



Welcome to Issue 127 of the Secondary and FE Magazine

This is a great time of year for real-life-inspired maths lessons: those spherical pumpkins asking to have their volume and surface areas calculated (or [something much more ambitious!](#)), the fireworks of Bonfire Night and Diwali prompting questions about projectiles and parabolas, and then some lessons on [star polygons](#) to encourage your pupils to be mathematical explorers, to develop their geometrical reasoning, and to leave you knee deep in Christmas classroom decorations. The real world has inspired this month's magazine too: modelling journeys with bearings, explaining HCFs with tilings, and tying together all sorts of seemingly disparate problems with the thread of proportional reasoning. We share a scheme of work to prepare sixth formers to learn about binomial hypothesis testing, and our roving cameras have found abundances of triangles. Does what's outside your classroom window sometimes stimulate activity inside? Let us know, by email info@ncetm.org.uk, or [@NCETMsecondary](https://twitter.com/NCETMsecondary) on Twitter.

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Here you will find a checklist of some of the recent, or still current, mathematical events featured in the news, by the media or on the internet: if you want a "heads up" on what to read, watch or do in the next couple of weeks or so, it's here. If you ever think that our heads haven't been up high enough and we seem to have missed something that's coming soon, do let us know: email info@ncetm.org.uk, or via Twitter, [@NCETMsecondary](https://twitter.com/NCETMsecondary).

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Getting lost in the autumn mist? We suggest how your pupils can find their bearings.

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A tree-ish diagram about teaching tree diagrams, and the other pre-requisites for Binomial hypothesis testing.

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Want to draw on maths research in your teaching but don't have time to hunker down in the library? Don't worry, we've hunkered for you: for this issue, the librarian has pulled together research about teaching ratio and proportion.

[It Stands to Reason](#)

No doubt your pupils can work out the HCF or LCM of two integers; but could they draw either of these for you?

[Eyes Down](#)

A picture to give you an idea: triangles everywhere.

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Heads Up

Whilst not wanting to define this article as mathematical gossip, it does come close! We've brought together news and current mathematical affairs, all in one place. We do hope it will interest you.



Happy Birthday [George Boole!](#)



The schools hosting the Shanghai exchange teachers are offering lots of events and PD opportunities in November: do contact your local host school, or your [Maths Hub Lead school](#), to find out what's happening in your area. Some "first impressions" from the NCETM team who visited Shanghai schools in September, and a flavour of how one school plans to work with their visiting teachers, can be viewed, and read [here](#).



Might your maths department be interested in participating in a free programme of professional development, with the aim of improving teachers' confidence in developing pupils' mathematical reasoning? Each of the 35 Maths Hubs is looking for five schools to take part in the project, which this year focuses on reasoning in Key Stage 3. [Find out more](#), including details of how to take part.



Among recent maths-related blogs that have caught our eye is an interesting [think-piece](#) on the new GCSE grades from the maths teacher and senior leader from an academy in West Yorkshire, on Twitter as [@workedgechaos](#). We'd be interested in your views on this important subject.



The integral presence of mathematics in so many aspects of everyday life is the focus of a three-day 'maths festival' at the Science Museum in London at the end of this month. It's called [What's Your Angle?](#) and is aimed specifically at 12-16-year-olds. Entrance is free, with admission on Friday 27 November specifically reserved for school groups.



ACME is seeking applications from 3-4 individuals to join an [expert panel](#) that will develop a project on the professional development learning journey for all teachers of mathematics. Applications should be submitted by **26 November**.



Two things to draw your attention to if any of your working week is devoted to students who've not yet got a GCSE Grade C or equivalent in maths.

- first, the free online course, [Citizen Maths](#), which has recently expanded its content significantly,
- and second, there's a new series of NCETM-run, one-day maths workshops for teachers and trainers in the FE and Skills sector, all within the first two weeks of December. Details and booking [here](#).



As the long winter nights draw in, we can treat ourselves by watching and listening to interesting programmes, which ideally are long enough to merit a cup of tea and a crumpet or two. So turn on the toaster and start with [In Our Time](#), BBC Radio 4 on 5 November, discussing whether $P=NP$, one of the most important unsolved questions in mathematics

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Building Bridges

Following on from the article in [Issue 121](#) that discussed the complexities of angle rules, a natural link is to the dreaded topic of bearings. How many of your GCSE pupils will proudly tell you about how they use a map compass when orienteering or on a Duke of Edinburgh expedition, but you know that as soon as they come into contact with 'maths bearings', they're already at 'perplexed' and are heading straight towards 'baffled'? They first met bearings in maths in Year 7 or 8, but that's now a distant memory: bearings are those little silver things that kept Marty McFly's skateboard wheels turning smoothly, aren't they?

