the whole image stays the same size. In the next three examples, as in example 1, the whole image grows, as larger and larger similar shapes are created. In both kinds of example continuation of the process for ever – 'to infinity' – can be imagined and discussed by your pupils. The next two examples are relatively easy to create because they can be drawn on a triangle-dotty grid.

**Example 4**

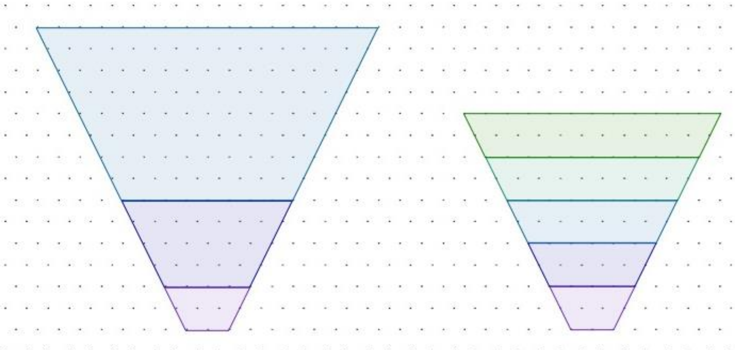
*Starting point:* a regular hexagon

The *recursive procedure* is simple to follow and describe – it is the kind of recursive rule that pupils might think of for themselves.

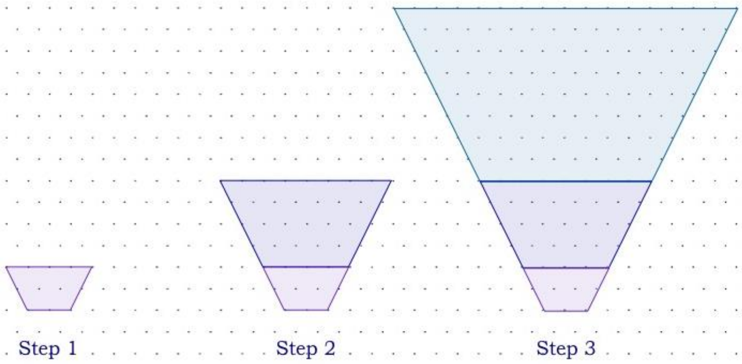


**Example 5**

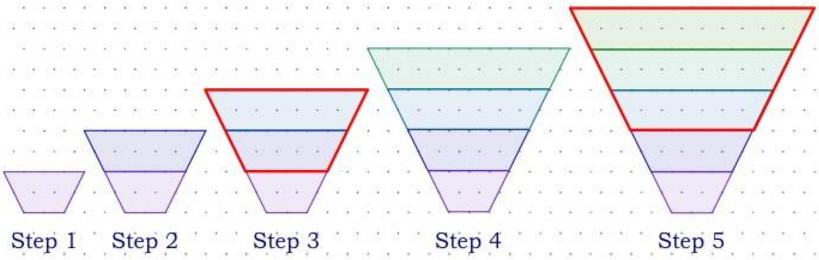
*Starting point:* an isosceles trapezium



The image on the left shows the effect of using a recursive procedure that produces a separate new similar trapezium at each step.



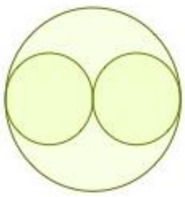

The image on the right, however, shows the kind of recursive procedure that pupils may think will create a new separate similar shape at each stage, but actually doesn't – it produces overlapping similar shapes, but not at every step. (Shapes similar to the starting trapezium shown in Step 1 are outlined in red.)




This kind of 'mistaken prediction' is a rich subject for discussion and learning: asking why the separate trapezia in the image on the right are not similar will prompt lots of worthwhile debate. If pupils use rules that generate shapes that aren't similar, they will get a much deeper understanding than if they only use rules that do: seeing "what it's not as well as what it is" is one of the key principles of conceptual variation.

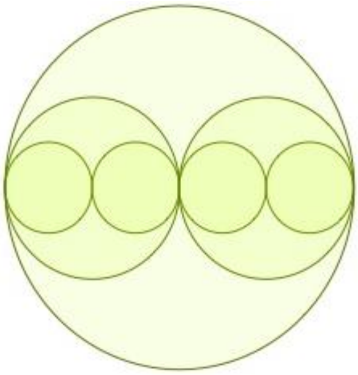
**Example 6**

*Starting point:* two touching identical circles fitting exactly in another circle …