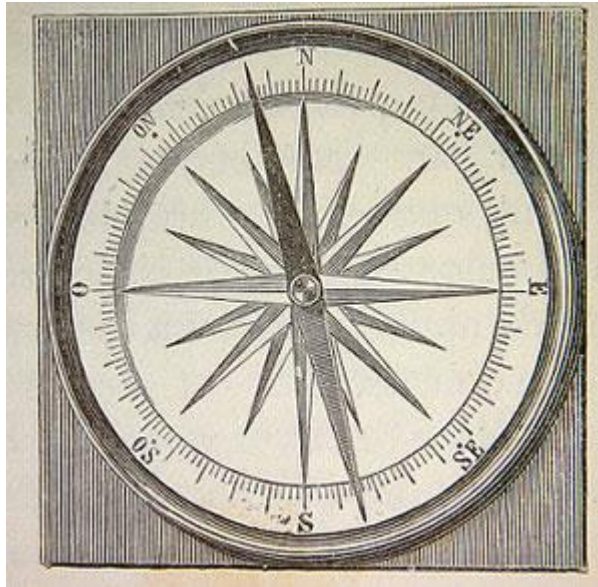
Let's start at the beginning: with the combined savvy of the whole class and a few hints, they should be able to recall the three key requirements about bearings:

- measure from North,
- measure clockwise,
- write the answer with three figures (which of course is not entirely true if the bearing is, say, 094.5, so we should clarify that we write 0 first when the bearing is less than 100: "94.5" is not a 3-figure bearing).

Given the simplicity of these three requirements, where do your pupils go wrong when they are faced with a question that contains bearings? Is it because of the wide range of topics that bearings seem to pop up in (trigonometry, construction, scale drawings, Pythagoras' Theorem), or is it the language used (especially 'from' and 'to'), or is it a consequence of their poor visualisation? Clearly using the map compass for orienteering didn't help much, probably because it's too remote from the maths lesson, and uses complicated equipment we don't have in the classroom – the physical experience of bearings-in-the-field doesn't have "stickiness" in the classroom. So what's to be done?

Treating Bearings as a standalone topic may well help: too often it's tagged on to the end of those other topics listed above, with our assumption that the procedural knowledge of bearings will be easy enough for our pupils to pick up and use, with a little prior reminding.

So, starting at the beginning: let's role play bearings, either with pupils standing around the room (or outdoor space), or using objects on their desks (with string connecting the two objects). But maybe that's not the beginning: let's recap the absolute basics. First, compass directions, aka the eight compass points N, E, S, W, NE, NW, SE, SW. A simple diagram to fill in with the letters and the bearings will be a good recap. At that point, three-figure bearings can be (re-)introduced using East or NE as a good example.



Then start placing objects (or pupils) in correct positions to model a bearing: one object (or pupil) being the centre of the compass and the other determining the line of the direction, and get them answering with more and more fluency. A single lesson should suffice, but recap of this skill in future lesson starters will be worthwhile (and fun!).

Don Steward has some lovely [bearing journeys](#) for students to draw on isometric paper, though I would recommend creating a couple of additional examples on square paper with the bearings being multiples of 45° , as well as those below.

Now your pupils should be ready for the big journey: to move from simple bearings-as-steps such as those above to measuring and describing the bearings between two objects A and B. It's important to ensure they see lots of examples of 'from' and 'to', and how these words relate to the journeys between the objects or points: they need to be able to tell you that the centre of the compass is placed at the start (where the journey is 'from'), and the compass arrow points towards the destination (where the journey is 'to').

Practical activities will help: on mini WBs, get the pupils to sketch some simple diagrams e.g. "the bearing of B from A is 045° ", the bearing "from A to B is 180° ". Now return to modelling using pupils or objects on desks, using the wording 'from', 'to', A and B. This will help your pupils visualise better than before, and not be thrown off course by the extra level of difficulty in the wording.

And now they're ready to attack those topics that often require understanding of bearings, with more confidence now that they have a secure grasp of the concepts ... but do keep recapping the three key requirements, the eight compass points (especially, to reinforce the leading 0 for bearings $< 100^\circ$), and do keep using the hands-on modelling in future lesson starters. This should help ensure they don't lose their way in the future.

Links to resources

- [A robot's journey using compass directions](#) (NRICH)
- [Introduction to Bearings](#) (TES Resources)
- [Bearings Worksheet](#) (TES Resources).



What activities do you use to teach bearings? Let us know (info@ncetm.org.uk or on Twitter [@NCETMsecondary](https://twitter.com/NCETMsecondary)) what they discover and how they reason about their observations.

You can find previous *Building Bridges* features [here](#).

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Sixth Sense

It's clear from reading examiners' reports that, for many AS and A2 students, binomial hypothesis testing is the topic that is least securely grasped compared to most of the other material usually assessed on Statistics 1 papers; conditional probability is probably (no pun intended) in second place. If students are to understand conceptually the logic of a binomial hypothesis test (and without that they will find it hard to answer any procedural question other than the most standard and straightforward), they need to have a fluent grasp of all the conceptual prerequisites. Next month we'll talk about teaching the test (just it, not "to" it!); but first, let's get the building blocks in order. I suggest [this tree-like diagram](#) (PDF).

By doing lots of similar examples, students should get to the position to generalise by themselves: if $P(\text{"success each time"}) = p$ and I carry out the experiment n times, and each time I do so I can assume that the outcome is independent of previous outcomes, then we write $P(\text{I get } r \text{ successes in total out of } n)$

as $P(X = r) = p^r (1-p)^{n-r} \times \frac{n!}{r!(n-r)!}$. I'd much rather have a scheme of work that allows me to do all

of this preparatory work than walk into an AS lesson and start by writing this last result on the board: the students having a secure understanding of the processes involved is far better than their learning mathematics as a series of rules of "things to do in a given situation". There should then be some time given to consolidating how to calculate, for example, $P(X \leq 2)$, $P(X < 4)$, $P(X \geq 13)$, $P(3 < X \leq 6)$, both using the formula and then by using the statistical tables: being familiar and confident with the tables and also the formula is a crucial skill to develop *separately from*, and *prior to*, hypothesis testing.

Finally, your students need to identify the structural characteristics of situations that are likely to suit modelling with a binomial distribution:

- an experiment is carried out n times
- each time, the experiment is either successful or not
- each time, the outcome of the experiment can be assumed to be independent of previous outcomes, and so we can say that there is a fixed $P(\text{"success each time"})$.

It's well worth looking at situations where one or more of these **aren't** likely to hold, to help students understand where they **are** likely to do so.

If you've never written a Scheme of Learning, this is the kind of thinking required, so have some sympathy with your Subject Lead! Why not pick another topic and ask yourself "what needs to be secure before I can teach this successfully?" and see if you can draw a similar picture? Share your examples with us, by email to info@ncetm.org.uk or by Twitter [@NCETMsecondary](https://twitter.com/NCETMsecondary).

You can find previous *Sixth Sense* features [here](#).

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From the Library

How do we explain the difference between ratio and proportion to our pupils? In what sense can the fraction $\frac{3}{4}$ be interpreted as a ratio, and if so is the corresponding ratio 3:4 or 3:1 – or both? Is it sometimes useful to think of a decimal as a ratio, and does it depend on the context: should a “trig ratio” such as $\sin 30^\circ = 0.5$ be written as a decimal or a fraction or a ratio or a scale factor? Is the gradient of a line a number or a ratio or a measure of the rate of change – and if the latter, what’s the “thing” that’s changing?

“Ratio, proportion and rates of change” has its own dedicated section amongst six in the new GCSE Mathematics Scope of Study (Department of Education, 2013). The related concepts, however, are present throughout mathematics, in number, algebra, geometry, statistics and probability. This article will look at the detail of one piece of research, and provide some pointers to various resources for exploring ratio and proportional reasoning.

As Watson *et al* point out in “Key Ideas in Teaching Mathematics” (2013, p. 42), ratio and proportion are extremely difficult to pin down as definitions, which gives an indication as to why pupils have such difficulty with the concepts. Commonly, ratio is described as the quotient of two numbers while proportion is the comparison of ratios. However, these definitions fail to convey the richness of mathematical experience. “Students learn ratio and proportional reasoning through repeated and varied experiences, over time, so that multiple uses of the words and the associated ideas and methods are met, used, and connected.”

[The Rational Number Project](#), based at the University of Minnesota, “advocates teaching fractions using a model that emphasizes multiple representations and connections among different representations” (see Figure 1).

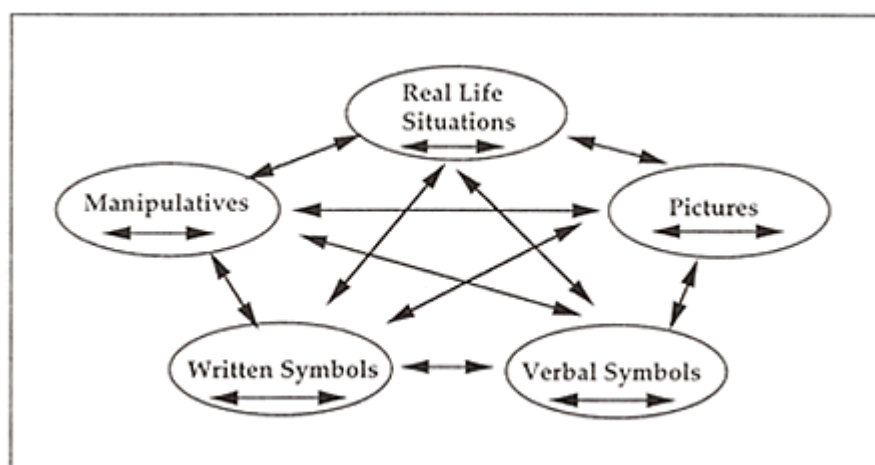


Figure 1. From Cramer (2003)