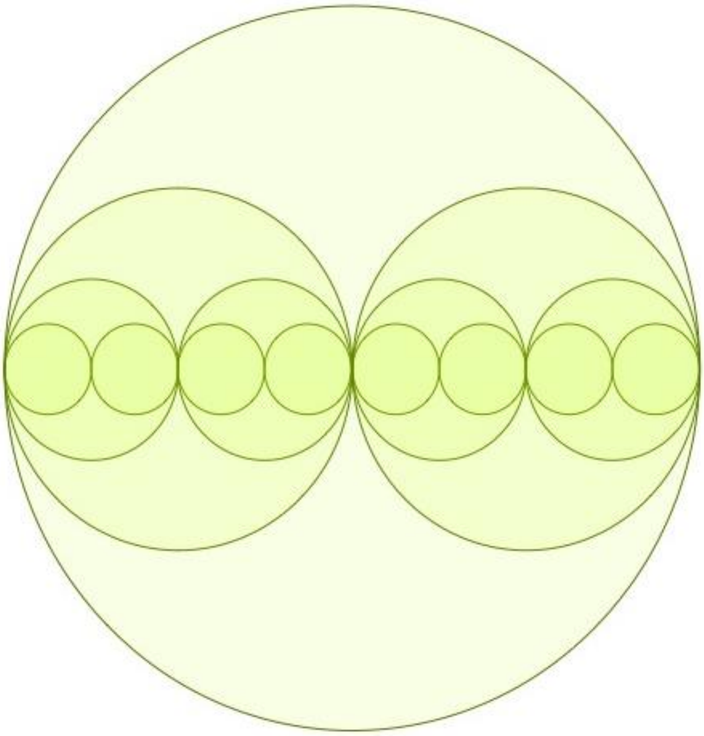


Pupils will understand the recursive procedure from their own, or given, diagrams, as follows …

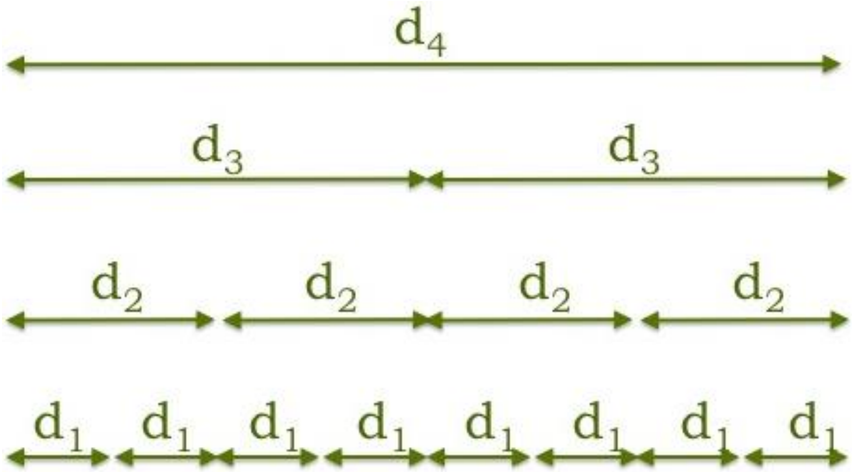
*Step 1*



*Step 2*



A powerful benefit of creating and investigating recursive procedures that produce similar shapes is that, depending on the shapes involved, pupils have opportunities to draw on their prior knowledge of a wide range of mathematical procedures and results. For example, in this circle situation they can use their knowledge of the relationship between the radius and the circumference of any circle to prove that in every 'row' of identical circles the sum of the circumferences equals the circumference of the largest single surrounding circle, perhaps starting with a diagram like this that shows the relationships between the diameters of the nested circles.




Your pupils should be able to explain why ALL circles are similar, and this would be a good opportunity to discuss it. This could lead to a rich discussion about length and area ratios: given two similar shapes, what's the same and what's different?

You can find previous *It Stands to Reason* features here

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# Eyes Down…Misconception or Mistake?

*by Mel Muldowney, a secondary school maths teacher working in the Midlands*

We know there are topics that students often mix up - so should these be taught close together with the aim of drawing our pupils' attention to their similarities and differences, or should they be taught separated as far as possible by a period of time and by other areas of the curriculum? Concepts such as area and perimeter or mean, median, mode and range are topics that are usually used in this discussion (what do you do? Let us know), but I've recently found evidence that frequency trees and probability tree diagrams should also be included.

Our department made the decision that there was a logical link between these two concepts, and so when I taught frequency trees, which is a new GCSE topic, to my year 10 group I made the link to probability trees. Finding the first incorrect answer in a recent assessment didn't trigger any alarm bells, but after three or four answers with the same (incorrect) solution I began to wonder what these students were thinking.

Emma carried out a survey of the number of homeworks completed by 32 students last week.

| Number of homeworks | Frequency | |
|---|---|---|
| 0 × | 3 | 3 |
| 1 × | 1 | 2 |
| 2 × | 12 | 24 ✓ |
| 3 × | 8 | 24 ✓ |
| 4 × | 6 | 24 ✓ |
| 5 × | 2 | 10 ✓ |
| | 32 | 87 |

Calculate the mean.

$87 \div 32 = 2.71875$

During our usual "post-mortem" of the assessment all of the students that dealt with the probability trees as you would a frequency tree claim not to have even considered that the sum of the probability in each "branch" must equal 1. However, when asked about why the probability of Wendy not winning at Hoopla was 0.6 one of them instantly stated "well, that's obvious, the probability of the events must equal 1". When asked what could have been done to check her thinking this student said "I suppose I could have checked it by working out the probability of ALL the combinations of outcomes which must also add up to 1".

Was it that these two topics were taught so close together that meant the students hadn't been able to form separate schema in their own minds? I'm not sure but it is something we will consider as we refine and revise our scheme of work.

*If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk with 'Secondary Magazine Eyes Down' in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we'll send you a £20 voucher.*

You can find previous *Eyes Down* features here

**Image credit:** page header by Alyssa L. Miller (adapted), 