[The project](#) has been running since 1979 and the associated website collates not only a large number of research papers but also provides sets of lessons, based on the research findings, with activities, comments and actions for teacher and student worksheets. One of the papers (Cramer and Post, 1993) explains how three types of task were developed to examine proportional reasoning:

(1) Missing value. This is based the work of Karplus *et al* (1974). Mr Tall measures 6 buttons high; Mr Small is 4 buttons in height, which is the same as 6 paperclips in height. How high is Mr Tall in paperclips?

(2) Numerical comparison. In these problems the pupil is to compare two given rates. A typical task might involve the comparison of orange drinks made by combining different ratios of concentrate and water and deciding which is the stronger mixture (Noelting, 1980).

(3) Qualitative prediction and comparison. "These types of problems require comparisons not dependent on specific numerical values ... Qualitative prediction and comparison problems require students to understand the meaning of proportions" (Cramer and Post, 1993). For example, "What will happen to the fraction $\frac{7}{8}$ if the top number gets smaller and the bottom number gets bigger?" (Heller *et al*, 1990).

Each problem type was posed in four different contexts: speed, scaling, mixture, and density. For example, in Figure 2, Problem 1 is a combination of missing value and speed, Problem 2 combines numerical comparison and scaling. Problems 3 and 4 both use qualitative type tasks – one uses prediction with mixture, the other comparison with density. Potentially this gives sixteen styles of question (given the two types of qualitative task).

Problem 1: Missing-value speed

Lisa and Rachel drove equally fast along a country road. It took Lisa 6 minutes to drive 4 miles. How long did it take Rachel to drive six miles?

Problem 2: Numerical comparison scaling

Anne and Linda are using different road maps of the city. On Anne's map a road 3 inches long is really 15 miles long. On Linda's map a road 9 inches long is really 45 miles long. Who is using the larger city map? a) Anne b) Linda c) Their maps are the same d) Not enough information to tell

Problem 3: Qualitative prediction mixture

If Nick mixed less lemonade mix with more water than he did yesterday, his lemonade drink would taste a) Stronger b) Weaker c) Exactly the same or) Not enough information to tell

Problem 4: Qualitative comparison density

Two friends hammered a line of nails into different boards. Bill hammered more nails than Greg. Bill's board was shorter than Greg's. On which board are the nails hammered closer together? a) Bill's board b) Greg's board c) Their nails are spaced the same d) Not enough information to tell

Rational Number Project problem types

Figure 2. From Cramer, K. & Post, T. (1983)



The researchers gave a number of problems to students in US grades seven and eight (equivalent in age to years 8 and 9 in England) and found four distinct solution strategies were used. These can be described as (Watson et al, p. 54)

- identifying a unit rate (“how many for one?”)
- identifying a scale factor (e.g. “3 times as many”)
- matching equivalent fractions (treating rates as fractions and using equivalent fractions to find the missing number)
- cross-multiplying.

The researchers found two particular factors stood out: the context of scaling was more problematic while, as might be expected, the complexity of the numerical relationships was critical. Whole number multiples suited the more intuitive unit rate and scale factor strategies and when faced with non-integral numerical relationships students often reverted to inappropriate additive methods. For example, in Problem 1 in Figure 2, an extra 2 miles might be interpreted as adding an extra 2 minutes onto the journey time, rather than *multiplying* by a factor of $3/2$. The cross multiplying strategy, while effective with non-integral ratios, had a tendency to be applied incorrectly.

The researchers recognise the need to start with the more intuitive strategies and familiar contexts, but emphasise the need to ensure that all strategies are experienced and that unfamiliar contexts and non-integral numerical relationships are encountered. They advocate the use of the qualitative problems on the basis that these require the understanding of concepts and do not allow for the blind application of a procedure. Furthermore, this type of qualitative reasoning is a necessary step in numeric problems before the setting up of the actual calculations.

The Nuffield website [Key Ideas in Teaching Mathematics](#), associated with the book of the same name (Watson *et al*, 2013), also has a section devoted to these concepts, [Ratio and proportional reasoning](#) (RPR for short). The authors treat the topic through eight themes, providing research-based guidance on issues to look out for and approaches to use. They also include a number of student activities, from SMILE, NRICH and Bowland, all available on the web, to address various teaching points. Their conclusion (p. 66) from the research is “Nothing suggests that one way of teaching is better than another” but “that learning RPR is a medium term project and not something that can be ‘sorted’ in a few lessons”. “Students learn ratio and proportional reasoning through repeated and varied experiences, over time, so that multiple uses of the words and the associated ideas and methods are met, used, and connected”.

To finish, an anecdote from the above book (pp. 60 – 1). “One of us recently observed a teacher ask a class of 12-year-olds what they knew about ratio. One student said ‘it is something to do with division but I cannot explain it very well’ and the teacher replied ‘well neither can I’, which seems a reasonable response given the complexities above.”

How do you explain ratio and proportion? What activities have your pupils found especially helpful? Which have prompted them to reason deeply and successfully? Let us know (info@ncetm.org.uk or Twitter [@NCETMsecondary](#)) what you try with your pupils, and what impact and outcomes you observe.

References

Cramer, K. (2003) [Using a translation model for curriculum development and classroom instruction](#). In Lesh, R., Doerr, H. (Eds.) *Beyond Constructivism. Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching*. Lawrence Erlbaum Associates, Mahwah, New Jersey.



Cramer, K. and Post, T. (1993). [Connecting Research To Teaching Proportional Reasoning](#). *Mathematics Teacher*, 86(5), 404-407.

Department for Education (2013) [Subject content and assessment objectives for GCSE in mathematics for teaching from 2015](#).

Heller, P., Post, T., Behr, M. and Lesh, R. (1990) Qualitative and Numerical Reasoning about Fractions and Rates by Seventh- and Eighth-Grade Students. *Journal for Research in Mathematics Education* 21, 388-402.

Karplus, E., Karplus, R. and Wollman, W. (1974) Intellectual Development Beyond Elementary School IV: Ratio, the Influence of Cognitive Style. *School Science and Mathematics* 74, 476-82.

Noelting, G. (1980) The Development of Proportional Reasoning and the Ratio Concept: Part 1-the Differentiation of Stages. *Educational Studies in Mathematics* 11, 217-53.

Nuffield Foundation [Key Ideas in Teaching Mathematics: Research-based guidance and classroom activities for teachers of mathematics](#).

[The Rational Number Project](#).

Watson, A., Jones, K. and Pratt, D. (2013) *Key Ideas in Teaching Mathematics: Research-based guidance for ages 9 – 19*. OUP.

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It Stands to Reason

Before reading on, take time to consider how you would explain convincingly your response(s) to this simple-but-rich question from [Learning and Doing Mathematics](#) by John Mason: *if a number is a factor of the product of some numbers, is it necessarily a factor of one of those numbers? Is it always / sometimes / never a factor of one of those numbers? Can we predict when it will be and when it won't?*

Pupils are more adept at, and more confident at, finding ways of solving mathematical problems when they develop the habit of looking for, understanding, and then exploiting, the underlying mathematical structure of the problem. With numerical problems, sometimes the most useful approach is to express a number as the (unique) product of its prime factors: for example 12 as $2^2 \times 3$, and 20 as $2^2 \times 5$. Being able to identify fluently the prime factors of any number, and from that deduce all its factors (not just the prime factors), is an important skill that often enables pupils to see possible routes to the solution of a numerical problem. Given that skill, which we assume in the following examples and suggestions, pupils can be prompted to find, talk about and then use, the underlying structure of the numbers expressed in terms of their factors and multiples. This is what we look at in this article. Our premise is that we can help pupils notice and exploit this structure by prompting them to think about structures that they should be able to see in visual images.

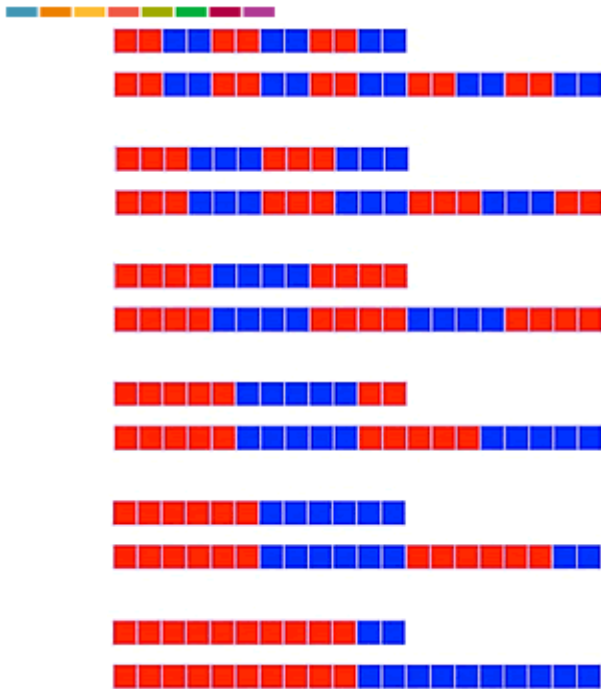
Strips of squares

These provide very simple images in which pupils can see facts and relationships involving factors. For example, suppose that the numbers 12 and 20 are represented by strips of 12 and 20 identical squares:



Now ask your pupils to use two contrasting colours in an effective and sensible way to show, on separate copies of the two strips together, factors of these two numbers.

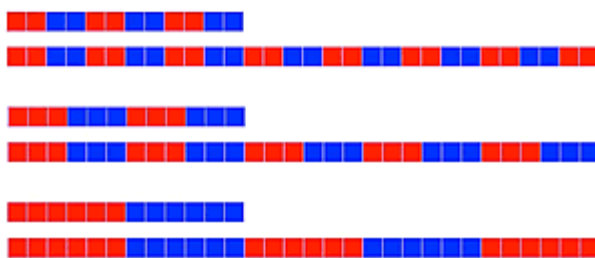
If they know that since $12 = 2^2 \times 3$, its factors are 1, 2, 3, 4, 6 and 12, and since $20 = 2^2 \times 5$ its factors are 1, 2, 4, 5, 10 and 20, hopefully they will colour the strips in twos, threes, fours, fives, sixes and tens, as we have done here:



Now ask them what they can deduce from their images, and when they articulate their conclusions encourage discussion. You want them to explain how their images show that the common factors of 12 and 20 (excluding 1) are 2 and 4, and therefore that the highest common factor of 12 and 20, $HCF(12, 20)$, is 4.

A very important conclusion is that the common factors of 12 and 20 are also factors of the difference between 12 and 20. Again, ask how the images show this.

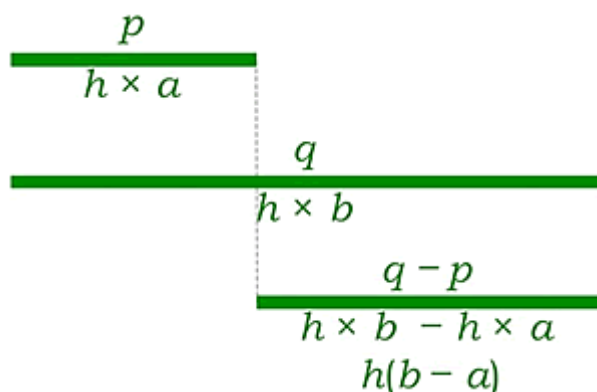
By exploring, in the same way, a variety of examples, including their own examples, such as 12 and 30 ...



... it is likely that pupils will be able (with some support, perhaps) to conjecture that the common factors of two numbers, including the highest common factor, are **necessarily** factors of their difference. In this example, all numbers that are factors of both 12 and 30 are also factors of their difference, so all the common factors of 12 and 30, including $HCF(12, 30)$, are factors of 18.)

Your pupils might now be ready to use algebraic notation to express, succinctly and generally, what they conjectured using their strips of squares about the HCF and the difference of two numbers. For example a clear summary using algebraic notation and supported by a sketch might look like this:

- Suppose $HCF(p, q) = h$, $p = a \times h$ and $q = b \times h$, where a and b have no common factors.
- Then $q - p = h(b - a)$, and so h is a factor of the difference of the two numbers:



They should be able to say in their own words why *any* number that is a common factor of two different numbers is also a factor of their sum and of their difference. It is one of the many facts about numbers that pupils with a deep understanding of relationships between numbers should be able to deduce and explain – but see how the pictorial approach makes this abstract idea much more accessible.

You want your pupils now to realise that they can **USE** this fact to help find the HCF of two numbers. For example, if they want to find the highest common factor of 738 and 750 they could now reason like this ...

1. $750 - 738 = 12$.
2. *The factors of 12 are 12, 6, 4, 3, 2, 1.*
3. *So $HCF(738, 750)$ is 12 or 6 or 4 or 3 or 2 or 1.*
4. *I want the highest of these numbers that is a factor of both 738 and 750.*
5. *Starting by dividing 738 by 12, I find that $738 = 12 \times 61 + 6$, so $HCF(738, 750)$ is not 12.*
6. *I then try the next highest, which is 6, and find that $738 = 6 \times 123$.*
7. *750 is 12 more than 738, so 6 is also a factor of 750 (or, $750 = 6 \times 125$).*
8. *Therefore $HCF(738, 750)$ is 6.*

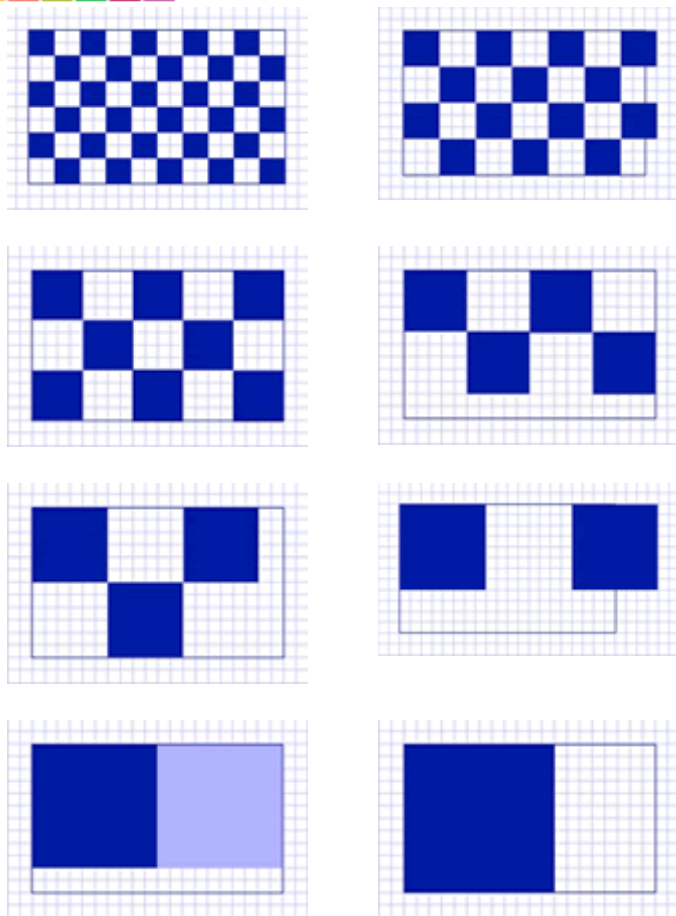
A powerful extension of this is to reason that the HCF of, say, 12 and 30 is not only a factor of $30 - 12$, but also a factor of ANY multiple of 30 – ANY non-equal multiple of 12, for example $30 - 2 \times 12 = 6$. This helps when the numbers are not close together: the HCF of 221 and 68 is the highest common factor of

$$\begin{aligned} 221 - 68 &= 153 \\ 221 - 2 \times 68 &= 85 \\ 221 - 3 \times 68 &= 17 \end{aligned}$$

and since 17 is prime, this tells us that the HCF of the two numbers must be 17: $221 = 17 \times 13$ and $68 = 17 \times 4$.

Square tiles in a rectangle

Your pupils will gain further insight into the structure of factors and multiples by exploring images of rectangles into which they try to fit identical square tiles. For example, they could investigate the common factors of 12 and 20 by trying to tile a 12-by-20 rectangle (drawn on a square grid) with identical square tiles. If they sketch diagrams such as these ...



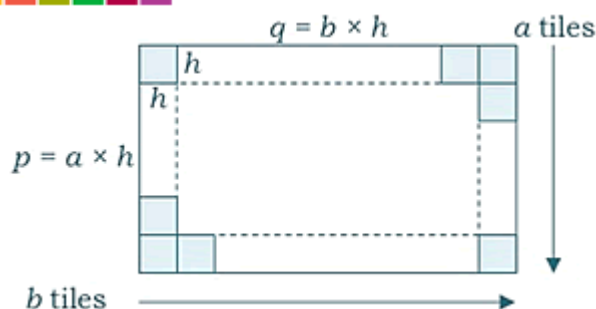
... they will see that, although within the rectangle they can **draw** squares of side-length 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12, they can only **tile the rectangle (without trimming any squares and so that there are no gaps)** with squares of side-length 2 or 4. Ask them to explain why this is a consequence of the fact that the only common factors of 12 and 20 (excluding 1) are 2 and 4. The argument you want them to propose and refine is something like: *If I can tile a rectangle with a square of a particular side-length, that is if I can completely cover the inside of the rectangle with identical copies of the square leaving no un-covered gaps, then the side-length of the square **must be** a factor of **both** the width **and** the height of the rectangle. So the side-lengths of the squares with which you can exactly tile a rectangle, leaving no gaps and without trimming any squares, are the common factors of the width and height of the rectangle - given in the same units of course.*

Once pupils understand, through devising and discussing a variety of (their own) examples,

- why it is possible (although not necessarily desirable!) to find the HCF of two numbers such as 12 and 20 by drawing square tiles in a rectangle
- that the side-length of the largest square tile with which you can completely cover a p -by- q rectangle, with no gaps, is the highest common factor of p and q ,

you can prompt their thinking to a much deeper level.

In order to do this, pupils need to think, and express ideas, generally for a while (as in the second 'understanding' above). You could display an image such as this ...



... and tell pupils that it shows a p -by- q rectangle filled exactly and completely with identical tiles of the largest possible side-length, h . Challenge them with the following questions about this image, which they could discuss **in the given order** for as long as necessary, perhaps first in pairs and then as a whole class:

- *What could p and q represent?*
- *What then would h represent?*
- *Why are there a tiles going downwards*
- *Why b tiles going across?*
- *What must be true about factors of a and b ?*
- *How can you express the total number of tiles in the rectangle (using given letters)?*

Once pupils have articulated, in their own (idiosyncratic) ways, correct answers about which they agree, such as ...

1. *p and q could represent two different whole numbers*
2. *h would then represent the highest common factor of p and q*
3. *There are a tiles going downwards because the height of the rectangle is p , and $p = a \times h$*
4. *There are b tiles going across because the width of the rectangle is q , and $q = b \times h$*
5. *a and b have no common factors (otherwise square tiles of side-length h would not be the largest square tiles that will fit in the rectangle)*
6. *The total number of tiles in the rectangle is $a \times b$.*

... it will be instructive to return to the previous numerical example by asking, and discussing pupils' answers to, the following question:

If you rearrange the largest tiles that will exactly cover a 12-by-20 rectangle so that they now form a new rectangle in which the height is the side-length of a single tile ...



... what does the length of the rectangle represent? Why?

We'll leave you to ponder this before the next issue! In the meantime your pupils might enjoy this assortment of problems from NRICH. If your pupils use reasoning along the lines we've developed in this article to solve these or other problems in ingenious ways please tell us about their methods, or send a photograph of their reasoning by email to info@ncetm.org.uk or tweet [@NCETMsecondary](https://twitter.com/NCETMsecondary).

Reference

Teaching mathematics at Secondary Level, Anthony D. Gardiner, The De Morgan Gazette 6 no 1, 2014



You can find previous [It Stands to Reason](#) features here

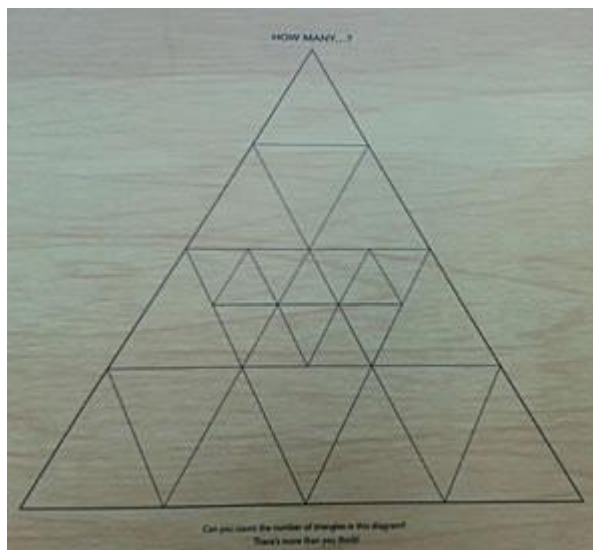
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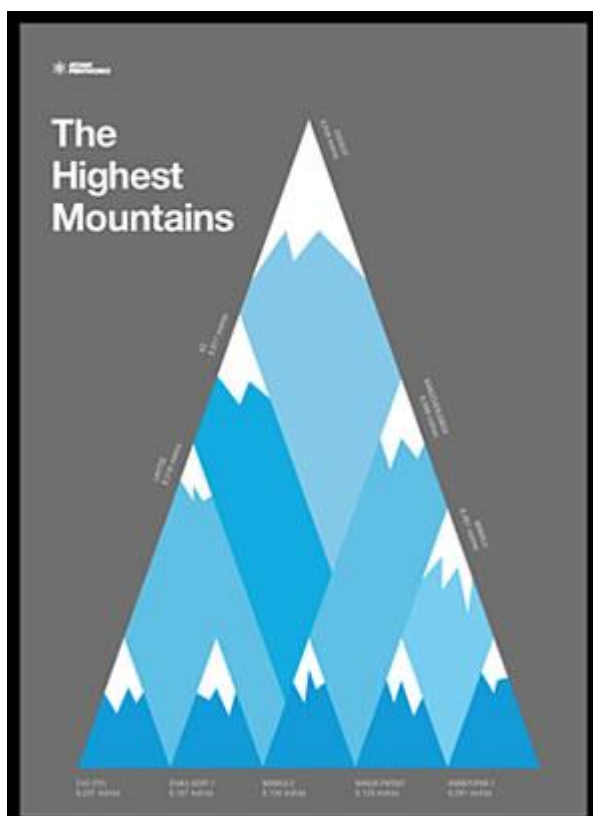


Eyes Down...

That puzzle-book chestnut (so autumnal!) “count the triangles within the triangle” has twice stared me in the face rather unexpectedly in the last couple of weeks: under my coffee on a table on a Grand Central train from King's Cross to York:



and on a poster (from Atomic Printworks) that now hangs above my desk...eyes up this time, not down.



Have you seen a maths puzzle where you least expected to?



If you have a thought-inducing picture, please send a copy (ideally, about 1-2Mb) to us at info@ncetm.org.uk with 'Secondary Magazine Eyes Down' in the email subject line. Include a note of where and when it was taken, and any comments on it you may have. If your picture is published, we'll send you a £20 voucher.

You can find previous *Eyes Down* features [here](#)

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